Entangled electronic states in multiple-quantum-dot systems

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We present an analytically solvable model of P colinear, two-dimensional quantum dots, each containing two electrons. Interdot coupling via the electron-electron interaction gives rise to sets of entangled ground states. These ground states have crystallike interplane correlations and arise discontinously with increasing magnetic field. Their ranges and stabilities are found to depend on dot size ratios, and to increase with P.

Recent experimental and theoretical interest in quantum-dot systems has opened up the fascinating area of highly correlated, few-body quantum phenomena in the traditionally large-N field of semiconductor physics. Complex ground state behavior as a function of magnetic field has been predicted for single two-dimensional (2D) quantum dots containing N electrons.[1] Even for as few as $N = 2$ electrons per 2D dot, "magic number" ground state transitions are predicted as a function of magnetic field as a result of the competition between (single-electron) confinement energy and (many-body) electron-electron interactions.[2,3] Remarkably, such transitions for $N = 2$ have recently been observed experimentally.[3,4] Arrays of coupled dots[5–8] have been attracting increasing attention, partly because of the possibility of application as ultrasmall logic gates.[9–11] Adjacent dots can be coupled by "optical wiring," i.e., coupled by the two-body electron-electron interaction between electrons in adjacent dots, which can be nonzero even in the *absence* of a single-body tunneling term.[12] Such few-electron coupled 2D systems are also interesting in that they can represent the small-N analogs of coupled, parallel 2D electron gases.[13] Given the complex ground state behavior in a single 2D dot as a function of B field, the coupled 2D dot system offers the interesting possibility of competition between electron correlations on the same dot (i.e., intradot interactions V_{intra}) and correlations between adjacent dots (i.e., interdot interactions V_{inter}).

This paper predicts the existence of entangled ground states with crystallike interdot correlations in multiple-quantum-dot systems. These ground states occur discontinously with increasing magnetic field, being interdispersed with states having negligible interdot correlations. The ranges and stabilities of the crystallike states depend on dot size ratios and *increase* with the number of dots P. Our model is solved analytically and consists of P colinear 2D quantum dots, each containing two electrons but not necessarily identical in size. The model considers a sufficient number of electrons as to contain both interdot and intradot electron-electron interactions, and yet still admit analytic solutions. These analytic solutions implicitly include mixing with all Landau levels.

Figure 1 shows our system for a pair of dots ($P = 2$). The dots are arranged vertically with separation s. This vertical geometry is of specific experimental interest given the possibility of fabrication via etching of a multiple-quantum-well structure (see Ref. 14 for $P = 2$). Following several single-dot studies,[1] we model each of the P dots by 2D (xy plane) parabolic potentials with a perpendicular magnetic field B (z direction) of sufficient strength to spin polarize the electrons. Electrostatic confinement in the z direction is taken to be sufficiently strong that the electrons are frozen in the lowest z subband. We take the electron-electron interaction potential to be of inverse-square form, $\frac{\beta}{r^2}$ where β is a positive parameter; this interaction for a single layer gives quantitatively similar results to the bare Coulomb interaction.[15,16] Interdot tunneling is assumed to be negligible. The Hamiltonian for the system of P dots is (with a symmetric gauge) $H = H_0 + V$ with

$$H_0 = \sum_{\alpha=1,2,\ldots,P;i=1,2} \left(\frac{\mathbf{p}_{\alpha,i}^2}{2m^*} + \frac{1}{2} m^* \omega_\alpha^2(B) |\mathbf{r}_{\alpha,i}|^2 + \frac{\omega_c}{2} L_{\alpha,i} \right). \tag{1}$$

$\mathbf{p}_{\alpha,i}$ and $\mathbf{r}_{\alpha,i}$ are the momentum and position of electron i in dot α. Each electron has effective mass m^* and z component of angular momentum $L_{\alpha,i}$. The cyclotron frequency is ω_c and $\omega_\alpha^2(B) = \omega_{0\alpha}^2 + \omega_c^2/4$. The electrostatic confining potential $\omega_{0\alpha}$ is in general different for each dot (N.B. the dot size $\sim \omega_{0\alpha}^{-1/2}$). The dominant interdot coupling is due to interactions between electrons on adjacent dots; we therefore take $V = V_{\text{intra}} + V_{\text{inter}}$

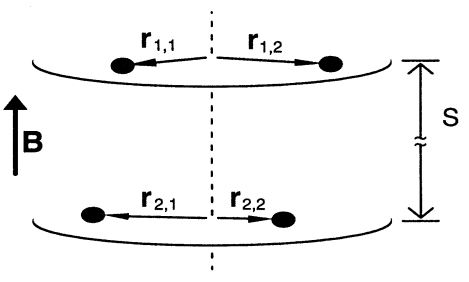

FIG. 1. Schematic illustration of the $P = 2$ dot system. Each dot contains two electrons.

