Stimulated Brillouin scattering in magnetized direct-gap semiconductors

R. Limaye and P. K. Sen
Department of Applied Physics, Shri Govindram Seksaria Institute of Technology and Science, Indore 452003, India
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Following the time-dependent perturbation technique for the density matrix, the phenomenon of stimulated Brillouin scattering (SBS) has been analytically studied in weakly noncentrosymmetric (NCS) crystals like InSb immersed in a moderately strong magnetostatic field. The twofold role of the magnetostatic field B in terms of the sharpening of the density of states and Landau-level splitting have been examined quantum mechanically. The extremely large value of the spectroscopic splitting factor $(g_c - g_v)$ and the very low electron and hole effective masses play the key roles in the significant enhancement in Brillouin susceptibility χ_B, even when B is reasonably low. The origin of the NCS effect has been assumed to be in the parity indefiniteness of the energy states in the crystals exhibiting lack of inversion symmetry. We have considered near-resonant band-to-band electron transition in the direct-gap crystals to reduce to a simple one-dimensional problem in the presence of the magnetostatic field. Numerical estimates have been made for the InSb crystal at 77 K, duly irradiated by a pulsed 5.3 μm CO laser. The SBS threshold is found to be well below the crystal damage threshold and χ_B is found to be appreciably large. The results support strongly the candidacy of InSb for establishing itself as a promising crystal for strong SBS and optical-phase conjugation while immersed in a moderate magnetostatic field.

I. INTRODUCTION

Interaction of an intense light beam, the so-called pump wave, with an active medium results in large optical amplification of the scattered wave at Stokes-shifted frequency. Such phase-coherent interactions are named stimulated scattering (SS) processes. Stimulated Raman and stimulated Brillouin scattering (SRS and SBS) in solids are the two nonlinear optical processes extensively investigated theoretically, as well as experimentally, since the advent of high power lasers. Among many technological applications of these SS processes, currently optical-phase conjugation (OPC) is the most sought after. Development of the concept of real-time holography by Gerritsen[1] and Stepanov, Ivakin, and Rubanov[2] on one hand, while wave-front reversal based upon stimulated Brillouin scattering[3] on the other, provided much impetus to the field of OPC. OPC via SBS is preferred over other competitive processes, since it requires a single pump wave along with a less stringent phase-matching condition, apart from its purity and high conversion efficiency.[4]

The stimulation for the study of SBS stems from the possibility of achieving coherent high power radiation, free from both angular and spatial distortions. It is well established that the origin of SBS lies in the third-order nonlinear optical susceptibility $\chi^{(3)}$ of the medium, also known as the Brillouin susceptibility χ_B. SBS occurs due to parametric coupling between the pump and the acoustic phonon mode duly generated by the same pump in the presence of a finite electrostriction in the crystalline solid medium.[5] For its onset SBS, like any other SS process, requires a threshold pump intensity, which is found to vary inversely with the Brillouin susceptibility χ_B.

In semiconductors, the application of an external magnetostatic field modifies the band structure and leads to formation of Landau levels and sharpening of the density of states. To the authors' knowledge, so far no systematic attempt has been made to investigate the dependence of χ_B on an applied magnetostatic field in a weakly noncentrosymmetric (NCS) crystal. Investigation of the role of an applied magnetostatic field in enhancing χ_B appears to be of considerable significance, when one treats the parametric process under a near-band-gap resonant laser excitation of a direct-gap material, like InSb, belonging to the III-V crystal class. Application of a large magnetostatic field has twofold implications on the nonlinear optical processes:

(1) Modification of electron energy levels due to spin degeneracy and formation of discrete Landau states. In narrow direct-gap semiconductors like InSb and InAs, the ratio of the electron and hole effective masses is very small (10^{-2}), hence Landau splitting becomes quite significant.[6,7]

(2) Sharpening of the density of states[7–9] leading to an enhancement in the optical susceptibility of the medium.

Thus an analytical study of the third-order nonlinear optical susceptibility of narrow direct-gap III-V semiconductors subjected to large magnetostatic fields is of fundamental significance. The present paper is devoted to such an analytical investigation, where the third-order nonlinearity gives rise to SBS in the crystalline medium.

In Sec. II we have obtained an expression for the Brillouin susceptibility χ_B, both in the absence and presence of the magnetic field, along with the threshold condition for the onset of SBS. The results are discussed in Sec. III, supported by numerical analysis. Important conclusions have been drawn in Sec. IV.

