Collective modes with a sound spectrum in layered superconductors

Robert A. Vardanian
Solid State Division, Institute of Radiophysics & Electronics, Armenian Academy of Sciences, Ashtarack-2, 378410, Armenia
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The collective-mode spectrum for a layered superconductor is considered here. The Hamiltonian describes electrons in a metallic layer in the effective-mass approximation and the coupling of neighboring layers by a Josephson-type term. In this model, electrodynamic equations are obtained that are valid in the whole range of the nonmagnetic-impurities concentration. The velocity values of collective modes for two principal directions of the wave vector (along and across the superlattice axis) are found.

I. INTRODUCTION

In conventional superconductors including nonmagnetic impurities, collective modes related with the order parameter phase oscillations were predicted in papers.[1-3] These oscillations, which weakly damp near the transition temperature were detected in the experiments of Carlson and Goldman[4] (the so-called "CG modes," see review[5] and refs. therein).

In this paper analysis of such modes is given in layered superconductors, such as transition-metal dichalcogenides intercalated by organic molecules, or in high-temperature oxides. Dynamic properties of electronic subsystem play an important role in our understanding of superconductivity in conventional superconductors, and they may serve as a key to the mechanism of high-temperature superconductivity (HTS). However, HTS materials differ from conventional superconductors in many ways.[6] Besides sufficiently high transition temperatures, these systems (e.g., La-Sr-Cu-O and Y-Ba-Cu-O) generally also have a pronounced layered structure, so that the interlayer distance $c \gg a,b$. The dispersion relation $\varepsilon(\mathbf{p})$ is highly anisotropic, where $\mathbf{p}=\{\mathbf{p}_\parallel,p_z\}$ is a quasimomentum ($\mathbf{p}_\parallel$ is a two-dimensional momentum); the z axis being chosen normal to the layers. As a first approximation, one can assume cylindrical Fermi surface, which corresponds to the neglect of interlayer transitions. However, the latter bring too small deviations of Fermi surface from cylindrical shape. Following Lawrence and Doniach,[7] we describe the conduction electrons of a layered superconductor by a Hamiltonian which treats the electrons within metallic layer in the effective-mass approximation and couples the neighboring layers by means of a Josephson-type term. In this model, the z dependence of the single-particle states has a tight-binding form, so that the dispersion relation is

$$\varepsilon(\mathbf{p})=\mathbf{p}_\parallel^2/2m+\eta(1-\cos p_z c)\ , \tag{1}$$

where m is the effective mass of electron in two-dimensional (2D) band, η is half-width of the energy band corresponding to motion across the layers, $|p_z|\le\pi/c$. This dispersion relation is a result of a cylinder-symmetric Fermi surface of an "hour-glass" shape, where the radius of the "waist" ($p_z=0$) is determined by the interlayer hopping probability η. Absence of hopping between the layers ($\eta=0$) would correspond to a strictly cylindrical Fermi surface of a 2D system. The dispersion law (1) was widely used in consideration of various properties both of conventional[8] and high-temperature[9] superconductors. It would be useful to generalize the consideration of collective modes for layered compounds, in particular for high-T_c superconducting oxides.

In this paper we consider the conditions of collective mode propagation both along and across the superlattice axis, on the basis of the BCS model. We assume the BCS s-wave intralayer pairing with one layer per unit cell, and neglect the pairing of electrons in different layers, in the spirit of the BCS theory for bulk superconductors, where interaction is assumed to be pointlike. Thus, this model describes a series of "two-dimensional" superconductors coupled only through the interlayer electron tunneling.