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where

$$V_{\text{intra}} = \sum_{\alpha=1,2,\ldots,P} \frac{\beta}{|\mathbf{r}_{\alpha,1} - \mathbf{r}_{\alpha,2}|^2} \tag{2}$$

and

$$V_{\text{inter}} = \sum_{\alpha=2,3,\ldots P} \sum_{i,j} \frac{\beta}{|\mathbf{r}_{\alpha,i} - \mathbf{r}_{(\alpha-1),j}|^2 + s^2} \,. \tag{3}$$

For $V_{\text{inter}} \to 0$, the exact eigenstates of H are products of P single-dot states.[15] For finite V_{inter}, no exact analytic solutions are known. Our approach is to Taylor expand V_{inter} under the assumption that $|\mathbf{r}_{\alpha,i} - \mathbf{r}_{(\alpha-1),j}|^2 < s^2$. Having solved the resulting problem, we can check from the analytic expressions for (e.g.) $\frac{r^2}{s^2}$ that any particular set of dot parameters is consistent with this assumption. Hence

$$V_{\text{inter}} = \frac{\beta}{s^2} \sum_{\alpha=2,3,\ldots,P} \sum_{i,j} \sum_{k=0}^{\infty} (-1)^k \times \left(\frac{|\mathbf{r}_{\alpha,i} - \mathbf{r}_{(\alpha-1),j}|^2}{s^2} \right)^k . \tag{4}$$

The exact analytic solution of H is now possible including terms of order $\beta r^2/s^4$ in V_{inter}; (analytic) perturbation theory is then employed for the $\beta r^4/s^6$ terms.

We now discuss the method explicitly for $P = 2$. Employing an orthogonal transformation, with coefficients depending on the relative dot sizes ω_1 and ω_2, allows exact solution of H including terms of order $\beta r^2/s^4$ in V_{inter}. The only nontrivial contributions to H to this order are those which depend on the relative positions of the electrons within each dot. The corresponding eigenstates contain quantum numbers m_α (the relative angular momentum between the two electrons in dot α). The ground and low-lying states of the $P = 2$ system have all other quantum numbers zero; these states can be labeled $|m_1, m_2\rangle$ (signifying the direct product of $|m_1\rangle$ and $|m_2\rangle$) and are used as a basis for the $\beta r^4/s^6$ perturbation. Figure 2(a) shows the ground state transitions as a function of B field in the limit $s \to \infty$ (i.e., $V_{\text{inter}} \to 0$) for the case of equal dot sizes ($\omega_1 = \omega_2$). As a demonstration of the (lack of) interdot correlation, the insets show the charge density in a given dot with the electrons in the other dot fixed opposite one another at the crosses. The plots are angularly symmetric (i.e., negligible interdot correlation). The radial localization increases as the ground state m_α increases (i.e., as $B \to \infty$, each dot approaches its own classical limit). Figure 2(b) shows the corresponding diagram for finite s. The main feature is that new entangled[17] states arise as a result of finite V_{inter} (i.e., including terms to order $\beta r^4/s^6$). The density plots show the strong interdot correlation which characterizes the new entangled ground states.

These entangled states can be easily understood for equal dot sizes. First note that in the exactly solved (i.e., order $\beta r^2/s^4$) system the states $|m_a, m_b\rangle$ and $|m_b, m_a\rangle$ are degenerate. Secondly, the perturbation (i.e., order $\beta r^4/s^6$ terms) only mixes $|m_a, m_b\rangle$ and $|m_c, m_d\rangle$ if $m_a - m_c = m_d - m_b = 0$ or ± 2. The mixed (i.e., entangled) states are therefore $(|m+2, m\rangle \pm |m, m+2\rangle)$, which we write as $|m+2, m\rangle_+$ and $|m+2, m\rangle_-$. It is remarkable that as B increases, the ground state switches back and forth between "pure" states ($m_1 = m_2$) and entangled states (N.B. the $|m, m\rangle$ states are now not strictly pure because of a very small nondegenerate perturbation mixing.) For large B ($B > 10$ T), the entangled states prevent the pure states from becoming ground states. As $B \to \infty$, the classical limit is reached of four point charges situated at the corners of a square when projected onto the xy plane. An alternative view of the formation of these entangled states is as a resonance phenomenon; for example, the state $|3,1\rangle$ can be thought of as continually exchanging energy with $|1,3\rangle$ via virtual photons (i.e., via the electromagnetic field representing the electron-electron interaction). This is equivalent to the resonant Forster process, which is well known as an energy transfer mechanism for biological molecules and proteins. In excitonic language, the states $|3,1\rangle$ and $|1,3\rangle$ correspond to the $|1,1\rangle$ vacuum plus an electron-hole excitation (exciton) of angular momentum 2 on dots 1 and 2, respectively; the formation of the entangled state $|3,1\rangle_-$ represents the resonance between these two adjacent excitons.