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II. BASIC FORMULATION OF BRILLOUIN SUSCEPTIBILITY χ_B

Here we address ourselves to the formulation of the Brillouin susceptibility χ_B for the Stokes' component of a scattered single-mode electromagnetic pump wave in the presence of an applied magnetostatic field B.

The generalized acoustic-phonon (AP) mode can be represented in a one-dimensional configuration following Sen and Sen[10] as

$$\frac{\partial^2 u(x,t)}{\partial t^2}-\frac{C_a}{d}\frac{\partial^2 u(x,t)}{\partial x^2}-2\Gamma\frac{\partial u(x,t)}{\partial t}=\frac{\langle F\rangle}{d} . \tag{1}$$

Here

$$u(x,t)=u\exp[i(k_v x-\omega_v t)] ; \tag{2}$$

u denotes the relative displacement of the nuclear positions within the crystal lattice. C_a and d represent the linear elastic modulus and homogeneous mass density of the crystal, respectively. Γ is the phenomenological damping parameter. $\langle F\rangle$ is the generalized force per unit volume experienced by the nuclei due to the pump wave. It is defined as[10]

$$\langle F\rangle=N\langle f\rangle =N[\langle f^{(1)}\rangle E+\langle f^{(2)}\rangle E^2+\langle f^{(3)}\rangle E^3+\cdots] . \tag{3}$$

Here N is the number of elementary cells per unit volume of the crystal, assuming each cell contains one electron in the absence of magnetic field.[11] We take $N=a_l^{-3}$ with a_l being the crystal-lattice constant.

The validity of the scattering of light by acoustic phonons (AP) is based upon Placzek's approximation[12] that the pump frequency is too high for the ions to be affected. With this approximation, one can neglect direct interaction between the pump and the ions. We adopt a semiclassical approach to the problem, where the electronic system is treated quantum mechanically while both the electromagnetic pump and acoustic fields are treated as classical waves.

The interaction Hamiltonian H_{int} in the one-dimensional configuration may be put as

$$H_{int}=H_{ev}+H_{er} , \tag{4}$$

where

$$H_{ev}=-fu\exp[i(k_v x-\omega_v t)] , \tag{5}$$

and

$$H_{er}=-\mu E_p\exp[i(k_p x-\omega_p t)]. \tag{6}$$

In the above equations, H_{er} and H_{ev} are the interaction Hamiltonians for the electron-radiation and electron–acoustic-field interactions, respectively; $\hat{\mu}$ is the dipole moment matrix operator. Both $\hat{\mu}$ and $\mathbf{E}_p$ are taken parallel to the X axis. We have also assumed that the undoped III-V semiconductors possess nondegenerate, isotropic, and parabolic band structures. The direct allowed electronic transitions at the same k value in the Brillouin zone occur between the topmost valence and the lowest conduction bands. The nonlinear optical effects arising due to the band nonparabolicity can be minimized by considering electronic transitions in the vicinity of the center of the first Brillouin zone at $k\cong\emptyset$.

For pump-initiated photoinduced electronic transitions between the initially completely occupied ground state $|g\rangle$ to the fully empty excited state $|n\rangle$ in the NCS system, we define the two operators $\hat{\mu}$ and $\hat{f}$ as

$$\hat{\mu}=\begin{bmatrix}\mu_{gg} & \mu_{gn}\\ \mu_{ng} & \mu_{nn}\end{bmatrix} \quad\text{and}\quad \hat{f}=\begin{bmatrix}f_{gg} & f_{gn}\\ f_{ng} & f_{nn}\end{bmatrix} , \tag{7}$$

with all matrix elements being finite due to parity indefiniteness[10] in a NCS crystal. We have adopted a density-matrix formalism which is one of the most convenient semiclassical treatments in the calculation of microscopic nonlinear optical susceptibilities, particularly when relaxations of excitations have to be dealt with.[13]

The equation of motion for the density matrix ρ can be expressed as

$$\frac{\partial\rho}{\partial t}=\frac{1}{i\hbar}[(H_0+H_{int}),\rho]+\left[\frac{\partial\rho}{\partial t}\right]_{relax} . \tag{8}$$