II. BASIC EQUATIONS

We shall use the Gor'kov equations[10] for the Green's functions to construct the theory of linear response of superconductor to longitudinal electric field.[11] Here, since the calculations will be carried out in an orbitrary gauge, with the use of two gauge-invariant potentials (scalar $\Phi=\phi+\dot{\chi}$ and vector $\mathbf{Q}=\mathbf{A}-\nabla\chi$), we shall need a set of two equations for determination of these quantities. The first equation is for current $j_\omega(\mathbf{k})$ [or charge density $\rho_\omega(\mathbf{k})$]. Here ω and $\mathbf{k}$ are, respectively, the frequency and wave vector of the electric field. As the second equation, we may use the continuity equation. However, since the latter is identical in a superconductor with the self-consistent equation for the phase of order parameter $\chi_\omega(\mathbf{k})$ ($\Delta_1-\Delta_1^*=4ie\chi$), we may use this equation, which significantly reduces the volume of calculations. Here and below we use the natural units $\hbar=c=k_B=1$. Thus, the necessary equations in the Matsubara technique are written in the form

$$j_\omega(\mathbf{k})=2eT\sum_\varepsilon\int vG^+_{1\varepsilon_+\varepsilon_-}(\mathbf{p}_+\mathbf{p}_-)\frac{d^3p}{(2\pi)^3}\ , \tag{2}$$

$$\Delta_1-\Delta_1^*=2|g|T\sum_\varepsilon\int[F_{1\varepsilon_+\varepsilon_-}(\mathbf{p}_+,\mathbf{p}_-) - F^+_{1\varepsilon_+\varepsilon_-}(\mathbf{p}_+,\mathbf{p}_-)]\frac{d^3p}{(2\pi)^3}\ . \tag{3}$$

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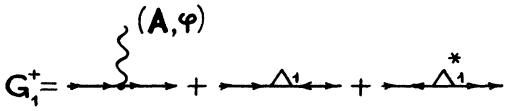

FIG. 1. First-order correction to the thermodynamically equilibrium Green's function G_0^+.

Here $|g|$ is the BCS effective electron-electron coupling constant, T is temperature, $\varepsilon_\pm=\varepsilon\pm\omega/2$, $\mathbf{p}_\pm=\mathbf{p}\pm\mathbf{k}/2$. We shall denote the vertex with electromagnetic potentials A and φ by a point with a wavy line ($\mathbf{k}\|A$). Then the graphical representation of the first-order correction G_1^+ to the free Green's function G_0^+ will assume the form shown in Fig. 1. Similar graphical representation exists also for the functions G_1^-, F_1^+, and F_1.

Equations (2) and (3) must be continued analytically in ε and ω to the real axis, and also averaged over the impurities. Here we shall use the technique developed by Gor'kov and Eliashberg,[12,13] without however imposing any limitation to the mean free path l, other than the condition $p_F l \gg 1$, which enables us to take only the ladder diagrams into account in averaging over the impurities. The technique of averaging over the positions of impurities, suggested by Gor'kov and Eliashberg,[13] allows us to obtain the result much more economically than when the well-known procedure[10] is used. It consists in that the averaging over the impurities of superconducting Green's function is reduced in a definite fashion to the averaging of Green's functions of the normal metal. In this case, four types of vertices are to be averaged for Eqs. (2) and (3)

$$\Gamma_i(\mathbf{k},\Omega_{mn})=\left\langle \int \frac{d^2\mathbf{p}_\parallel}{(2\pi)^2}\int_{-\pi/c}^{\pi/c}\frac{dp_z}{2\pi}f_i(v)G^R_{\xi_m}(\mathbf{p}_+) \times G^A_{\xi_n}(\mathbf{p}_-)\right\rangle_{\text{imp}}+\text{c.c.} \qquad (4)$$

$$i=1,2,3,4\ .$$

Here in agreement with Ref. 13

$$G^{R(A)}_\xi(\mathbf{p})=\frac{1}{\xi-\varepsilon(\mathbf{p})+\mu\pm l/2\tau} \qquad (5)$$

is the retarded (advanced) Green's function of the normal metal, μ is the Fermi energy, and τ is the lifetime of electron due to scattering on impurities, and

