Gap-function anisotropy and collective modes in a bilayer superconductor with Cooper-pair tunneling

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We study the gap-function anisotropy and collective-mode spectrum ($\omega \leq 2\Delta$) of the Anderson-Chakravarty model for high-T_c oxides based on Cooper-pair tunneling in a bilayer (with strength T_J). For both s-wave and d-wave pairing in the layers, the shape of the gap around the Fermi surface is strongly dependent on T_J. Besides the usual Anderson-Bogoliubov phase and Littlewood-Varma amplitude modes, we find branches (optical phononlike modes) involving fluctuations of the relative phase and amplitude of the order parameters of the two layers. These modes are the dynamic signature of the Cooper-pair tunneling model since their energy and damping depends critically on the relative magnitudes of the Cooper-pair tunneling strength (T_J) and the pairing interaction (g), but not significantly on the symmetry of the pairing. The generalization to a trilayer system (which can arise in Bi, Tl, and Hg copper oxides) is briefly discussed.

I. INTRODUCTION

Recently Anderson[1] has introduced a mechanism in which Cooper-pair tunneling between nearby CuO_2 sheets leads to a strong enhancement of T_c in the oxide superconductors. Assuming s-wave pairing, this bilayer pair tunneling model has been shown by Anderson and co-workers[2–4] to give rise to an anisotropic s-wave gap function $\Delta_{\mathbf{p}}$.

In this paper, we use a slightly generalized form to study the gap-function anisotropy and collective oscillations of the bilayer order parameter for both s-wave and d-wave pairing in the layers. As expected, the shape of the gap function on the Fermi surface is strongly dependent on the magnitude of T_J. If the pair tunneling strength T_J is fairly large, the gap is very small in a large region around d-wave node positions $p_y = \pm p_x$ [the Γ-$X(Y)$ lines], compared to the pure d-wave gap without pair tunneling ($T_J = 0$). Experiments to look for this using high-resolution angular-resolved photoemission spectroscopy[5] in cuprates would be of interest. Perhaps of greatest interest, we find collective modes involving out-of-phase fluctuations in the relative phase and amplitude of the order parameters in the two different layers. The energy and damping of these out-of-phase modes depends critically on whether the pairing (g) in a given sheet is larger or smaller than the Cooper-pair tunneling strength (T_J), but not on the symmetry assumed for the pairing. These modes are thus the characteristic dynamic signature of the Cooper-pair tunneling model discussed in Refs. 1–4. A Brief Report of these results has already been published[6] for the case of s-wave pairing and isotropic Josephson coupling T_J in a bilayer.

In the present paper, we do not discuss the correctness of the Cooper-pair (Josephson) tunneling model for high-T_c layered materials. As in Ref. 6, we work within this model and use it to discuss the gap-function anisotropy and order-parameter fluctuations in a superconducting bilayer as a function of T_J.

In Sec. II, we discuss the gap-function anisotropy in a superconducting bilayer with Cooper-pair tunneling, assuming either s-wave or d-wave pairing in the layers. In Sec. III, we develop the formalism to deal with time-dependent fields and work out the collective-mode branches. These appear as poles of various correlation functions involving the fluctuations of density, as well as phase and amplitude of the order parameters. In Sec. IV, the dispersion relation and damping of all the in-phase and out-of-phase collective modes are studied. In Sec. V, we summarize our main conclusions. In the Appendix, we discuss the analogous results for collective modes in a *trilayer* system. For simplicity, in this paper we only treat the $T = 0$ case (for finite temperature, calculations similar to those given in Ref. 7 can be performed).

II. FORMALISM AND GAP ANISOTROPY

The total effective Hamiltonian $K = H - \mu N$ of our coupled bilayer model is

$$K = K_0 + V_{11} + V_{12}, \quad (1)$$

where the noninteracting part is

$$K_0 = \sum_{\mathbf{p},\sigma,i} \epsilon_{\mathbf{p}} a^\dagger_{i,\mathbf{p},\sigma} a_{i,\mathbf{p},\sigma} + \sum_{\mathbf{p},\sigma,i\neq j} \bar{t}(\mathbf{p}) a^\dagger_{i,\mathbf{p},\sigma} a_{j,\mathbf{p},\sigma}. \quad (2)$$

Here $a_{i,\mathbf{p},\sigma}$ and $a^\dagger_{i,\mathbf{p},\sigma}$ are the usual destruction and creation operators, respectively, for electrons on layer i and the layer kinetic energy (measured with respect to Fermi energy) is $\epsilon_{\mathbf{p}} = p^2/2m - \mu$. The intralayer interactions (pairing and Coulomb) are denoted by V_{11} and the interlayer interactions by V_{12}. Following the arguments of Refs. 1–4, we omit single-particle hopping between layers due to the second term in (2), although it will give rise to pair tunneling in higher order (see below).

Within the time-dependent Hartree-Fock-Gor'kov (mean-field) approximation for a perturbed system, the intralayer two-particle interaction V_{11} is described by[6,7]

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$$\delta V_{11} = \frac{1}{2} \sum_{\substack{\mathbf{p},\mathbf{p}',\mathbf{q} \\ \sigma,i}} \Big[2v_{2D}(\mathbf{q}) \langle a^\dagger_{i,\mathbf{p}+\mathbf{q},\sigma} a_{i,\mathbf{p},\sigma} \rangle \sum_{\sigma'} a^\dagger_{i,\mathbf{p}'-\mathbf{q},\sigma'} a_{i,\mathbf{p}',\sigma'} - 2g_Z(\mathbf{p},\mathbf{p}') \langle a^\dagger_{i,\mathbf{p},\sigma} a_{i,\mathbf{p}+\mathbf{q},\sigma} \rangle a^\dagger_{i,\mathbf{p}'+\mathbf{q},\sigma} a_{i,\mathbf{p}',\sigma}$$
$$+ g(\mathbf{p},\mathbf{p}') \langle a^\dagger_{i,\mathbf{p},\sigma} a^\dagger_{i,-\mathbf{p}+\mathbf{q},-\sigma} \rangle a_{i,-\mathbf{p}'+\mathbf{q},-\sigma} a_{i,\mathbf{p}',\sigma} + g(\mathbf{p},\mathbf{p}') \langle a_{i,-\mathbf{p}+\mathbf{q},-\sigma} a_{i,\mathbf{p},\sigma} \rangle a^\dagger_{i,\mathbf{p}',\sigma} a^\dagger_{i,-\mathbf{p}'+\mathbf{q},-\sigma} \Big] , \quad (3)$$

where $v_{2D}(\mathbf{q}) = 2\pi e^2/q$ is the Coulomb interaction between electrons in a given layer, $g_Z(\mathbf{p},\mathbf{p}')$ is a short-range interaction in the zero-sound (particle-hole) channel, and $g(\mathbf{p},\mathbf{p}')$ is the pairing interaction in the Cooper (particle-particle) channel. The latter is responsible for the superconductivity in a layer. We will approximate the short-range particle-hole interaction $g_Z(\mathbf{p},\mathbf{p}')$ as a constant g_Z for simplicity. For $\mathbf{p}$, $\mathbf{p}'$ close to the Fermi momentum, the pairing interaction in (3) is assumed to have the separable form (with $g > 0$)

$$g(\mathbf{p},\mathbf{p}') \equiv -g f(\mathbf{p}) f(\mathbf{p}'), \quad (4)$$

where we have defined the (real) function

$$f(\mathbf{p}) = \begin{cases} 1 & \text{for } s\text{-wavepairing}, \\ \cos(2\phi) & \text{for } d_{x^2-y^2}\text{-wavepairing}. \end{cases} \quad (5)$$

Here ϕ is the direction of $\mathbf{p}$ on the two-dimensional (2D) Fermi surface. The average $\langle \hat{A} \rangle$ in (3) is determined self-consistently by the full mean-field-approximation (MFA) Hamiltonian [see (23)]. In (3), we only include the long-range Coulomb interaction in the self-consistent *Hartree* term and the short-range interactions in the exchange or *Fock* terms. For a more detailed discussion, see Ref. 7.

In an analogous way, the time-dependent mean-field approximation for interlayer interaction V_{12} in the p-h channel is given by[6]

$$\delta V_{12}^{p\text{-}h} = \frac{1}{2} \sum_{\substack{\mathbf{p},\mathbf{p}',\mathbf{q} \\ \sigma,\sigma',i\neq j}} 2v_\perp(\mathbf{q}) \langle a^\dagger_{i,\mathbf{p}+\mathbf{q},\sigma} a_{i,\mathbf{p},\sigma} \rangle \times a^\dagger_{j,\mathbf{p}'-\mathbf{q},\sigma'} a_{j,\mathbf{p}',\sigma'}, \quad (6)$$

where we only include the *interlayer* Coulomb interaction[8] $v_\perp(\mathbf{q}) = v_{2D}(\mathbf{q}) e^{-qd}$ (where d is the distance between layers) in the Hartree term. The contribution from the exchange terms are ignored due to the absence of single-quasiparticle tunneling ($\langle a^\dagger_i a_j \rangle = 0$) as well as the absence of Cooper pairs formed by pairing of electrons in different layers ($\langle a^\dagger_i a^\dagger_j \rangle = \langle a_i a_j \rangle = 0$). Finally, the mean-field approximation for the interlayer interaction V_{12} in the p-p channel is given by

$$\delta V_{12}^{p\text{-}p} = -\frac{1}{2} \sum_{\substack{\mathbf{p},\mathbf{p}',\mathbf{q} \\ \sigma,i\neq j}} \big[T_J(\mathbf{q},\mathbf{p},\mathbf{p}') \langle a^\dagger_{i,\mathbf{p},\sigma} a^\dagger_{i,-\mathbf{p}+\mathbf{q},-\sigma} \rangle a_{j,-\mathbf{p}'+\mathbf{q},-\sigma} a_{j,\mathbf{p}',\sigma}$$
$$+ T_J(\mathbf{q},\mathbf{p},\mathbf{p}') \langle a_{j,-\mathbf{p}+\mathbf{q},-\sigma} a_{j,\mathbf{p},\sigma} \rangle a^\dagger_{i,\mathbf{p}',\sigma} a^\dagger_{i,-\mathbf{p}'+\mathbf{q},-\sigma} \big] , \quad (7)$$

where only the Josephson coupling for the Cooper-pair tunneling contributes. The physical content of (7) reduces to the well-known Lawrence-Doniach theory[9] of layered superconductors in the Ginzburg-Landau region.