Here H_0 and H_{int} are unperturbed and interaction Hamiltonians of the system, respectively. The second term on the right-hand side of (8) represents relaxations of excitations. Expanding ρ as

$$\rho=\rho^{(0)}+\rho^{(1)}+\rho^{(2)}+\cdots , \tag{9}$$

one obtains equations of motion of the density operator ρ of various orders as

$$\frac{\partial}{\partial t}[\rho^{(0)}]=\frac{1}{i\hbar}[H_0,\rho^{(0)}] , \tag{10a}$$

$$\frac{\partial}{\partial t}[\rho^{(1)}]=\frac{1}{i\hbar}[H_0,\rho^{(1)}]+\frac{1}{i\hbar}[H_{int},\rho^{(0)}]+\left[\frac{\partial\rho^{(1)}}{\partial t}\right]_{relax} , \tag{10b}$$

and

$$\frac{\partial}{\partial t}[\rho^{(2)}]=\frac{1}{i\hbar}[H_0,\rho^{(2)}]+\frac{1}{i\hbar}[H_{int},\rho^{(1)}]+\left[\frac{\partial\rho^{(2)}}{\partial t}\right]_{relax} . \tag{10c}$$

The density-matrix elements of various orders for the material system can be had from Eqs. (10) as

$$\rho_{ng}^{(1)}(\omega_p)=\frac{H_{ng}(\omega_p)}{\hbar[\omega_p-\omega_{ng}-i\Gamma]} , \tag{11a}$$

$$\rho_{ng}^{(2)}(\omega_s)=\frac{H_{ng}(\omega_p)[H_{nn}(\omega_v)-H_{gg}(\omega_v)]}{[\hbar^2(\omega_s-\omega_{ng}-i\Gamma)(\omega_p-\omega_{ng}-i\Gamma)]}+\frac{H_{gn}(\omega_v)[H_{nn}(\omega_p)-H_{gg}(\omega_p)]}{[\hbar^2(\omega_s-\omega_{ng}-i\Gamma)(\omega_v-\omega_{ng}-i\Gamma)]} , \tag{11b}$$

and

$$\rho_{gg}^{(2)}(\omega_v)=-\rho_{nn}^{(2)}(\omega_v)=\frac{H_{gn}(\omega_s)H_{ng}(\omega_p)}{\hbar^2(\omega_v-i\Gamma)}\times\left[\frac{1}{(\omega_s-\omega_{ng}-i\Gamma)}-\frac{1}{(\omega_p-\omega_{ng}-i\Gamma)}\right], \tag{11c}$$

with $\omega_s=(\omega_p-\omega_v)$ and $\mathbf{k}_s=\mathbf{k}_p-\mathbf{k}_v$ for the Stokes' mode of the scattered electromagnetic wave. In obtaining Eq. (11), we have assumed for the sake of mathematical simplicity that the relaxation frequency in Eq. (11) is identical to one introduced in Eq. (1) for the generalized AP mode.

The ensemble averages of the generalized force and induced dipole moment of an arbitrary order j are defined as

$$\langle f^{(j)}\rangle=\mathrm{Tr}(\pm\rho^{(j)}), \tag{12a}$$

$$\langle \mu^{(j)}\rangle=\mathrm{Tr}(\mu\rho^{(j)}), \tag{12b}$$

with $j=1,2,3,\ldots$.

Using Eqs. (4)–(12), and on mathematical simplifications, one may find

$$\langle f^{(1)}\rangle\langle E\rangle=\frac{f_{gn}\mu_{ng}E_v}{\hbar\Omega_v}, \tag{13}$$

$$\langle f^{(2)}\rangle\langle E^2\rangle=\frac{|\mu_{ng}|^2(f_{nn}-f_{gg})E_pE_s^*}{\hbar^2\Omega_s\Omega_p}+\frac{\mu_{ng}f_{gn}(\mu_{nn}-\mu_{gg})\omega_vE_pE_s^*}{\hbar^2\Omega_s\Omega_p\Omega_v}, \tag{14}$$

$$\langle \mu^{(1)}\rangle=\frac{|\mu_{ng}|^2E_p}{\hbar\Omega_p}, \tag{15}$$

and

$$\langle \mu^{(2)}\rangle=\frac{|\mu_{ng}|^2(f_{nn}-f_{gg})u^*E_p}{\hbar^2\Omega_s\Omega_p}+\frac{\mu_{ng}f_{gn}(\mu_{nn}-\mu_{gg})\omega_vE_pu^*}{\hbar^2\Omega_s\Omega_p\Omega_v}, \tag{16}$$

with

$$\Omega_{p,s,v}=\omega_{p,s,v}-\omega_{ng}-i\Gamma .$$

The second-order force component duly generated by the pump represented by Eq. (14) has two components: the first term represents the contribution of the centrosymmetry (CS), while the second term represents the additional contribution arising due to the noncentrosymmetry (NCS) property of the system. It may be remembered that the matrix elements μ_{nn}, μ_{gg}, f_{gn}, and f_{ng} are finite only in NCS crystals.[10]