$$f_1=1,\ f_2=(v\cdot n),\ f_3=v,\ f_4=v(v\cdot n);\ n=A/A\ ; \qquad (6)$$

$$v=\partial\varepsilon(p)/\partial p,\quad v=(v_\parallel,v_z);\quad v_z=v_\perp\sin(p_z c)\ ,$$

$$v_\perp=\max v_z=\eta c\ .$$

The bracket $\langle\cdots\rangle_{\text{imp}}$ denotes an average over the impurities of the two Green's functions product. In the diagrams, we denote the potential of the impurity by a cross. The crosses which refer to the same impurity are connected by a dashed line. Then, in the ladder approximation, for vertices Γ_2 and Γ_4 we have the integral equations represented by diagrams in Fig. 2.

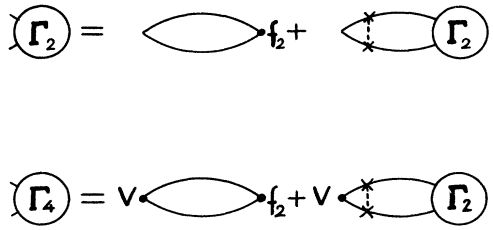

FIG. 2. Diagrammatic representation of the integral equations of two Green's functions product in the presence of impurities for vertices Γ_2 and Γ_4.

Omitting the calculations, we write down the result for two orientations of wave vector $\mathbf{k}_\sigma=(\mathbf{k}_\parallel,\mathbf{k}_\perp)$, along and across the layers:

$$\Gamma_1^\sigma=N(0)\left[\frac{2\pi\beta(a_\sigma)}{Dk_\sigma^2-i\Omega_{mn}}+\text{c.c.}\right], \qquad (7)$$

$$\Gamma_3^\sigma=n\Gamma_2^\sigma=-nN(0)v_\sigma\left[\frac{i\pi a_\sigma g(a_\sigma)}{Dk_\sigma^2-i\Omega_{mn}}+\text{c.c.}\right], \qquad (8)$$

$$\Gamma_4^\sigma=-nN(0)v_\sigma^2\left[\frac{i\pi\Omega_{mn}\tau}{Dk_\sigma^2-i\Omega_{mn}}+\text{c.c.}\right]. \qquad (9)$$

Here the following notations are used

$$\begin{aligned} &g(a_\sigma)=2[1-\beta(a_\sigma)]/a_\sigma^2;\ \beta(a_\sigma)=(1+a_\sigma^2)^{-1/2};\\ &a_\sigma=k_\sigma v_\sigma\tilde\tau;\ \tilde\tau=\tau(1-i\Omega_{mn}\tau)^{-1};\\ &D=D_\sigma g(a_\sigma)(1-i\Omega_{mn}\tau)^{-2};\ \Omega_{mn}=\xi_m-\xi_n\ ;\\ &D_\parallel=v_\parallel^2\tau/2;D_\perp=\eta^2c^2\tau/2\ ; \end{aligned} \qquad (10)$$

and

$$1/\tau=2\pi n_{\text{imp}}N(0)V_0^2\ , \qquad (11)$$

where n_{imp} is the concentration of impurities, $N(0)=m/2\pi c$ is the single-particle density of states at the Fermi surface for a single spin in layered superconductor, V_0 is the constant of impurity potential. Note that we assumed $\eta\ll\mu=p_F^2/2m$ to obtain expressions (7)–(9). Therefore integration over $\mathbf{p}_\parallel$ was carried out near the two-dimensional Fermi surface.[8]

Upon averaging over the impurities and analytic continuation in ε and ω, with account of (7)–(9), as well as expressions for thermodynamically equilibrium Green's functions of superconductor[10]

$$G^\pm_{0\varepsilon}(\xi_\mathbf{p})=\mp\frac{\varepsilon\pm\xi_\mathbf{p}}{\xi_\mathbf{p}^2-\xi_\varepsilon^2};F_{0\varepsilon}(\xi_\mathbf{p})=F^+_{0\varepsilon}(\xi_\mathbf{p})=\frac{\Delta}{\xi_\mathbf{p}^2-\xi_\varepsilon^2}\ ;$$

$$\xi_\varepsilon=(\varepsilon^2-\Delta^2)^{1/2};\ \xi_\mathbf{p}=\varepsilon(\mathbf{p})-\mu\simeq v_F(|\mathbf{p}_\parallel|-\mathbf{p}_F)\ .$$