While Chakravarty and Anderson[3] argue that the quasiparticle momentum should be conserved during tunneling (i.e., $\mathbf{p} = \mathbf{p}'$) in (7), we treat a generalized version of the Cooper-pair interlayer interaction in which only the center-of-mass momentum of the Cooper pair ($\mathbf{q}$) is conserved. We allow the quasiparticle momentum to be spread out just as in the intralayer pairing interaction [see the last two terms in (3)]. As in Ref. 6, we use the separable form [with $T_J(\mathbf{q}) \geq 0$]

$$T_J(\mathbf{q},\mathbf{p},\mathbf{p}') = T_J(\mathbf{q}) \bar{t}(\mathbf{p}) \bar{t}(\mathbf{p}') \quad (8)$$

for quasiparticle momenta $\mathbf{p}$ and $\mathbf{p}'$ close to the Fermi surface (some sort of BCS cutoff is needed, for the usual reasons). In (8), $\bar{t}(\mathbf{p})$ is the interlayer single-particle tunneling amplitude introduced in (2), normalized with a maximum value of unity.[10] Our ansatz (8) reduces to the expression of Anderson and co-workers[2–4] when one imposes their stronger constraint that the quasiparticle momentum should be conserved in (7),

$$T_J(\mathbf{q}=0,\mathbf{p},\mathbf{p}') = T_J(\mathbf{q}=0)[\bar{t}(\mathbf{p})]^2 \delta_{\mathbf{p},\mathbf{p}'}. \quad (9)$$

Following Refs. 2–4, we use the specific form

$$\bar{t}(\mathbf{p}) = \cos^2(2\phi)\,\mathrm{sgn}[f(\mathbf{p})] \quad (10)$$

in our calculations, where $\cos^2(2\phi)$ corresponds to the continuum limit of $[\cos(p_x a) - \cos(p_y a)]^2$. The factor $\mathrm{sgn}[f(\mathbf{p})]$ in $\bar{t}(\mathbf{p})$ is redundant for s-wave pairing case (since $f = 1$) but is crucial for the d-wave pairing case where $f = \cos(2\phi)$ can change sign. This factor $\mathrm{sgn}[f(\mathbf{p})]$ ensures that the Cooper pairs involved in (7) have the same symmetry as those in (3). This in turn guarantees that the amplitude of the resulting gap function $|\Delta_\mathbf{p}|$ will have the symmetry of the CuO_2 layer electronic band structure.

The generalized time-dependent mean-field approximation summarized by (3)–(7) allows one to give a complete description of the Cooper-pair dynamics (see also

Ref. 7). These equations can be used to find the self-consistent BCS gap equation (we recall that the order parameter describes the *static* mean field arising from pairs with center-of-mass momentum $\mathbf{q} = 0$)

$$\Delta_{i,\mathbf{p}} = -\sum_{\mathbf{p}'} g(\mathbf{p},\mathbf{p}')\langle a_{i,-\mathbf{p}',-\sigma} a_{i,\mathbf{p}',\sigma}\rangle + \sum_{\mathbf{p}'} T_J(\mathbf{q}=0,\mathbf{p},\mathbf{p}')\langle a_{j,-\mathbf{p}',-\sigma} a_{j,\mathbf{p}',\sigma}\rangle. \quad (11)$$

The indices i and j in (11) (with $i \neq j$) are redundant since the anomalous pair average in the both layers must be identical by symmetry, with

$$\langle a_{1,-\mathbf{p},-\sigma} a_{1,\mathbf{p},\sigma}\rangle = \langle a_{2,-\mathbf{p},-\sigma} a_{2,\mathbf{p},\sigma}\rangle = \Delta_{\mathbf{p}}\chi_{\mathbf{p}}(T), \quad (12)$$

where $\chi_{\mathbf{p}}(T) \equiv (1/2E)\tanh(\beta E/2)$, $\beta = 1/k_B T$, and the BCS quasiparticle spectrum is $E_{\mathbf{p}} = \sqrt{\epsilon_{\mathbf{p}}^2 + |\Delta_{\mathbf{p}}|^2}$. Using (4) and (8) in (11), we find

$$\Delta_{\mathbf{p}}(T) = \Delta_0(T) f(\mathbf{p}) + \Delta_1(T)\bar{t}(\mathbf{p}), \quad (13)$$

where we have self-consistently defined

$$\Delta_0(T) \equiv g\sum_{\mathbf{p}}^{\omega_c} \Delta_{\mathbf{p}} f(\mathbf{p})\chi_{\mathbf{p}}(T),$$
$$\Delta_1(T) \equiv T_J\sum_{\mathbf{p}}^{\omega_c} \Delta_{\mathbf{p}}\bar{t}(\mathbf{p})\chi_{\mathbf{p}}(T), \quad (14)$$

and $T_J \equiv T_J(\mathbf{q}=0)$. One should, in general, use a different Fermi surface cutoff ω_c for g and T_J.

Inserting (13) into (14), one finds

$$\Delta_0(T) = \frac{g\Delta_1(T)\sum_{\mathbf{p}}^{\omega_c}\bar{t}(\mathbf{p})f(\mathbf{p})\chi_{\mathbf{p}}(T)}{1 - g\sum_{\mathbf{p}}^{\omega_c}[f(\mathbf{p})]^2\chi_{\mathbf{p}}(T)} \quad (15)$$

and

$$\Delta_1(T) = \frac{T_J\Delta_0(T)\sum_{\mathbf{p}}^{\omega_c}\bar{t}(\mathbf{p})f(\mathbf{p})\chi_{\mathbf{p}}(T)}{1 - T_J\sum_{\mathbf{p}}^{\omega_c}[\bar{t}(\mathbf{p})]^2\chi_{\mathbf{p}}(T)}. \quad (16)$$

Since $\bar{t}(\mathbf{p})$ is defined in (10) to have the same sign as $f(\mathbf{p})$, and g and T_J are positive, the only *consistent* solutions of (15) and (16) are such that Δ_0 and Δ_1 have the same sign (with our sign convention, we have $\Delta_0, \Delta_1 \geq 0$). Moreover, one can see from the definition of (14) that in a limited sense, $\Delta_0 \sim g$ and $\Delta_1 \sim T_J$. While there exists additional solutions of (15) and (16) with different signs for Δ_0 and Δ_1, these solutions are unphysical since one can show they involve discontinuities of these gap functions on the Fermi surface.[11] The anisotropic features of the gap function given by (11)–(16) are qualitatively quite similar to the one discussed in Ref. 2 based on (9).

The self-consistency of the gap function (13)–(16) requires that

$$(1 - gf_{20})(1 - T_J f_{02}) - gT_J f_{11}^2 \simeq 1 - gf_{20} - T_J f_{02} = 0, \quad (17)$$

where we have defined

$$f_{\ell k} \equiv \sum_{\mathbf{p}}^{\omega_c}[f(\mathbf{p})]^{\ell}[\bar{t}(\mathbf{p})]^k\chi_{\mathbf{p}}(T). \quad (18)$$

In the second line in (17), we make use of the fact that to a good approximation for the BCS weak-coupling limit, $f_{02}f_{20} - f_{11}^2 \simeq 0$. The gap function $\Delta_{\mathbf{p}}$ enters through $E_{\mathbf{p}}$ in $\chi_{\mathbf{p}}(T)$. As an example, when one assumes $T_J(\mathbf{q},\mathbf{p},\mathbf{p}') = T_J$ [i.e., $\bar{t}(\mathbf{p}) = 1$] and *s*-wave pairing [i.e., $f(\mathbf{p}) = 1$], choosing the same cutoff frequency ω_c for g and T_J allows one to trivially solve (17) to obtain the expected result

$$k_B T_c \sim \hbar\omega_c \exp\left[-\frac{1}{N(0)(g+T_J)}\right], \quad (19)$$

where the 2D density of states at the Fermi surface is $N(0) = m/\pi$. It is clear that T_c is enhanced drastically if T_J is large compared to g.