One defines the third-order-induced polarization as

$$P^{(3)}=\epsilon_0\chi^{(3)}|E_p|^2E_s=\mathcal{N}\langle\mu^{(2)}\rangle|E_p|^2E_s, \tag{17}$$

where $\mathcal{N}$ is the number of transition electron-hole pairs per unit volume of the crystal. One can find the third-order optical susceptibility of the crystal at the Stokes-shifted frequency from the above formulations given by

$$\chi^{(3)}=\frac{N\mathcal{N}}{\epsilon_0d\hbar^4D^*|\Omega_s|^2|\Omega_p|^2}\left[|\mu_{ng}|^4(f_{nn}-f_{gg})^2+\frac{\omega_v^2}{\omega_{ng}^2+\Gamma^2}\times|\mu_{ng}|^2f_{gn}^2(\mu_{nn}-\mu_{gg})^2-\frac{2|\mu_{ng}|^3f_{gn}(\mu_{nn}-\mu_{gg})(f_{nn}-f_{gg})\omega_v}{(\omega_{ng}^2+\Gamma^2)^{1/2}}\right] \tag{18}$$

where

$$D^*=[-\omega_v^2+k^2v_s^2-2i\Gamma\omega_v] .$$

In Eq. (18), we define $v_s=(C_a/d)^{1/2}$ as the acoustic velocity in the crystal. The third-order optical susceptibility represented by Eq. (18) has its origin in the electrostriction, and, hence, one may more appropriately call it the Brillouin susceptibility. Accordingly, we can safely take $\omega_v\ll\omega_p$ and $\omega_p\sim\omega_{ng}$ such that $\omega_p\sim\omega_s\sim\omega_{ng}\gg\omega_v,\Gamma$. Thus Eq. (18) simplifies in absence of magnetic field B to the form

$$\chi^{(3)}_{(B=0)}=\frac{N\mathcal{N}}{\epsilon_0d\hbar^4D^*(\omega_p-\omega_{ng})^4}\left[|\mu_{ng}|^4(f_{nn}-f_{gg})^2+\frac{|\mu_{ng}|^2f_{gn}^2(\mu_{nn}-\mu_{gg})^2\omega_v^2}{\omega_p^2}-\frac{2|\mu_{ng}|^3f_{gn}(\mu_{nn}-\mu_{gg})(f_{nn}-f_{gg})\omega_v}{\omega_p}\right]. \tag{19}$$

Equation (19) reveals that the Brillouin susceptibility comprises three components, among which the first term remains finite irrespective of the symmetry properties of the crystal. In other words, the Brillouin susceptibility in a centrosymmetric crystal is represented by the first component. The remaining two parts have their origins in the NCS properties of the active medium.

Next, we explore critically the role of an applied magnetostatic field B on $\chi^{(3)}$, keeping in view its twofold contribution in terms of Landau-level formation and sharpening of the density of states.

The Landau-level formation modifies the conduction- and valence-band energies as follows:[6]

$$E_c = E_{c0} + \hbar^2 k'^2/2m_c + (n' + \tfrac{1}{2})\hbar\omega_{cc} + g_c \beta_m M_J B \quad (20)$$

and

$$E_v = E_{v0} + \hbar^2 k^2/2m_v - (n + \tfrac{1}{2})\hbar\omega_{cv} + g_v \beta_m M_J B \ ; \quad (21)$$

suffixes c and v denote conduction and valence bands, respectively. E_{c0} and E_{v0} are the energies at the center of the lowest conduction band and highest valence (heavy-hole) band. m_{cv}, g_{cv}, and $\omega_{cc,cv}$ $(=|eB/m_{cv}|)$ are the effective masses, g factors, and cyclotron frequencies of electrons in conduction band and holes in valence band, respectively. M_J $(=+\frac{1}{2})$ is the spin degeneracy quantum number, while β_m $(=eh/2m_0)$ is a Bohr magneton. Application of the magnetostatic field B splits the three-dimensional band into a series of one-dimensional subbands. For direct allowed transitions in a magnetic field, the selection rules make $n=n'$ and $k=k'$. Hence for finite B, the transition energy defined as $\hbar\omega_{kB} = E_c - E_v$ can be obtained as