Equations (2) and (3) assume the following form (we have omitted the orientation index σ of the wave vector):

$$\mathbf{j}_\omega(\mathbf{k})=2e^2\int\frac{d\varepsilon}{2\pi i}\int\frac{d\xi_m d\xi_n}{(2\pi i)^2}(\xi_m^2-\xi_+^2)^{-1}(\xi_n^2-\xi_-^2)^{-1}$$
$$\times\{A[(\varepsilon_++\xi_m)(\varepsilon_-+\xi_n)+\Delta^2]\Gamma_4+\varphi[(\varepsilon_++\xi_m)(\varepsilon_-+\xi_n)-\Delta^2]\Gamma_3+2\chi\Delta^2[\varepsilon_++\xi_m+\varepsilon_-+\xi_n]\Gamma_3\}\ , \tag{12}$$

$$\Delta_1-\Delta_1^*=2i\lambda\int\frac{d\varepsilon}{2\pi i}\int\frac{d\xi_m d\xi_n}{(2\pi i)^2}(\xi_m^2-\xi_+^2)^{-1}(\xi_n^2-\xi_-^2)^{-1}$$
$$\times\{eA\Omega_{mn}\Delta\Gamma_2+e\varphi\omega\Delta\Gamma_1+(\Delta_1-\Delta_1^*)(\xi_m\xi_n-\varepsilon_+\varepsilon_-+\Delta^2)\Gamma_1\}\ , \tag{13}$$

where $\lambda=|g|N(0)$, $\xi_+=\xi_{\varepsilon+}$, $\xi_-=\xi_{\varepsilon-}$. Here and below, all integrals over ε must be understood as analytically continued, i.e.,

$$\int d\varepsilon F(\xi_+,\xi_-)=\int d\varepsilon\left\{\tanh\frac{\varepsilon_+}{2T}F(\xi_+^R,\xi_-^R)-\tanh\frac{\varepsilon_-}{2T}F(\xi_+^A,\xi_-^A)-\left[\tanh\frac{\varepsilon_+}{2T}-\tanh\frac{\varepsilon_-}{2T}\right]F(\xi_+^R,\xi_-^A)\right\}\ , \tag{14}$$

where

$$\xi_\varepsilon^R=-(\xi_\varepsilon^A)^*=\begin{cases}\mathrm{sgn}\varepsilon(\varepsilon^2-\Delta^2)^{1/2}, & \varepsilon^2>\Delta^2\ ,\\ i(\Delta^2-\varepsilon^2)^{1/2}, & \varepsilon^2<\Delta^2\ .\end{cases} \tag{15}$$

Expressions (12) and (13) are easily integrated over ξ_m and ξ_n, by calculating the corresponding residues. Using the following relations

$$\Omega[\varepsilon_+/\xi_+-\varepsilon_-/\xi_-]=\omega[1-(\varepsilon_+\varepsilon_-+\Delta^2)/\xi_+\xi_-]$$
$$\Omega[1-(\varepsilon_+\varepsilon_--\Delta^2)/\xi_+\xi_-]=\omega[\varepsilon_+/\xi_+-\varepsilon_-/\xi_-] \tag{16}$$

as well as

$$\int d\varepsilon[\varepsilon_+/\xi_+-\varepsilon_-/\xi_-]=4\omega;\quad\int d\varepsilon\Omega/(\xi_+\xi_-)=4\lambda^{-1}\ , \tag{17}$$