The gap functions (13) on the Fermi surface (at $T = 0$) can be written in more explicit forms which bring out the physics: for *s*-wave pairing,

$$\Delta_{\mathbf{p}} = \Delta_0 + \Delta_1\cos^2(2\phi), \quad (20)$$

and for *d*-wave pairing,

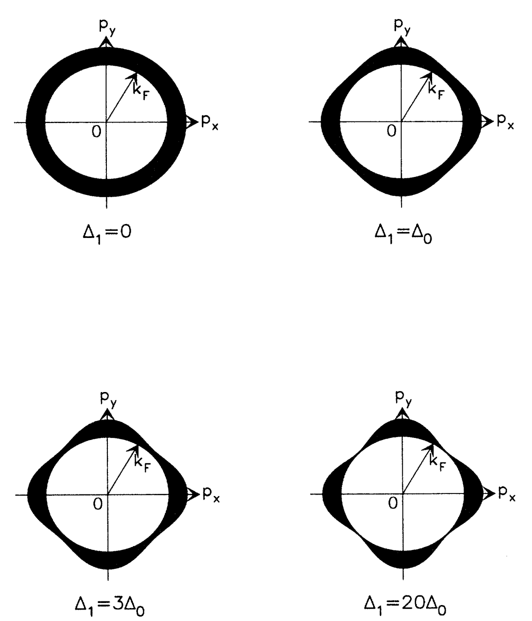

FIG. 1. Sketch of the order parameter $\Delta_{\mathbf{p}}$ (black shaded area) as given by (20), for a superconducting bilayer with Cooper-pair tunneling and *s*-wave pairing on the layer. The maximum of the amplitude for the gap function is taken to be the same for all values of Δ_0 and Δ_1.

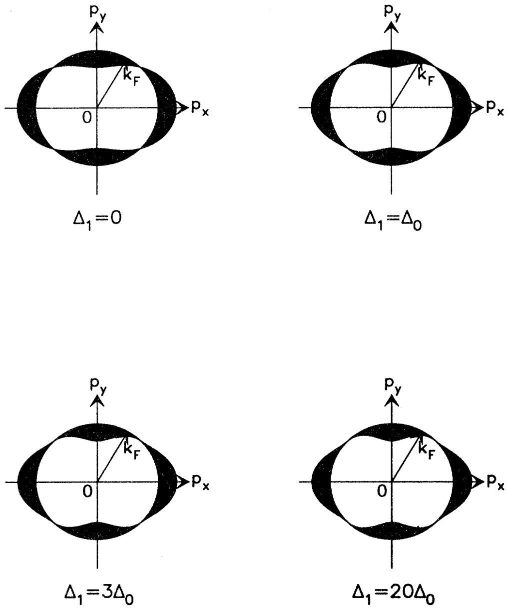

FIG. 2. Sketch for $\Delta_{\mathbf{p}}$ as given by (21), for d-wave layer pairing. See Fig. 1.

$$\Delta_{\mathbf{p}} = \Delta_0 \cos(2\phi) + \Delta_1 \cos^2(2\phi)\mathrm{sgn}[\cos(2\phi)]. \tag{21}$$

In Figs. 1 and 2, we plot $\Delta_{\mathbf{p}}$ for s-wave and d-wave intralayer pairing by varying the relative size of Δ_0 and Δ_1. For convenience of illustration, the maximum of the gap function is taken to be the same in all cases. Our main interest here is the relative variation of the gap function over different regions of the Fermi surface, which we take to be circular. We note in both the s-wave and d-wave pairing cases, the variation of the gap around the Fermi surface is strongly dependent on the relative size of Δ_0 and Δ_1 (or, equivalently, the relative size of g and T_J). In particular, the gap is seen to be very small in a fairly large region around d-wave node positions $p_y = \pm p_x$ [the Γ-$X(Y)$ lines] when $\Delta_1 \gg \Delta_0$ (or when the pair tunneling strength T_J is fairly large), as compared to the d-wave gap without the pair tunneling effect ($T_J = 0$). This behavior is a characteristic feature of strong pair tunneling and could be used as an experimental test of the latter.

III. DERIVATION OF RESPONSE FUNCTIONS

As discussed in Ref. 6, in discussing collective modes, it is useful to define the following operators associated with the ith layer:[12]

$$\begin{aligned}
\rho_{i,\mathbf{q}} &= \frac{1}{\sqrt{2}} \sum_{\mathbf{p},\sigma} a^\dagger_{i,\mathbf{p},\sigma} a_{i,i\mathbf{p}+\mathbf{q},\sigma},\\
\Phi_{i,\mathbf{q},\sigma} &= \frac{1}{\sqrt{2}} \sum_{\mathbf{p}} f(\mathbf{p})(a^\dagger_{i,\mathbf{p},\sigma} a^\dagger_{i,-\mathbf{p}-\mathbf{q},-\sigma}\\
&\quad - a_{i,-\mathbf{p}+\mathbf{q},-\sigma} a_{i,\mathbf{p},\sigma}),\\
A_{i,\mathbf{q},\sigma} &= \frac{1}{\sqrt{2}} \sum_{\mathbf{p}} f(\mathbf{p})(a^\dagger_{i,\mathbf{p},\sigma} a^\dagger_{i,-\mathbf{p}-\mathbf{q},-\sigma}\\
&\quad + a_{i,-\mathbf{p}+\mathbf{q},-\sigma} a_{i,\mathbf{p},\sigma}),\\
\tilde{\Phi}_{i,\mathbf{q},\sigma} &= \frac{1}{\sqrt{2}} \sum_{\mathbf{p}} \bar{t}(\mathbf{p})(a^\dagger_{i,\mathbf{p},\sigma} a^\dagger_{i,-\mathbf{p}-\mathbf{q},-\sigma}\\
&\quad - a_{i,-\mathbf{p}+\mathbf{q},-\sigma} a_{i,\mathbf{p},\sigma}),\\
\tilde{A}_{i,\mathbf{q},\sigma} &= \frac{1}{\sqrt{2}} \sum_{\mathbf{p}} \bar{t}(\mathbf{p})(a^\dagger_{i,\mathbf{p},\sigma} a^\dagger_{i,-\mathbf{p}-\mathbf{q},-\sigma}\\
&\quad + a_{i,-\mathbf{p}+\mathbf{q},-\sigma} a_{i,\mathbf{p},\sigma}).
\end{aligned} \tag{22}$$

For a given layer, $\Phi_{\mathbf{q}}$ and $\tilde{\Phi}_{\mathbf{q}}$ represent the phase fluctuations, while $A_{\mathbf{q}}$ and $\tilde{A}_{\mathbf{q}}$ represent the amplitude fluctuations and $\rho_{\mathbf{q}}$ is the usual density fluctuation. In terms of these operators, the mean-field approximation (3)–(7) ($\delta V_2 = \delta V_{11} + \delta V_{12}$) for the effect of the two-particle interactions can be written as[6,7]

$$\begin{aligned}
\delta V_2 = \sum_{\mathbf{q},i\neq j} \Big[&[2v_{2\mathrm{D}}(\mathbf{q}) + g_Z] \langle \rho^\dagger_{i,\mathbf{q}} \rangle \rho_{i,\mathbf{q}} + 2v_\perp(\mathbf{q}) \langle \rho^\dagger_{i,\mathbf{q}} \rangle \rho_{j,\mathbf{q}}\\
&- g\Big(\langle \Phi^\dagger_{i,\mathbf{q}} \rangle \Phi_{i,\mathbf{q}} + \langle A^\dagger_{i,\mathbf{q}} \rangle A_{i,\mathbf{q}} \Big)\\
&- T_J(\mathbf{q}) \Big(\langle \tilde{\Phi}^\dagger_{i,\mathbf{q}} \rangle \tilde{\Phi}_{j,\mathbf{q}} + \langle \tilde{A}^\dagger_{i,\mathbf{q}} \rangle \tilde{A}_{j,\mathbf{q}} \Big) \Big],
\end{aligned} \tag{23}$$

where we assume that the spin up and spin down give the same contribution.

A perturbing Hamiltonian δV_1 can be expressed in terms of the operators in (22) as

$$\begin{aligned}
\delta V_1 = \sum_{\mathbf{q},i} [&\delta\zeta(\mathbf{q},\omega)\rho_{i,\mathbf{q}} + \delta\eta(\mathbf{q},\omega)\Phi_{i,\mathbf{q}} + \delta\eta^*(\mathbf{q},\omega) A_{i,\mathbf{q}}\\
&+ \delta\tilde{\eta}(\mathbf{q},\omega)\tilde{\Phi}_{i,\mathbf{q}} + \delta\tilde{\eta}^*(\mathbf{q},\omega)\tilde{A}_{i,\mathbf{q}}],
\end{aligned} \tag{24}$$

where the external perturbation $\delta\zeta$ couples into the density operators $\rho_{i,\mathbf{q}}$, the symmetry-breaking field $\delta\eta$ couples into $\Phi_{i,\mathbf{q}}$, etc. Using linear response theory to calculate the effect of δV_1 in (24), in conjunction with a mean-field approximation for the two-particle interaction in (23) appropriate to a bilayer superconductor with Josephson tunneling, we obtain[7,13]

$$\begin{aligned}
\delta \hat{O}_i &= \sum_j \Big[\chi_{O_i\rho_j}\delta\zeta + \chi_{O_i\Phi_j}\delta\eta + \chi_{O_iA_j}\delta\eta^* + \chi_{O_i\tilde{\Phi}_j}\delta\tilde{\eta} + \chi_{O_i\tilde{A}_j}\delta\tilde{\eta}^* \Big]\\
&\overset{\mathrm{MFA}}{=} \sum_{j'\neq j} \big(\chi^0_{O_i\rho_j} \{ \delta\zeta + [2v_{2\mathrm{D}}(\mathbf{q}) + g_Z]\delta\rho_j + 2v_\perp(\mathbf{q})\delta\rho_{j'} \} + \chi^0_{O_i\Phi_j}(\delta\eta - g\delta\Phi_j) + \chi^0_{O_iA_j}(\delta\eta^* - g\delta A_j)\\
&\quad + \chi^0_{O_i\tilde{\Phi}_j}[\delta\tilde{\eta} - T_J(\mathbf{q})\delta\tilde{\Phi}_{j'}] + \chi^0_{O_i\tilde{A}_j}[\delta\tilde{\eta}^* - T_J(\mathbf{q})\delta\tilde{A}_{j'}] \big) ,
\end{aligned} \tag{25}$$

where the various response functions in (25) are defined in terms of the ten operators $(\hat{O}, \hat{P})$ in (22) for given values of $\mathbf{q}$ and σ,

$$\chi_{\hat{O}\hat{P}}(\mathbf{q}, i\Omega_n) = -\int_0^\beta d\tau e^{i\Omega_n \tau}\langle \hat{\mathrm{T}}_\tau \hat{O}_{\mathbf{q}}(\tau)\hat{P}^\dagger_{\mathbf{q}}(0)\rangle. \qquad (26)$$