$$\hbar\omega_{kB} = \hbar\omega_g + \frac{\hbar^2 k^2}{2m_r} + (n + \tfrac{1}{2})\hbar\Omega_c + (g_c - g_v)\beta_m M_J B \quad (22)$$

for a direct allowed transition at state $|k\rangle$.

If the applied magnetic field is sufficiently large, one may choose $n=0$ Landau subbands. The contribution of the spin degeneracy factor $(g_c - g_v)\beta_m M_J B$ is considerably large in direct narrow-gap semiconductors like InSb due to the large value of $(g_c - g_v)$ $(=-44$ for the InSb crystal). $\Omega_c (=\omega_{cc} + \omega_{cv})$ is the cyclotron frequency of the electron-hole pair. In the present analysis, we have restricted ourselves only to the limit of high magnetic field. Consequently, we select $n=n'=0$ such that the transition frequency $\omega_{k\mathbf{B}}$ is given by

$$\omega_{kB} = \omega_{gB} + \frac{\hbar k^2}{2m_r} \quad (23)$$

with

$$\omega_{gB} = \omega_g + \frac{\Omega_c}{2} + (g_c - g_v)\beta_m M_J B/\hbar \ . \quad (24)$$

Here, ω_{gB} is the renormalized band-gap frequency of the crystal in the presence of the magnetic field. The implication of sharpening of the density of states in the Landau subbands may be analyzed as follows.

The number of electron states within a Landau subband with a given quantum number n in the range k to $k+dk$ is $[eBV/2\pi^2\hbar]dk$, with $dk = \pi/L$, and L being the interaction length. We have incorporated a factor of 2 for the two spin states of the electrons.[6]

On incorporating the sharpening of the density of states, the induced third-order polarization in the medium in the presence of magnetostatic field may be defined as

$$P^{(3)}_{(B\neq 0)} = \frac{eB}{2\pi^2\hbar}\int_{-\infty}^{+\infty} \langle \mu^{(2)}(k)\rangle dk \ , \quad (25)$$

remembering that the transition dipole density $\mathcal{N}$ in the absence of magnetic field can be given by $1/v\sum_{\mathbf{k}}$. For thickly populated electron states, one can replace the summation by a k integral.[7]

One can obtain the third-order optical susceptibility of the crystal at the Stokes-shifted frequency, making use of Eqs. (19), (23), and (25) as

$$\chi^{(3)}_{B\neq 0} = \frac{NeB}{2\pi^2\epsilon_0 d\hbar^5 D^*}\left[|\mu_{ng}|^4 (f_{nn} - f_{gg})^2 + \frac{|\mu_{ng}|^2 f_{gn}^2 (\mu_{nn} - \mu_{gg})^2 \omega_v^2}{\omega_p^2} - \frac{2|\mu_{ng}|^3 f_{gn}(\mu_{nn} - \mu_{gg})(f_{nn} - f_{gg})\omega_v}{\omega_p} \right] \int_{-\infty}^{+\infty} \frac{dk}{(\omega_p - \omega_{kB})^4} \ . \quad (26)$$

The renormalized interband transition frequency may be defined as

$$\omega_{k\mathbf{B}} = \hbar[a^2 + k^2]/2m_r \ , \quad (27a)$$

with $a^2 = 2m_r\omega_{g\mathbf{B}}/\hbar$. Also, we take

$$(\omega_p - \omega_{k\mathbf{B}}) = -\hbar[k^2 + g^2]/2m_r \ , \quad (27b)$$

where $g^2 = 2m_r[\omega_{g\mathbf{B}} - \omega_p]/\hbar$. Equations (26) and (27) yield

$$\chi^{(3)}_{B\neq 0} = \frac{5Ne}{32\pi\epsilon_0 d D^* \hbar^5}\left[\frac{2m_r}{\hbar}\right]^{1/2} \frac{B}{(\omega_{gB} - \omega_p)^{7/2}} \times \left[|\mu_{ng}|^4 (f_{nn} - f_{gg})^2 + \frac{|\mu_{ng}|^2 f_{gn}^2 (\mu_{nn} - \mu_{gg})^2 \omega_v^2}{\omega_p^2} - \frac{2|\mu_{ng}|^3 f_{gn}(\mu_{nn} - \mu_{gg})(f_{nn} - f_{gg})\omega_v}{\omega_p} \right] \ . \quad (28)$$