we obtain, after straightforward but cumbersome calculations

$$\mathbf{j}_\omega(\mathbf{k})=\frac{\sigma}{4}\int\frac{d\varepsilon}{1-i\Omega\tau}\left[Q\Omega\left[1-\frac{\varepsilon_+\varepsilon_-+\Delta^2}{\xi_+\xi_-}\right]-\mathbf{k}\Phi\left[\frac{\varepsilon_+}{\xi_+}-\frac{\varepsilon_-}{\xi_-}\right]\right]\frac{g(a)}{Dk^2-i\Omega}\ , \tag{18}$$

$$DikQ\int\frac{d\varepsilon}{\xi_+\xi_-}\frac{\Omega}{1-i\Omega\tau}\frac{g(a)}{Dk^2-i\Omega}+\omega\Phi\int\frac{d\varepsilon}{\xi_+\xi_-}\frac{\beta(a)}{Dk^2-i\Omega}=0 \tag{19}$$

in which g, β, and D are given by formulas (10), $\Omega=\xi_++\xi_-$, and

$$Q=A-\nabla\chi,\quad\Phi=\varphi+\dot\chi \tag{20}$$

are the generalized gauge-invariant vector and scalar potentials.

III. COLLECTIVE MODES

The equations for current (18) and order parameter phase (19) are valid in the whole range of nonmagnetic impurities concentration, as well as for any orientations of the wave vector $\mathbf{k}$, both along and across superlattice axis. The conductivity σ is connected with diffusion coefficient by the Einstein's relation[14]

$$\sigma_{\parallel,\perp}=2e^2N(0)D_{\parallel,\perp}\ . \tag{21}$$

In our model, the natural anisotropy parameter is

$$\gamma=(\sigma_\perp/\sigma_\parallel)^{1/2}=(D_\perp/D_\parallel)^{1/2}=\eta c/v_\parallel\ll1\ . \tag{22}$$

Further, we shall consider the solution of the set of equations (18) and (19) in the case when the current of normal electrons is equal and oppositely directed to the current of superconducting pairs, so that the total current $\mathbf{j}_\omega(\mathbf{k})=0$. Below we limit our consideration by a dirty limit and weak spatial dispersion

$$D_\sigma k_\sigma^2\ll\omega\ll\Delta\ll\tau^{-1}\ . \tag{23}$$

In this case $\beta(a)=g(a)=1$ and the system of homogeneous equations for Q and Φ assumes the form

$$\omega\mathbf{Q}\int d\varepsilon[1-(\varepsilon_+\varepsilon_-+\Delta^2)/\xi_+\xi_-]-\mathbf{k}\Phi\int d\varepsilon[1-(\varepsilon_+\varepsilon_--\Delta^2)/\xi_+\xi_-]=0\ ,$$
$$iD\mathbf{k}Q\int\frac{d\varepsilon}{\xi_+\xi_-}+\omega\Phi\int\frac{d\varepsilon}{\xi_+\xi_-}\frac{1}{\xi_++\xi_-}=0\ . \tag{24}$$

Evaluating integrals over energy ε near the transition temperature T_c and setting the determinant of system (24) equal to zero, for the dispersion law of modes we obtain

$$\omega^2\left[1+\frac{\delta}{\pi\Delta}\ln\frac{8\Delta}{\omega}\right]-s^2k^2+i\delta\omega\left[1+\frac{s^2k^2}{\pi\delta\Delta}\ln\frac{8\Delta}{\omega}\right]=0\ , \tag{25}$$

where $\delta=\pi\Delta^2/2T$, $N_s/N=\delta\tau$ is a ratio of superconducting pair numbers to that of normal carriers,[5] and $s=(2D\Delta)^{1/2}$ is the wave velocity. As it follows from (25), in the case of weak attenuation ($\delta\ll\omega$) and under the neglect of pair breaking, i.e., when

$$\max\{\tau_\varepsilon^{-1},\delta\}<\omega<\min\{\Delta,\pi\Delta/\ln(8\Delta/\omega)\} \tag{26}$$

the acoustic dispersion law is valid. For two principal

orientations of wave vector, the velocity $s_\sigma=(2D_\sigma\Delta)^{1/2}$ according to (10) is given by

$$s_\parallel=v_\parallel(\Delta\tau)^{1/2} \text{ and } s_\perp=\eta c(\Delta\tau)^{1/2} . \quad (27)$$