We drop the spin label σ in Φ_i, A_i and $\tilde{\Phi}_i$, $\tilde{A}_i$ for simplicity.

One has to solve a 10×10 random-phase-approximation-like matrix equation which summarizes the above coupled equations (25). These can be represented schematically by[6]

$$\chi = \chi^0 - \chi^0 \mathcal{V}\chi = (\mathcal{I} + \chi^0\mathcal{V})^{-1}\chi^0, \qquad (27)$$

where $\mathcal{I}$ is the 10×10 unit matrix and the matrix elements χ^0_{rs} with $r, s = (1, ..., 10) \equiv (\rho_{1,\mathbf{q}}, ..., \tilde{A}_{2,\mathbf{q}})$, represent the response functions for noninteracting BCS quasiparticles. The general structure of (27) shows that the *complete* collective mode spectrum of the system will be given by the zeros of the secular determinant, namely,

$$\det(\mathcal{I} + \chi^0\mathcal{V}) = 0. \qquad (28)$$

A detailed calculation gives

$$\chi^0 = \begin{pmatrix} A_{00} & 0 & -2c_{10} & 0 & 0 & 0 & -2c_{01} & 0 & 0 & 0 \\ 0 & A_{00} & 0 & -2c_{10} & 0 & 0 & 0 & -2c_{01} & 0 & 0 \\ -2c_{10} & 0 & -B_{20} & 0 & 0 & 0 & -B_{11} & 0 & 0 & 0 \\ 0 & -2c_{10} & 0 & -B_{20} & 0 & 0 & 0 & -B_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & -C_{20} & 0 & 0 & 0 & -C_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & -C_{20} & 0 & 0 & 0 & -C_{11} \\ -2c_{01} & 0 & -B_{11} & 0 & 0 & 0 & -B_{02} & 0 & 0 & 0 \\ 0 & -2c_{01} & 0 & -B_{11} & 0 & 0 & 0 & -B_{02} & 0 & 0 \\ 0 & 0 & 0 & 0 & -C_{11} & 0 & 0 & 0 & -C_{02} & 0 \\ 0 & 0 & 0 & 0 & 0 & -C_{11} & 0 & 0 & 0 & -C_{02} \end{pmatrix}, \qquad (29)$$

where we have defined $(\ell, k = 0, 1, 2)$ (Refs. 6 and 14)

$$\begin{aligned} A_{\ell k}(\mathbf{q}, i\Omega_n) &= \int \frac{d\mathbf{p}}{(2\pi)^2}\big[f(\mathbf{p})\big]^\ell \big[\bar{t}(\mathbf{p})\big]^k \frac{E+E'}{2EE'}\frac{EE' - \epsilon\epsilon' + \Delta_{\mathbf{p}}\Delta_{\mathbf{p+q}}}{(i\Omega_n)^2 - (E+E')^2}, \\ B_{\ell k}(\mathbf{q}, i\Omega_n) &= \int \frac{d\mathbf{p}}{(2\pi)^2}\big[f(\mathbf{p})\big]^\ell \big[\bar{t}(\mathbf{p})\big]^k \frac{E+E'}{2EE'}\frac{-(EE' + \epsilon\epsilon' + \Delta_{\mathbf{p}}\Delta_{\mathbf{p+q}})}{(i\Omega_n)^2 - (E+E')^2}, \\ C_{\ell k}(\mathbf{q}, i\Omega_n) &= \int \frac{d\mathbf{p}}{(2\pi)^2}\big[f(\mathbf{p})\big]^\ell \big[\bar{t}(\mathbf{p})\big]^k \frac{E+E'}{2EE'}\frac{-(EE' + \epsilon\epsilon' - \Delta_{\mathbf{p}}\Delta_{\mathbf{p+q}})}{(i\Omega_n)^2 - (E+E')^2}, \\ c_{\ell k}(\mathbf{q}, i\Omega_n) &= \int \frac{d\mathbf{p}}{(2\pi)^2}\big[f(\mathbf{p})\big]^\ell \big[\bar{t}(\mathbf{p})\big]^k \frac{-i\Omega_n\Delta_{\mathbf{p}}}{2E}\frac{1}{(i\Omega_n)^2 - (E+E')^2}. \end{aligned} \qquad (30)$$

Here $i\Omega_n$ is the usual Bose Matsubara frequency, $E \equiv E_{\mathbf{p}}$, $E' \equiv E_{\mathbf{p+q}}$, and $\epsilon \equiv \epsilon_{\mathbf{p}}$, $\epsilon' \equiv \epsilon_{\mathbf{p+q}}$. The corresponding interaction matrix $\mathcal{V}$ in (27) is found to be

$$\mathcal{V} = \begin{pmatrix} -2v_{2\mathrm{D}} - g_Z & -2v_\perp & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2v_\perp & -2v_{2\mathrm{D}} - g_Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & g & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_J & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & T_J & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_J \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_J & 0 \end{pmatrix}. \qquad (31)$$

Using (29) and (31) in (27), one can show that the determinant of the denominator factorizes,

$$\det(\mathcal{I} + \chi^0\mathcal{V}) = D_+ D_- D'_+ D'_-, \qquad (32)$$

where we have defined

$$\begin{aligned} D_\pm \equiv &\Big[1 - \big(2v_{2\mathrm{D}} \pm 2v_\perp + g_Z\big)A_{00}\Big]\Big[\big(1 - gB_{20}\big)\big(1 \mp T_J B_{02}\big) \mp gT_J B_{11}^2\Big] \\ &+4\Big[2v_{2\mathrm{D}} \pm 2v_\perp + g_Z\Big]\Big[gc_{10}^2\big(1 \mp T_J B_{02}\big) \pm T_J c_{01}^2\big(1 - gB_{20}\big) \pm 2gT_J c_{10}c_{01}B_{11}\Big], \end{aligned} \qquad (33)$$

$$D'_{\pm} \equiv \Big[1 - gC_{20}\Big]\Big[1 \mp T_J C_{02}\Big] \mp g T_J C_{11}^2. \tag{34}$$

This means that there are four possible collective-mode branches in a superconducting bilayer which includes interlayer Cooper-pair tunneling, given by the solutions of $\mathrm{Re}D_{\pm}(\mathbf{q},\omega) = 0$ and $\mathrm{Re}D'_{\pm}(\mathbf{q},\omega) = 0$. To gain more insight into these collective-mode branches, we consider the response functions for the simple case of isotropic T_J and s-wave pairing [this means $f(\mathbf{p}) = 1$ and $\bar{t}(\mathbf{p}) = 1$ in (30)]. In this case, $\tilde{\Phi} = \Phi$ and $\tilde{A} = A$ and all $x_{\ell k}$ for $x = A, B, C, c$ and $\ell, k = 0, 1, 2$ reduce to $x_{00} \equiv x_0$. The functions $D_{\pm}$ and $D'_{\pm}$ in (33) and (34) then simplify to the results quoted in Ref. 6,

$$D_{\pm}(\mathbf{q}, i\Omega_n) \equiv \Big[1 - \Big(2v_{2\mathrm{D}} \pm 2v_{\perp} + g_Z\Big)A_0\Big]\Big[1 - \Big(g \pm T_J\Big)B_0\Big] + 4\Big(2v_{2\mathrm{D}} \pm 2v_{\perp} + g_Z\Big)\Big(g \pm T_J\Big)c_0^2\ , \tag{35}$$

$$D'_{\pm}(\mathbf{q}, i\Omega_n) \equiv 1 - \Big(g \pm T_J\Big)C_0. \tag{36}$$

Solving (27), we find the following nonvanishing correlation functions:

$$\begin{aligned}
\chi_{\rho_1\rho_1} &= \chi_{\rho_2\rho_2} = \frac{[1-(2v_{2\mathrm{D}}+g_Z)E_-]E_+[1-(g-T_J)B_0][1-(g+T_J)B_0]+4T_Jc_0^2}{D_+D_-}\ ,\\
\chi_{\rho_1\rho_2} &= \chi_{\rho_2\rho_1} = \frac{2v_{\perp}E_-E_+[1-(g-T_J)B_0][1-(g+T_J)B_0]-4T_Jc_0^2}{D_+D_-}\ ,\\
\chi_{\rho_1\Phi_1} &= \chi_{\rho_2\Phi_2} = \chi_{\Phi_1\rho_1} = \chi_{\Phi_2\rho_2}\\
&= -2\frac{c_0\{1-(2v_{2\mathrm{D}}+g_Z)A_0-gB_0+[(2v_{2\mathrm{D}}+g_Z)g+2v_{\perp}T_J](A_0B_0+4c_0^2)\}}{D_+D_-}\ ,\\
\chi_{\rho_1\Phi_2} &= \chi_{\rho_2\Phi_1} = \chi_{\Phi_2\rho_1} = \chi_{\Phi_1\rho_2}\\
&= -2\frac{c_0\{2v_{\perp}A_0+T_JB_0-[(2v_{2\mathrm{D}}+g_Z)T_J+2v_{\perp}g](A_0B_0+4c_0^2)\}}{D_+D_-}\ ,\\
\chi_{\Phi_1\Phi_1} &= \chi_{\Phi_2\Phi_2}\\
&= -\frac{[1-gF_-]F_+[1-(2v_{2\mathrm{D}}-2v_{\perp}+g_Z)A_0][1-(2v_{2\mathrm{D}}+2v_{\perp}+g_Z)A_0]+8v_{\perp}c_0^2}{D_+D_-}\ ,\\
\chi_{\Phi_1\Phi_2} &= \chi_{\Phi_2\Phi_1}\\
&= -\frac{T_JF_-F_+[1-(2v_{2\mathrm{D}}-2v_{\perp}+g_Z)A_0][1-(2v_{2\mathrm{D}}+2v_{\perp}+g_Z)A_0]-8v_{\perp}c_0^2}{D_+D_-}\ ,\\
\chi_{A_1A_1} &= \chi_{A_2A_2} = -\frac{C_0(1-gC_0)}{D'_+D'_-}\ ,\\
\chi_{A_1A_2} &= \chi_{A_2A_1} = -\frac{T_JC_0^2}{D'_+D'_-},
\end{aligned} \tag{37}$$

where we have defined

$$\begin{aligned}
E_{\pm} &\equiv A_0 - \frac{4(g\pm T_J)c_0^2}{1-(g\pm T_J)B_0}\ ,\\
F_{\pm} &\equiv B_0 - \frac{4(2v_{2\mathrm{D}}\pm 2v_{\perp}+g_Z)c_0^2}{1-(2v_{2\mathrm{D}}\pm 2v_{\perp}+g_Z)A_0}.
\end{aligned} \tag{38}$$

In the usual way with imaginary frequency Green's functions, we analytically continue $i\Omega_n$ to the real frequency axis $\omega + i0^+$.

We note the following important features exhibited by the response functions in (37). One finds that $\chi_{\rho_i\rho_j}(\mathbf{q},\omega)$, $\chi_{\Phi_i\Phi_j}(\mathbf{q},\omega)$, and $\chi_{\rho_i\Phi_j}(\mathbf{q},\omega)$ (for $i,j = 1,2$) all have poles when $D_+ = 0$ and $D_- = 0$. This shows that the zeros of $\mathrm{Re}D_{\pm}$ given by (33) and (35) correspond to collective modes associated with fluctuations of the charge density as well as the *phase* of the Cooper-pair order parameter. Moreover, these are coupled to each other and are strongly modified by the long-range intralayer $v_{2\mathrm{D}}$ and interlayer $v_{\perp}$ Coulomb interaction [as shown explicitly by (33) and (35)]. We refer to these two zeros of $\mathrm{Re}D_{\pm} = 0$ as phase modes I and II. In contrast, the response functions $\chi_{A_iA_j}(\mathbf{q},\omega)$ (for $i,j = 1,2$) involving the amplitude operators only have poles corresponding to $D'_+ = 0$ and $D'_- = 0$. This shows that the zeros of $\mathrm{Re}D'_{\pm}$ given by (34) and (36) correspond to collective modes associated with Cooper-pair *amplitude* fluctuations. These two amplitude modes are not coupled to the charge or phase fluctuations and are unaffected by the Coulomb interaction [as shown explicitly by (34) and (36)]. We refer to these as amplitude modes I and II.

We remark that even in the absence of interlayer pairing tunneling ($T_J = 0$), the interlayer Coulomb interaction $v_{\perp}(\mathbf{q})$ in (35) renormalizes the phase modes of the individual layers (see also Appendix A of Ref. 14).

IV. COLLECTIVE-MODE DISPERSION RELATIONS AND DAMPING

A. *p-h* excitation and damping

When one deals with an anisotropic tunneling strength as given in (8) with $\bar{t}(\mathbf{p})$ assumed to be of the form (10),

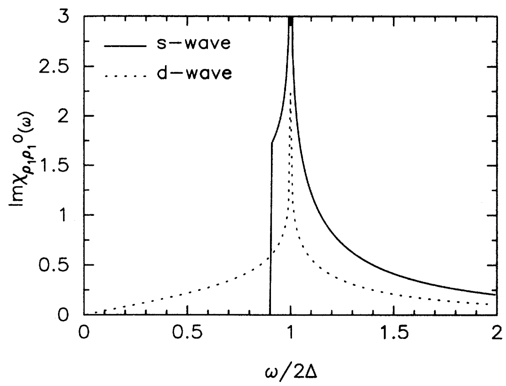

FIG. 3. The frequency dependence of the imaginary part of the noninteracting density response function as given by (39) and (30), with $\Delta_1 = 0.1\Delta_0$ (or $\Delta_0/\Delta = 10/11$), for both s-wave and d-wave layer pairing. For d-wave pairing, there is finite spectral weight at all frequencies.

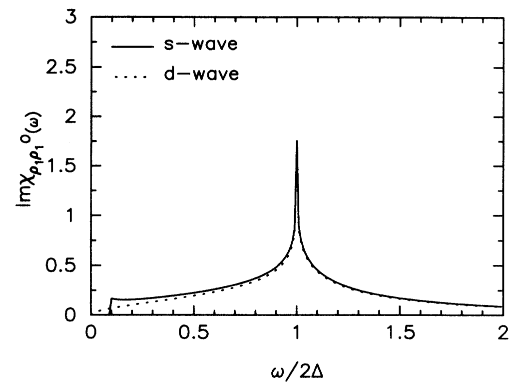

FIG. 4. The same plot as in Fig. 3, for $\Delta_1 = 10\Delta_0$ (or $\Delta_0/\Delta = 1/11$).

one obtains an anisotropic gap function for *both* s-wave and d-wave *intralayer* pairing. Moreover, the collective modes found within 2Δ (where $\Delta \equiv \Delta_0 + \Delta_1$ denotes the *maximum* value of the gap) can be damped due to the p-h pair breaking which is always possible because of the vanishing of the energy gap. We evaluate the imaginary part of the density response function for two noninteracting BCS quasiparticles [see (37) after setting v_{2D}, $v_\perp$, g_Z, g, and T_J equal to zero]

$$\mathrm{Im}\chi^0_{\rho_1\rho_1}(\mathbf{q},\omega) \sim \mathrm{Im}A_0(\mathbf{q},\omega), \tag{39}$$

where $A_0 \equiv A_{00}$ is defined in (30). The $\mathbf{q} = 0$ results are plotted in Figs. 3 and 4 with two different ratios of Δ_1/Δ_0, for both s-wave and d-wave intralayer pairing.

The main difference between the s-wave and d-wave pairing as shown in Figs. 3 and 4 are easily understood from the gap-function anisotropy (see Figs. 1 and 2). Since Δ_0 is the minimum value of the gap $\Delta_\mathbf{p}$ in the s-wave case, the p-h spectral weight has a threshold $\omega = 2\Delta_0$. This implies for the s-wave pairing that the collective modes with energy $\omega < 2\Delta_0$ are well defined, but are damped when $\omega > 2\Delta_0$. In contrast, the p-h spectral weight develops at $\omega = 0$ for d-wave case since the minimum gap vanishes. Thus the collective modes in the d-wave pairing case are always damped. However, we note that in the low-frequency region, the p-h spectral weight (and hence the damping) is still small even in the d-wave pairing case. Therefore low-energy collective modes are always well defined, for both s-wave or d-wave pairing. We also observe from Fig. 4 that when Δ_1 is large compared to Δ_0 (i.e., $T_J \gg g$), as expected, the p-h spectral weight is very similar for both kinds of pairing.

B. Collective-mode spectrum

In Ref. 6, we have already given a detailed discussion of collective modes of a bilayer, assuming s-wave pairing in a given layer and isotropic tunneling strength T_J $[\bar{t}(\mathbf{p}) = 1]$. These results bring out the essential physics of the collective-mode spectrum of the bilayer superconductor model. In this section, we generalize our earlier analysis to deal with an anisotropic tunneling strength T_J, treating *either* s-wave or d-wave pairing in the layers. Due to the strong p-h damping in the high-frequency region ($\omega \gtrsim 2\Delta$), we will focus on collective modes in the low-frequency region ($\omega \ll 2\Delta$). We ignore the wave vector dependence and set $T_J(\mathbf{q}) = T_J$ in (33) and (34).