Equation (28) represents the Brillouin susceptibility of the NCS crystal in the presence of a large magnetostatic field. The same equation can be employed to estimate the exclusive contribution of the NCS property to the third-order susceptibility in a CS system as

$$\frac{\chi^{(3)}_{\rm NCS}}{\chi^{(3)}_{\rm CS}}=1-2s+s^2 , \tag{29}$$

where

$$s=\left|\frac{f_{gn}}{f_{nn}-f_{gg}}\right|\left|\frac{\mu_{nn}-\mu_{gg}}{\mu_{ng}}\right|\left[\frac{\omega_v}{\omega_p}\right] , \tag{30}$$

s being a measure of the NCS contribution. It is obvious from Eqs. (29) and (30) that $\chi^{(3)}_{\rm NCS}$ is always less than $\chi^{(3)}_{\rm CS}$. Thus one may infer that the NCS property of the crystal brings down $\chi^{(3)}$. This can be related to the well-known phenomenon of the retardation effect in a NCS crystal.

A. SBS threshold

In order to examine the occurrence of SBS with a finite gain mechanism in the active medium, we consider the electromagnetic wave equations for the pump E_p as well as the backscattered Stokes' mode E_s under the slowly varying envelope approximation (SVEA) given by

$$\frac{\partial E_p}{\partial x}=\alpha_I E_p+\frac{i\omega^2}{2kc^2}\chi_B|E_s|^2E_p \tag{31}$$

and

$$\frac{\partial E_s}{\partial x}=-\alpha_I E_s-\frac{i\omega^2}{2kc^2}\chi_B|E_p|^2E_s , \tag{32}$$

respectively. In the above equations, we have included the effect of the intensity-dependent background absorption coefficient α_I at frequency $\omega_p \sim \omega_s = \omega$ (say), since $\omega_v \ll \omega_p$ and $\omega_s=\omega_p-\omega_v$. Following the standard approach as discussed by Bose, Aghamkar, and Sen,[14] the Brillouin mode can be represented by

$$E_s(x)=E_s(L)\exp\left[\alpha_I(L-x)+\frac{k}{2\alpha_I}e^{2\alpha_I x}\{1-e^{2\alpha_I(L-x)}\}\right] \tag{33}$$

where $k=[\omega^2\chi_{\rm B}|E_p|^2/2kc^2]$.

The onset of SBS corresponds to the fact that $E_s(x)$ is equal to $E_s(L)$, L being the crystal thickness. For $E_s(x)=E_s(L)$, one obtains

$$\alpha_I(L-x)+\frac{k}{2\alpha_I}e^{2\alpha_I x}\{1-e^{2\alpha_I(L-x)}\}=0 . \tag{34}$$

From (34) it is evident that the gain constant depends on the intensity-dependent absorption coefficient α_I and parameter k for a semiconductor wave guide a few μm in thickness, irradiated by slightly off-resonant lasers, we may assume $2\alpha_I x<1$ and $2\alpha_I(L-x)<1$. On mathematical simplification, we obtain

$$k=\alpha_I . \tag{35}$$

Equation (35) enables one to calculate the threshold value of the excitation intensity $I_{p\rm th}$ required for the onset of SBS given by

$$I_{p\rm th}=\frac{n_0\epsilon_0c^3k\alpha}{\omega^2|\chi_{\rm B}|} \tag{36}$$

where we have used the relation $I_p=1/2[n_0\epsilon_0c|E_p|^2]$, n_0 being the crystal background refractive index at the pump frequency ω.

B. SBS gain characteristics

The SBS gain behavior of the NCS crystal in the presence of a large magnetostatic field is analyzed for excitation intensity I_p well above $I_{p\rm th}$. The Brillouin gain constant is given by[15]

$$g_B=-\frac{k}{2\epsilon_l}(\chi_{Bi})|E_0|^2 , \tag{37}$$

where $\chi_{\rm B\it i}$ is the imaginary part of $\chi^{(3)}$, E_0 is the electric-field amplitude above threshold.