The damping δ of collective modes near transition temperature T_c is small. It is easy to see that the ratio $s_\perp/s_\parallel$ is expressed through the same anisotropy parameter γ. In inequality (26) τ_ε is the inelastic relaxation time. The considered quasiequilibrium solution is valid when the condition $\omega\tau_\varepsilon\gg1$ holds and the dynamic pair-breaking effect is absent ($\omega<\Delta$). Therefore, the wave numbers satisfy the following inequality $\omega\sim(D_\sigma k_\sigma^2\Delta)^{1/2}<\Delta$, i.e.,

$$k_{\parallel,\perp}<(\Delta/D_{\parallel,\perp})^{1/2}=\xi_{\parallel,\perp}^{-1} ,$$

where $\xi_\parallel$ and $\xi_\perp=\gamma\xi_\parallel$ are, respectively, the coherence lengths parallel and normal to the layers.

IV. CONCLUSION

The issue of collective modes propagation in superconductors was widely discussed in the literature, starting from the creation of microscopic BCS theory. The excellent review on this topic has been made by Martin.[15]

As shown by Bogoliubov[16] and Anderson,[17] if the Coulomb interaction is neglected, the longitudinal collective acoustic-type oscillations with frequencies within the energy gap must originate in the electron system of superconductor. However, when the Coulomb interaction is taken into account, these oscillations turn into plasma oscillations,[18] which do not differ from those in normal metals. Strictly speaking, such a situation may occur only in superconductors with infinite conductivity in the normal state.

The mechanism considered in Refs. 1–3 and in the present work, substantially differs from the standard treatment.[15] Note that the modes obtained in the present paper are caused by velocity oscillations of the superfluid pairs (order-parameter phase). Meanwhile, the Bogoliubov-Anderson modes are due to oscillations of density $N_s\sim|\Delta|^2$, namely to oscillations of order parameter modulus. In addition, due to Landau-type collisionless attenuation in clean superconductors ($\omega\tau\gg1$), the collective modes treated in this work and in Refs. 1–3 are damped. Considering the equation of self-consistence for $\Delta_1+\Delta_1^*$, it is easy to find that in our case var $N_s\sim\Delta_1+\Delta_1^*=0$.

Above we limited our consideration only by the dirty case, when $\Delta\tau\ll1$, which according to (28) means that $l\ll\xi$. Though Eqs. (18) and (19) of linear response to the longitudinal electrical field are valid for any impurity concentrations, their application to HTS oxides, in the so-called clean limit $\Delta\tau\gg1$ (or $\xi\ll l$) should be made carefully. Because of the small coherence length, as well as complex band structure of some HTS materials, such as Y-Ba-Cu-O,[19] their multigap structure becomes very essential.[20]

Experimental methods of the so-called "CG-modes" observation are considered in detail by Schön.[5] Taking into account the typical values of parameters, one may say that the obtained results are applicable not only to conventional superconductors, but also to La-Sr-Cu-O-type HTS compounds.[21] One should note that La-Sr-Cu-O has a relatively simple band structure described by the one-band model.[20,21] For HTS Y-Ba-Cu-O-type oxides the simple theory presented here is not applicable without some modification. However, the development of the theory for two-band structures goes beyond the scope of this paper and requires special consideration.

Finally, we note that experimental revealing of anisotropy parameter $\gamma=(s_\perp/s_\parallel)$, which reflects the Fermi surface topology will allow us to unite the set of anisotropy properties displayed both in normal and superconducting states of layered superconductors. It is necessary also to point out the more expressed anisotropy of parameters of the normal state $D_\perp/D_\parallel=\sigma_\perp/\sigma_\parallel=\gamma^2$, as compared to that of the superconducting state $\xi_\perp/\xi_\parallel=s_\perp/s_\parallel=\gamma$.