1. Phase modes

We first ignore the Coulomb interaction ($v_{2D} = v_\perp = 0$) and study the phase modes in a neutral superconductor. Using (33), the phase modes I and II are found to be given by the zeros of

$$D_\pm(\mathbf{q},\omega) = \Big[1 - g_Z A_{00}\Big]\Big[\big(1 - gB_{20}\big)\big(1 \mp T_J B_{02}\big) \mp gT_J B_{11}^2\Big] + 4g_Z\Big[gc_{10}^2\big(1 \mp T_J B_{02}\big) \pm T_J c_{01}^2\big(1 - gB_{20}\big) \pm 2gT_J c_{10}c_{01}B_{11}\Big]. \tag{40}$$

Before solving the above complicated equation, we note that numerical calculation shows that (for small $\mathbf{q}$ and ω) $[B_{20}B_{02} - B_{11}^2] \sim [c_{10}^2 B_{02} + c_{01}^2 B_{20} - 2c_{10}c_{01}B_{11}] \propto \omega^4 \to 0$. This allows us to reduce (40) to

$$D_\pm(\mathbf{q},\omega) = \Big[1 - g_Z A_{00}\Big]\Big[1 - gB_{20} \mp T_J B_{02}\Big] + 4g_Z\Big[gc_{10}^2 \pm T_J c_{01}^2\Big]. \tag{41}$$

Rewriting the $B_{\ell k}$ in (30) as

$$B_{\ell k}(\mathbf{q},\omega) = f_{\ell k} - Q_{\ell k}(\mathbf{q},\omega), \tag{42}$$

where the functions $f_{\ell k}$ have been defined earlier in (18) and the functions $Q_{\ell k}$ are defined by

$$Q_{\ell k}(\mathbf{q},\omega) \equiv \int \frac{d\mathbf{p}}{(2\pi)^2}\left[f(\mathbf{p})\right]^{\ell}\left[\bar{t}(\mathbf{p})\right]^{k} \times \frac{E+E'}{4EE'}\frac{\omega^2-(\epsilon'-\epsilon)^2-(\Delta_{\mathbf{p}+\mathbf{q}}-\Delta_{\mathbf{p}})^2}{\omega^2-(E+E')^2}. \tag{43}$$

Using (42) and (43) and recalling the gap equation in (17), we can rewrite the key factor in (41),

$$1 - gB_{20} \mp T_J B_{02} = \begin{cases} gQ_{20} + T_J Q_{02} & \text{(I)},\\ gQ_{20} - T_J Q_{02} + \dfrac{2x}{x+y} & \text{(II)}, \end{cases} \tag{44}$$

where the upper sign gives branch I and the lower sign gives branch II. The constant $y \equiv f_{20}/f_{02}$ equals 8/3 for s-wave and 4/3 for d-wave pairing, if the same BCS cutoff frequencies are applied for both g and T_J. We treat the ratio

$$x \equiv \frac{T_J}{g} \tag{45}$$

as an adjustable quantity, as a way of exploring how the qualitative features of the order-parameter fluctuations in the bilayer superconductor model vary.

In the limit of small q, where we can use $\epsilon' - \epsilon \simeq \mathbf{v}_F\cdot\mathbf{q}$ and $\Delta_{\mathbf{p}+\mathbf{q}} \simeq \Delta_{\mathbf{p}}$, we need only keep the q dependence in the term $\epsilon' - \epsilon$ in $Q_{\ell k}$ [see (43)] and set $q = 0$ elsewhere in (41). This gives

$$\begin{aligned} Q_{\ell k} &= \frac{N(0)}{(2\Delta)^2}\left[\omega^2 - \frac{1}{2}v_F^2 q^2\right] I_{\ell k0}(\omega)\ ,\\ A_{\ell k} &= N(0) I_{\ell k2}(\omega)\ ,\\ c_{\ell k} &= -\frac{N(0)}{2(2\Delta)}\omega I_{\ell k1}(\omega), \end{aligned} \tag{46}$$

with the functions $I_{\ell km}(\omega)$ defined by

$$I_{\ell km}(\omega) \equiv \frac{1}{4}\int_{-\infty}^{\infty} d\bar{\epsilon}\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{[f(\mathbf{p})]^{\ell}[\bar{t}(\mathbf{p})]^{k}[h(\mathbf{p})]^{m}}{\bar{E}(\bar{\omega}^2-\bar{E}^2)}, \tag{47}$$

and $\bar{\omega} \equiv \omega/2\Delta$, $\bar{\epsilon} \equiv \epsilon/\Delta$, and $\bar{E} \equiv E/\Delta$. It is convenient to write $\Delta_{\mathbf{p}} \equiv \Delta h(\mathbf{p})$, where the anisotropic functions $h(\mathbf{p})$ for s-wave and d-wave pairing are given in (20) and (21), respectively. Using these results in (41), we obtain [with $N(0)g_Z \to g_Z$, $N(0)g \to g$, and $N(0)T_J \to T_J$]

$$\begin{aligned} D_{\pm}(\mathbf{q},\omega) = &\frac{1}{(2\Delta)^2}\left[gI_{200} \pm T_J I_{020}\right]\\ &\times\left[\omega^2 - \frac{1}{2}v_F^2q^2\left(1 - g_Z I_{002}\right)\right]\\ &+\begin{cases} 0 & \text{(I)},\\ \left(1 - g_Z I_{002}\right)\dfrac{2x}{x+y} & \text{(II)}, \end{cases} \end{aligned} \tag{48}$$

in which we have used the approximation $\omega^2(I_{002}I_{200} - I_{101}^2) \sim \omega^2(I_{002}I_{020} - I_{011}^2) \simeq 0$. While the solutions for branch I and II phase modes share the same form for both s-wave and d-wave pairing [as given in (48)], there is a crucial difference between s-wave and d-wave pairing built into the functions $I_{\ell km}(\omega)$. The latter involve a different function $f(\mathbf{p})$ (and hence a different anisotropic gap function $\Delta_{\mathbf{p}}$) for s-wave and d-wave pairing.

To be more explicit, one finds that independent of x, the branch I phase mode ($\mathrm{Re}D_+ = 0$) is given by ($\mathrm{Re}I_{002} \simeq -1/2$)

$$\begin{aligned} \omega^2 &= \frac{1}{2}v_F^2q^2\left(1 + g_Z|I_{002}|\right)\\ &\simeq \frac{1}{2}v_F^2q^2\left(1 + \frac{g_Z}{2}\right). \end{aligned} \tag{49}$$

This always exists in the region $\omega < 2\Delta$ and corresponds to the well-known 2D Anderson-Bogoliubov phonon mode.[14] The particle-hole channel pairing interaction g_Z only modifies the phonon velocity in (49). This low-energy mode is well defined for both s-wave and d-wave pairing since $\mathrm{Im}I_{002}(\omega) \simeq 0$ at low ω.

The preceding result for the in-phase phase mode is for a *neutral* bilayer superconductor model. When we include the Coulomb interactions $v_{2\mathrm{D}}$ and $v_\perp$ in (33), since $(v_{2\mathrm{D}} + v_\perp) \simeq 4\pi e^2/q$ in the limit of $q \to 0$, the phase I (Anderson-Bogoliubov) mode in (49) is renormalized into

$$\omega^2 = \omega_{2\mathrm{D}}^2 + \frac{1}{2}v_F^2q^2\left(1 + \frac{g_Z}{2}\right), \tag{50}$$

where $\omega_{2\mathrm{D}}^2 \equiv 2\pi(2n)e^2q/m$ ($n = k_F^2/2\pi$ is the 2D number density). This first term dominates the right-hand side (rhs) of (50), which is seen to correspond to a 2D-like plasmon mode of the two layers moving in phase.

In contrast, the branch II phase mode ($\mathrm{Re}D_- = 0$) has a solution which it is convenient to write in the form

$$\omega^2 = \left[\omega_0^2(\omega) + \frac{1}{2}v_F^2q^2\right]\left(1 + \frac{g_Z}{2}\right), \tag{51}$$

where we have defined the frequency-dependent energy gap as

$$\omega_0^2(\omega) \equiv \frac{(2\Delta)^2}{g}\frac{2x}{\mathrm{Re}[(x+y)(xI_{020} - I_{200})]}. \tag{52}$$

When the Coulomb interaction is included in (33), making use of the fact that for small $\mathbf{q}$, where $(v_{2\mathrm{D}} - v_\perp) \simeq 2\pi e^2 d$, we obtain, in place of (51),[15]

$$\omega^2 = \left[\omega_0^2(\omega) + \frac{1}{2}v_F^2q^2\right]\left(1 + \frac{1}{2}g_Z + 2me^2d\right), \tag{53}$$

where ω_0 is as defined in (52). The last term in (53) is equal to $2d/a_0$, where a_0 is the usual Bohr radius. This term is typically much larger than unity in the copper oxides.