From Eqs. (19) and (28), it may be noted that $\chi_{\rm B}$ is an imaginary quantity given by $\chi_{{\rm B}i}=-|\chi_{\rm B}|$ for dispersionless acoustic-wave propagation with $\omega_v=k_v v_s$.[16] Consequently, the characteristic behavior of the Brillouin gain can be studied analytically in the crystal as a function of various system parameters such as B and I_p. As is evident from Eq. (37), the SBS gain characteristics can be studied simply by considering the nature of the dependence of $|\chi_{\rm B}|$ on the system parameters, assuming that the estimates are made for excitation intensities well above the SBS threshold.

III. RESULTS AND DISCUSSION

The theoretical formulations as presented in Sec. II are analyzed numerically in this section to study the nature of dependence of the threshold intensity and the Brillouin susceptibility $|\chi_{\rm B}|$ on the applied magnetostatic field B. The semiconductor bulk crystal used for this purpose is InSb, which is a narrow direct-gap semiconductor with nearly cubic zinc-blende structure. The stimulation for the selection of InSb for the SBS analysis stems from the extensive technological applications it has already found for itself in modern optoelectronics. This crystal has demonstrated giant optical nonlinearities when irradiated by near-resonant lasers with photon energies in the vicinity of the crystal band-gap energy. In order to achieve large SBS gain, we have considered the irradiation of InSb by 5.3-μm pulsed CO lasers with pulse durations in the nanosecond regime such that the possibility of optical damage of InSb crystal can be avoided.

The relevant material parameters of InSb are as follows. The lattice temperature is taken to be 77 K. $E_g=0.23$ eV, $\epsilon_l=15.8$, $n_0=3.975$, $k=10^6$ m^{-1}, $d=5.8\times10^3$ K gm^{-3}, $\Gamma=10^9$ s^{-1}, $\omega=3.55\times10^{14}$ s^{-1}, $\alpha=10^4$ m^{-1}, $v_s=4\times10^9$ m s^{-1}, $M_J=\frac{1}{2}$, lattice constant $a_l=6.48\times10^{-10}$ m, $\beta_m=9.274\times10^{-24}$, and $(g_c-g_v)=-44.$[17] We have studied the variation of $|\chi_{\rm B}|$, with applied magnetostatic field B in Fig. 1, for an InSb

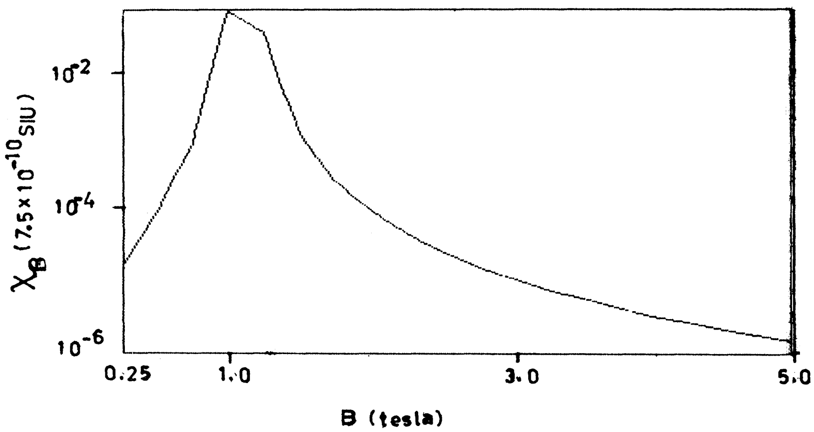

FIG. 1. Variation of the Brillouin susceptibility χ_B with the magnetostatic field B in InSb crystal at 5.3-μm pump wavelength.

crystal. Since the role of the NCS property is almost negligible in this crystal, we have considered only the centrosymmetric component of $|\chi_B|$ in Eq. (28) for this analysis. Quite interestingly, this figure exhibits the critical nature of the dependence of the Brillouin susceptibility on B. For $B \sim 1.0$ to 1.4 T, $|\chi_B|$ is quite large with the peak value of $|\chi_B| \sim 7.5 \times 10^{-11}$ Système International (SI) units being available in the resonant region at a moderate magnetic field of 1.0 T. On increasing B further, $|\chi_B|$ saturates to a value around 2.75×10^{-15} SI units for $B > 5$ T in the InSb-CO system.