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[1]R. A. Vardanian and S. G. Lisitsyn, Zh. Eksp. Teor. Fiz. **69**, 1267 (1975) [Sov. Phys. JETP **42**, 648 (1975)].

[2]A. Schmid and G. Schön, Phys. Rev. Lett. **34**, 941 (1975).

[3]S. N. Artemenko and A. F. Volkov, Zh. Eksp. Teor. Fiz. **69**, 1764 (1975) [Sov. Phys. JETP **42**, 896 (1975)].

[4]R. V. Carlson and A. M. Goldman, Phys. Rev. Lett. **34**, 11 (1975); J. Low Temp. Phys. **27**, 67 (1976).

[5]G. Schön, in *Nonequilibrium Superconductivity,* edited by D. N. Langenberg and A. I. Larkin (North-Holland, Amsterdam, 1985), pp. 589–641.

[6]D. M. Ginsberg, in *Physical Properties of High Temperatures Superconductors,* edited by D. M. Ginsberg (World Scientific, Singapore, 1989), Chap. 1.

[7]W. E. Lawrence and S. Doniach, *Proceedings of the Twelfth International Conference on Low Temperatures Physics,* Kyoto, Japan, 1970 (Academic Press of Japan, Kyoto, 1971), p. 361.

[8]R. R. Gerhardts, Phys. Rev. B **9**, 2945 (1974); R. A. Klemm, A. Luther, and M. R. Beasley, *ibid.* **12**, 877 (1975); R. A. Klemm and K. Scharnberg, *ibid.* **24**, 6361 (1981).

[9]W. Pint, E. Langman, and E. Schachinger, Physica C **157**, 415 (1989); T. Schneider and M. R. Sörensen, Z. Phys. B **80**, 331 (1990); *ibid.* **81**, 3 (1990); C. Jiang and J. P. Carbotte, J. Low Temp. Phys. **87**, 95 (1992); Phys. Rev. B **48**, 3853 (1993).

[10]A. Abrikosov, L. Gor'kov, and I. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Englewood Cliffs, NJ, 1963).

[11]The dielectric response of layered superconductor to the transversal electromagnetic fields has been discussed by J. J. Chang and D. Scalapino, Phys. Rev. B **40**, 4299 (1989).

[12]L. Gor'kov and G. Eliashberg, Zh. Eksp. Teor. Fiz. **54**, 612

(1968) [Sov. Phys. JETP **27**, 328 (1968)].
[13]L. Gor'kov and G. Eliashberg, Zh. Eksp. Teor. Fiz. **56**, 1297 (1969) [Sov. Phys. JETP **29**, 698 (1969)].
[14]See, for example, A. Abrikosov, *Fundamentals of the Theory of Metals* (North-Holland, Amsterdam, 1988).
[15]P. Martin, in *Superconductivity,* edited by R. D. Parks (Dekker, New York, 1969), pp. 371–391.
[16]N. N. Bogoliubov, Nuovo Cimento **7**, 794 (1958).
[17]P. W. Anderson, Phys. Rev. **110**, 827 (1958).
[18]Note that recently a Bogoliubov-Anderson-like mode has been considered in layered superconductors by H. A. Fertig and S. Das Sarma, Phys. Rev. Lett. **65**, 1482 (1990); Phys. Rev. B **44**, 4480 (1991). They have shown that for wave vector directions close to the superlattice axis, as well as when tunneling between layers is ignored, the plasmon mode remains below the superconducting gap edge.
[19]W. Pickett, Rev. Mod. Phys. **61**, 433 (1989).
[20]V. Z. Kresin and S. A. Wolf, Physica C **169**, 476 (1990).
[21]V. Z. Kresin and S. A. Wolf, Phys. Rev. B **41**, 4278 (1990).