The detailed behavior of this branch II phase mode given by (53) depends very much on the self-consistent value of $\omega_0^2(\omega)$ in (52). We find that to have a small positive value of ω_0^2 (i.e., $\omega_0 \ll \Delta$), one needs a small ratio $x \ll 1$ ($T_J \ll g$), which implies $\Delta_1 \ll \Delta_0$. For $\omega_0 \ll \Delta$, we have [$\mathrm{Im}I_{020}(\omega) \simeq \mathrm{Im}I_{200}(\omega) \simeq 0$ for low ω]

$$\mathrm{Re}I_{020} = \begin{cases} -\dfrac{3}{16} \ (s), \\ -\dfrac{1}{4} \ (d), \end{cases} \qquad \mathrm{Re}I_{200} = \begin{cases} -\dfrac{1}{2} \ (s), \\ -\dfrac{1}{2} \ (d), \end{cases} \tag{54}$$

where s and d denote s-wave and d-wave pairing in the layers. Substituting (54) into (52), we see that for small x, $\omega_0^2(\omega)$ is given by [this corrects (27) in Ref. 6]

$$\omega_0^2 \simeq \begin{cases} (2\Delta)^2 \dfrac{3x}{2g} \ (s), \\ (2\Delta)^2 \dfrac{3x}{g} \ (d). \end{cases} \tag{55}$$

This means that the branch II phase mode exhibits an energy gap $\omega_0 \propto \sqrt{x}$ in the $\mathbf{q} = 0$ limit and for small x. Since the region $\omega \ll 2\Delta$ is being considered, for consistency (53) is only valid for $x \lesssim 0.04$ for s-wave and $x \lesssim 0.02$ for d-wave pairing. We remark that in the regime where $x \gg 1$, this branch II phase mode has a dispersion relation similar to (53) but with $\omega_0 > 2\Delta$. In this case, this mode is strongly damped due to pair breaking.

Anderson and co-workers[1–4] argue that T_J can be much larger than g in the oxide superconductors. It can be seen from (51)–(55) that the branch-II-phase collective-mode energy spectrum and the damping depend critically on the relative magnitude of the Cooper-pair tunneling strength T_J and the pairing interaction g (i.e., the ratio x), but not significantly on the *nature* of the pairing. As we noted in Ref. 6, this branch II phase mode is analogous to one first discussed by Leggett[16] in a simple model of a two-band bulk superconductor. Leggett also pointed out that such a fluctuation in the relative phase of *two* order parameters was physically the analog of excitonlike modes in a bulk s-wave superconductor with an attractive interaction in the d-wave channel.[17]

2. Amplitude modes

We next turn to (34) and discuss the two amplitude modes given by $\mathrm{Re}D'_\pm = 0$. These are unaffected by the Coulomb interaction. The discussion for the branch I and II amplitude modes are similar to the analysis of branch I and II phase modes given above. Since $[C_{20}C_{02} - C_{11}^2] \propto \omega^4 \to 0$, $D'_\pm$ in (34) is reduced to

$$D'_\pm(\mathbf{q}, \omega) = 1 - gC_{20} \mp T_J C_{02}. \tag{56}$$

Following the expansion for the functions $B_{\ell k}$ in (42), we rewrite the $C_{\ell k}$ in (30) as

$$C_{\ell k}(\mathbf{q}, \omega) = f_{\ell k} - P_{\ell k}(\mathbf{q}, \omega), \tag{57}$$

where $P_{\ell k}$ is defined as

$$P_{\ell k}(\mathbf{q}, \omega) = \int \frac{d\mathbf{p}}{(2\pi)^2} [f(\mathbf{p})]^\ell [\bar{t}(\mathbf{p})]^k \frac{E + E'}{4EE'} \times \frac{\omega^2 - (\epsilon' - \epsilon)^2 - (\Delta_{\mathbf{p+q}} + \Delta_{\mathbf{p}})^2}{\omega^2 - (E + E')^2}. \tag{58}$$

This is similar to $Q_{\ell k}$ in (43), apart from having the uncanceled term $(\Delta_{\mathbf{p+q}} + \Delta_{\mathbf{p}})$ in place of $(\Delta_{\mathbf{p+q}} - \Delta_{\mathbf{p}})$, which plays a key role in amplitude modes. $D'_\pm$ in (56) thus can be represented as [using (17)]

$$D'_\pm = \begin{cases} gP_{20} + T_J P_{02} & \text{(I)}, \\ gP_{20} - T_J P_{02} + \dfrac{2x}{x + y} & \text{(II)}, \end{cases} \tag{59}$$

where the parameters x and y are as given in (44).

In the limit of small q, we can approximate the functions $P_{\ell k}$ in (58) as

$$P_{\ell k} = \frac{N(0)}{(2\Delta)^2} \left[\left(\omega^2 - \frac{1}{2} v_F^2 q^2 \right) I_{\ell k 0}(\omega) - (2\Delta)^2 I_{\ell k 2}(\omega) \right], \tag{60}$$

where $I_{\ell k m}(\omega)$ are given in (47). As a result, we obtain [with $N(0)g \to g$ and $N(0)T_J \to T_J$]

$$D'_\pm(\mathbf{q}, \omega) = \frac{1}{(2\Delta)^2} \left[gI_{200} \pm T_J I_{020} \right] \left[\omega^2 - \frac{1}{2} v_F^2 q^2 \right] - \left[gI_{202} \pm T_J I_{022} \right] + \begin{cases} 0 & \text{(I)}, \\ \dfrac{2x}{x + y} & \text{(II)}. \end{cases} \tag{61}$$

Again, the branch I and II amplitude modes share the same form for both s-wave and d-wave pairing [as given by the zeros of (61)].

One finds that the dispersion relation of the branch I amplitude mode ($\mathrm{Re}D'_+ = 0$) can be written in the form

$$\omega^2 = \omega_{1+}^2(\omega) + \frac{1}{2} v_F^2 q^2, \tag{62}$$

where again it is convenient to define an energy-dependent gap

$$\omega_{1+}^2(\omega) \equiv (2\Delta)^2 \frac{\mathrm{Re}[I_{202}(\omega) + xI_{022}(\omega)]}{\mathrm{Re}[I_{200}(\omega) + xI_{020}(\omega)]} > 0. \tag{63}$$

This mode corresponds to when the Cooper-pair amplitudes in the two layers oscillate in phase and is the analog of the well-known amplitude mode of a bulk s-wave superconductor.[18] Thus we can interpret ω_{1+}^2 in (63) as related to the energy needed to break up a Cooper pair, which should correspond to some appropriate Fermi surface average of $(2\Delta_{\mathbf{p}})^2$. The value of ω_{1+} is clearly dependent on the relative magnitude of T_J and g (i.e., x), as well as Δ_0 and Δ_1. One may verify that when $x \to 0$

(and $\Delta_1 \ll \Delta_0$), corresponding to a pure *s*-wave (or *d*-wave) superconductor without any pair tunneling, we have $\omega_{1+}^2(\omega) = 4\Delta^2$ (Ref. 18) for *s*-wave and $3\Delta^2$ for *d*-wave pairing.[7] This in turn shows that in this limit, the branch I amplitude mode has energy at $\omega = 2\Delta$ for *s*-wave and $\omega = \sqrt{3}\Delta$ for *d*-wave pairing cases. These modes are strongly damped since the energies are near 2Δ, with strong *p*-*h* damping. For large value of x, the gap functions become highly anisotropic and we have not been able to find a solution of $\omega^2 = \omega_{1+}^2(\omega)$ for either *s*-wave or *d*-wave pairing.

In contrast, for the branch II amplitude modes ($\text{Re}D'_- = 0$), one finds that the zeros of (61) are given by

$$\omega^2 = \omega_g^2(\omega) + \frac{1}{2}v_F^2 q^2, \tag{64}$$

where again it is convenient to introduce

$$\omega_g^2(\omega) \equiv \omega_0^2(\omega) + \omega_{1-}^2(\omega) \tag{65}$$

and ω_0^2 is given by (52) and ω_{1-}^2 is given by [compare with (63)]

$$\omega_{1-}^2(\omega) \equiv (2\Delta)^2 \frac{\text{Re}[I_{202}(\omega) - xI_{022}(\omega)]}{\text{Re}[I_{200}(\omega) - xI_{020}(\omega)]} > 0. \tag{66}$$

Evaluating (52), we find $\omega_0^2 < 0$ for $x \gg 1$, while ω_{1-}^2 is always positive. In this regard, the appropriate value of the energy gap ω_g in (65) can be much smaller than 2Δ if $\omega_0^2 \to -\omega_{1-}^2$. In the regime $x \ll 1$, the branch II amplitude mode always has an energy gap $\omega_g > 2\Delta$. As a result, it is strongly damped and of less interest.

V. CONCLUDING REMARKS

In this paper, we have extended our recent discussion[14,7] of the order-parameter fluctuations in a 2D superconductor to the case of a superconducting bilayer. Some of these results were reported in Ref. 6. Following recent work,[1–4] we allowed Cooper-pair tunneling between the two layers but no coherent single-particle tunneling. The order-parameter dynamics of this bilayer model was worked out in a time-dependent Hartree-Fock-Gor'kov mean-field approximation, within the usual weak-coupling BCS scenario. We considered the case of either *s*-wave[6] or *d*-wave pairing in the two superconducting layers and derived approximate self-consistent equations for the gap function in Sec. II.

In the rest of the paper, we discuss the fluctuations of these static solutions using linear response theory. As expected, we find two kinds of collective mode in our coupled bilayer model. The complex order parameters (phase and amplitude) of the two layers can either fluctuate in phase or out of phase with respect to each other. In addition to the well-known in-phase Anderson-Bogoliubov phase and Littlewood-Varma amplitude modes, we predict out-of-phase modes. The energies of these modes are strongly dependent on whether the tunneling strength T_J is larger or smaller than the layer pairing interaction g. We compare the results for *s*-wave and $d_{x^2-y^2}$-wave pairing in the layers and find that these out-of-phase collective modes are basically the same in both cases in the low-frequency region ($\omega \ll \Delta$).