The knowledge of $|\chi_B|$ enables one to calculate the threshold value of the excitation intensity I_{pth} required for the onset of gain behavior of the backscattered Stokes' component of the Brillouin mode. The variation of I_{pth} with applied magnetic field B is depicted in Fig. 2, where we have used Eq. (36) with absorption coefficient $\alpha = 10^4$ m^{-1} at $\omega = 3.55 \times 10^{14}$ s^{-1}. One observes the threshold intensity $I_{pth} \sim 720$ KW cm^{-2} at 0.25 T. The corresponding electric field turns out to be 11.7×10^5 V m^{-1}, which is well below the optical damage threshold for the InSb crystal, as discussed earlier. On increasing B, I_{pth} falls very sharply and attains the minimum value of 100 W cm^{-2} for $B = 1.0$ T. For $B > 1.0$ T, I_{pth} increases and finally saturates to a value ~ 7.0 MW cm^{-2} at $B > 5$ T. The characteristic B dependence of I_{pth} may be ascribed to the resonance established between the e-h pair cyclotron-energy-incorporated crystal band gap and the pump photon energy in addition to the sharpening of the density of states. The Landau-level splitting with a very large spectroscopic splitting factor of $|g_c - g_v| \sim 44$ plays a decisive role in the giant enhancement of $|\chi_B|$ even at a low magnetic field, viz., $B = 1$ T.

The results reveal that the InSb-CO system subjected to a moderate magnetostatic field could be a promising combination for the study of optical phase conjugation by SBS. This investigation has already been undertaken, and will be the subject matter of a future publication.

IV. CONCLUSIONS

The present paper deals with an analytical investigation of SBS in a weakly NCS crystal like InSb immersed in a magnetostatic field. The following important conclusions can be drawn on the basis of the above study.

(1) We have followed a semiclassical approach in which the third-order optical susceptibility is formulated by the density-matrix method, while the threshold for the onset of SBS and the Brillouin gain mechanism have been analyzed following the classical electromagnetic approach. The induced third-order polarization is assumed to be due solely to electrostriction.

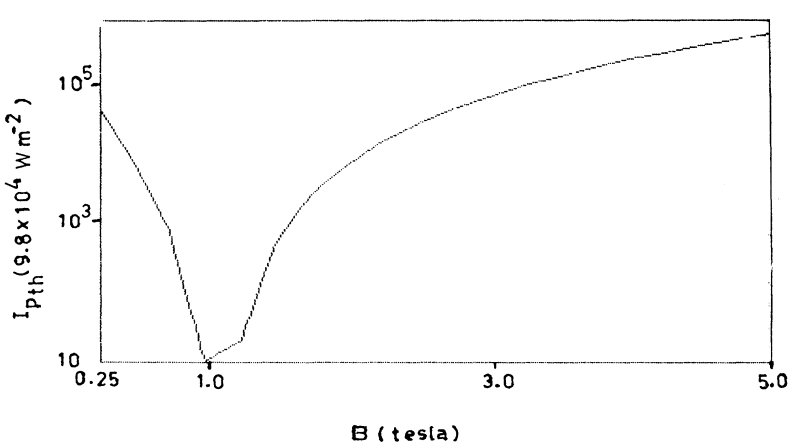

FIG. 2. Dependence of the threshold excitation intensity for SBS on the applied magnetostatic field B in InSb crystal at 5.3-μm radiation wavelength.

(2) The effect of the application of magnetic field on the third-order optical susceptibility of a narrow-gap semiconductor is analyzed quantum mechanically by incorporating the Landau-level splitting of the energy states and the sharpening of the density of states.

(3) The resonance peak shifts to the frequency $\omega_{gB}=\omega_g+\Omega_c/2+(g_c-g_v)\beta_m M_J B/\hbar$ in the presence of the magnetostatic field B. A significant rise in $|\chi_B|$ can be observed even at moderate magnetic fields in the range of 1–2 T. The corresponding threshold intensity falls to a very low value 100 $\mathrm{W\,cm^{-2}}$ in the sharp resonant transition regime $\omega_p \sim \omega_{gB}$. The value of I_{pth} is well below the optical damage threshold of an InSb bulk crystal under nanosecond pulse excitation regime.

(4) It has been shown that NCS properties effectively reduce the third-order optical susceptibility, and hence the SBS process becomes partially weakened.

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