The out-of-phase modes are a feature of the two-layer model and thus of special experimental interest as an indication of the role of interlayer Cooper-pair tunneling.[6] The new out-of-phase fluctuations in phase and amplitude may be described as an "internal dynamic Josephson effect."[16] The out-of-phase phase mode of two coupled order parameters was discussed at length by Leggett[16] in the context of a two-band (*s*- and *d*-electron) model of a bulk superconductor. Much of Leggett's analysis and discussion can be applied to our bilayer model.

We also generalize the discussion to a trilayer system, which can arise in Bi, Tl, and Hg Cooper oxides (see the Appendix). As expected, there are three collective-mode branches for both phase and amplitude fluctuations exhibited in the trilayer system. In addition to the branch I which corresponds to the fluctuations in which order parameters on all three layers oscillate in phase, we find another two "out-of-phase" branch modes. One branch, in analogy with the branch-II out-of-phase modes in bilayer system, involves only the out-of-phase fluctuations of the order parameter in the outer layers, while the order parameter in the middle layer is undisturbed. Another branch corresponds to fluctuations in which the order parameters of the two outer layers oscillate in phase, while the order parameter of the middle layer oscillates out of phase to the two side layers. The analog of these three branches have been discussed in Ref. 8 in the normal phase ($T > T_c$).

We have not considered the excitonlike modes[17] in this paper. However, when *s*-wave and *d*-wave pairing interactions are both present (with one larger than the other), there will be excitonlike modes (two branches) in addition to the amplitude and phase modes discussed here for bilayers. For further discussion of such excitonlike mode in a two-dimensional *d*-wave superconductor, see Ref. 7.

In Ref. 6, we briefly discussed experimental techniques which might be used to study the out-of-phase modes we have predicted to exist in a superconducting bilayer. One needs a probe which is sensitive on a scale of the separation d of the two sheets. One possibility is inelastic light scattering, the theory of which has been extensively developed for semi-infinite metallic superlattices in the normal state (see, for example, Ref. 19). The intensity of this Raman scattering involves a weighted sum of the density response functions χ_{ρ_1,ρ_1}, χ_{ρ_2,ρ_2}, and χ_{ρ_1,ρ_2}, as given by (37) for a superconducting bilayer. Similarly the electron energy loss scattering (EELS) cross section involves a (different) weighted sum of these response functions.[19] In a future publication,[20] we will use the results of the present paper to calculate these cross sections, which are sensitive to the spacing of the sheets and show resonances associated with the zeros of $D_\pm$ in (33).

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APPENDIX: TRILAYER SYSTEM

The generalization from a bilayer to trilayer system is straightforward. In this appendix, we briefly sketch the final results (for more details, see Ref. 11). We consider the isotropic T_J and s-wave pairing case, which contains the essential physics. We consider Cooper-pair tunneling only between adjacent layers but include the Coulomb interaction between all layers. The various physical quantities (e.g., density of states, Fermi energy, gap, pairing interaction, etc.) are taken to be the same for both outer layers (by symmetry), but allowed to be different in the middle layer.[21,22] In this appendix, we use the index 1 for either of the two outer layers and 0 for the middle layer.

Following our analysis for the bilayer system, we find the matrix for the various BCS noninteracting response functions [compare with (29)]

$$\chi^0 = \begin{pmatrix} A_1 & 0 & 0 & -2c_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_0 & 0 & 0 & -2c_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_1 & 0 & 0 & -2c_1 & 0 & 0 & 0 \\ -2c_1 & 0 & 0 & -B_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2c_0 & 0 & 0 & -B_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2c_1 & 0 & 0 & -B_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -C_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -C_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -C_1 \end{pmatrix}, \tag{A1}$$

where various functions in (A1) involving outer or middle layers are analogous to (30) with $\ell = k = 0$ and $E \to E_i = \sqrt{\epsilon_i^2 + |\Delta_i|^2}$ $(i = 0, 1)$, etc., corresponding to middle $(i = 0)$ or outer $(i = 1)$ layers. The interaction matrix $\mathcal{V}$ is found to be [compare with (31)]

$$V = \begin{pmatrix} -2v_1 & -2v_{10} & -2v_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ -2v_{10} & -2v_0 & -2v_{10} & 0 & 0 & 0 & 0 & 0 & 0 \\ -2v_{11} & -2v_{10} & -2v_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_{C1} & T_J & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & T_J & g_{C0} & T_J & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & T_J & g_{C1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & g_{C1} & T_J & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & T_J & g_{C0} & T_J \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_J & g_{C1} \end{pmatrix}, \tag{A2}$$

where, for the layer i, $2v_i \equiv 2v_{2D} + g_{Zi}$ is the sum of the intralayer Coulomb Hartree interaction and short-range exchange p-h interaction and g_i is the p-p channel pairing interaction; $v_{10}(\mathbf{q}) = v_{2D}(\mathbf{q})e^{-qd}$ is the Coulomb interaction between adjacent layers and $v_{11}(\mathbf{q}) = v_{2D}(\mathbf{q})e^{-2qd}$ is the Coulomb interaction between the two outer layers (see also Ref. 8).

Using (A1) and (A2) in the mean-field analysis of Sec. III, we find

$$\det(\mathcal{I} + \chi^0\mathcal{V}) = DD_-D'D'_-, \tag{A3}$$

where

$$\begin{aligned} D \equiv & \left\{1 - 2\left[v_0A_0 + (v_1 + v_{11})A_1\right] + 4\left[v_0(v_1 + v_{11}) - 2(v_{01})^2\right]A_0A_1\right\}\left[\left(1 - g_0B_0\right)\left(1 - g_1B_1\right) - 2T_J^2B_0B_1\right] \\ & -8(g_0g_1 - 2T_J^2)\left[(v_1 + v_{11})B_0c_1^2 + v_0B_1c_0^2\right] + 8\left[v_0g_0c_0^2 + 4v_{01}T_Jc_0c_1 + (v_1 + v_{11})g_1c_1^2\right] \\ & -16\left[v_0(v_1 + v_{11}) - 2(v_{01})^2\right]\left[g_0(1 - g_1B_1)A_1c_0^2 + g_1(1 - g_0B_0)A_0c_1^2\right. \\ & \left.+2T_J^2(A_0B_0c_1^2 + A_1B_1c_0^2) - 4(g_0g_1 - 2T_J^2)c_0^2c_1^2\right], \end{aligned} \tag{A4}$$

$$D_- \equiv \left[1 - 2(v_1 - v_{11})A_1\right]\left[1 - g_1B_1\right] + 8(v_1 - v_{11})g_1c_1^2, \tag{A5}$$

$$D' \equiv (1 - g_0C_0)(1 - g_1C_1) - 2T_J^2C_0C_1, \tag{A6}$$

$$D'_- \equiv 1 - g_1C_1. \tag{A7}$$

It is clear from (A3) that the collective modes will be given by the zeros of the real parts of these functions. As three layers are involved, one expects to have three collective-mode branches for both phase and amplitude fluctuations of the order parameter. In this appendix, we summarize our main results for the trilayer system (in the absence of superconductivity, these reduce to those of Ref. 8).

The factors D_- and D'_- in (A5) and (A7), in analogy with the branch-II out-of-phase modes in bilayer system, involve only the fluctuations of the order parameter in the outer layers (i.e., only layers with index 1). These correspond to the phase and amplitude fluctuations in which the order parameters on the two outer layers have out-of-phase fluctuations while the middle layer is undisturbed (we call this branch II). To be more explicit, the solutions of $\mathrm{Re}D_- = 0$ correspond to a phase fluctuation and $\mathrm{Re}D'_- = 0$ correspond to an amplitude fluctuation. Both exhibit an optical-phonon dispersion relation, although with a different energy gap. The size of the energy gap strongly depends on the relative magnitude of T_J and the pairing interaction g_i, just as we found for the out-of-phase branch-II phase and amplitude modes in a bilayer system in Sec. IV.

In contrast, the zeros of D and D' in (A4) and (A6) give rise to another set of phase and amplitude modes. Of these two branches, we find[11] that one corresponds to the fluctuations in which order parameters on *all three* layers oscillate in phase (we refer to this as branch I). Another branch corresponds to fluctuations in which the order parameters of the two outer layers oscillate in phase, while the order parameter of the *middle* layer oscillates out of phase to the two side layers (we refer to this as branch III). Due to all the order parameters on three layers oscillating in phase, the branch I phase fluctuation [i.e., the well-known Anderson-Bogoliubov phonon mode with $\omega \sim q$] in a neutral superconductor (which is independent of T_J and g_i) will be shifted to a 2D plasmon ($\omega \sim \sqrt{q}$) by the Coulomb interaction. In contrast, the branch III phase mode, involving the out-of-phase oscillations of the order parameter on the two outer layers relative to the middle layer, has an optical-phonon dispersion relation. Again, the size of the energy gap of the branch III optical phonons is strongly dependent on the relative magnitude of T_J and the pairing interaction g_i.

For the amplitude modes, which are not affected by the Coulomb interaction, we also find an optical-phonon-like dispersion relation for both branches I and III (similar to the amplitude modes in the bilayer system). The magnitude of the energy gap of the branch I amplitude mode, which depends on T_J and g_i, corresponds to an appropriate Fermi surface average of $2|\Delta_{\mathbf{p}}|$, which is the minimum energy required to break up a Cooper pair.[11]

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