Figures 3(i a) and 3(i b) show in detail the region near the $|1,1\rangle$ and $|3,3\rangle$ crossover discussed above. Figures 3(ii a) and 3(ii b) show the corresponding region for the case of unequal dot sizes ($\omega_1 \neq \omega_2$). In this case $|m_a, m_b\rangle$ and $|m_b, m_a\rangle$ are no longer degenerate. However, by keeping $|\omega_1 - \omega_2| << \omega_1$, we can arrange that the two

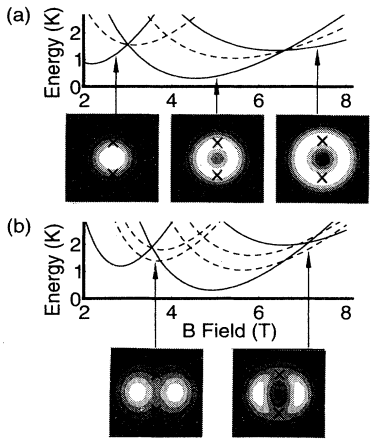

FIG. 2. (a) Energies (in Kelvin relative to an arbitrary zero) of low-lying eigenstates $|m_1, m_2\rangle$ as a function of B for $P = 2$ dot system in limit $s \to \infty$. Lowest curve at a given B corresponds to the ground state. From left to right the solid lines represent successive ground states $|1,1\rangle$, $|3,3\rangle$, and $|5,5\rangle$; dashed lines correspond to the degenerate pairs ($|3,1\rangle$, $|1,3\rangle$) and ($|5,3\rangle$, $|3,5\rangle$). Contour plots are ground state charge densities in a given dot with electrons in the other dot fixed at the crosses. (b) As (a), but with *finite* s. The degenerate pairs from (a) split to form entangled states; the dashed lines to the left represent new states $|3,1\rangle_+$ and $|3,1\rangle_-$; those to the right are $|5,3\rangle_+$ and $|5,3\rangle_-$. States $|3,1\rangle_-$ and $|5,3\rangle_-$ become ground states; their charge densities are shown.

states are energetically much closer to one another than to any other state with which they can mix. Under these conditions we can again solve the problem analytically; the entangled states are $(p|m+2,m\rangle \pm q|m,m+2\rangle)$ where p,q are unequal and depend on the energies of the component states. Figure 3(ii b) shows that the stability of the entangled ground states, i.e., the gap between the entangled ground states and the other competing states, is greater than the corresponding gap in Fig. 3(i b). The gap scales approximately as $\sqrt{\epsilon^2+(\Delta E)^2}$ where ϵ is the gap in the equal dot system [Fig. 3(i b)] and ΔE is the energy difference between component states [i.e., the separation of the dashed lines in Fig. 3(ii a)]. The results of Figs. 3(i) and 3(ii) are analogous to linear and quadratic Stark effects; in both Figs. 3(i) and 3(ii) we essentially have two-level systems with the former being degenerate while the latter is nondegenerate.

Figures 3(iii a) and 3(iii b) consider the case of $P=3$ equal-size dots. Exact analytic treatment to order $\beta r^2/s^4$ proceeds as above. The ground and low-lying states are now characterized by three nonzero quantum numbers describing the relative angular momentum between electrons on each of the three dots; we label these states $|m_1,m_2,m_3\rangle$. Just as for $P=2$ with $\omega_1=\omega_2$ to this order (i.e., $\beta r^2/s^4$) we find degeneracies: $|m_a,m_b,m_c\rangle$ is degenerate with $|m_c,m_b,m_a\rangle$. In the limit of large s all other states formed by permuting m_a,m_b,m_c would also be degenerate. Having exactly solved for $P=3$ to order $\beta r^2/s^4$, we again turn to perturbation theory to treat the terms of order $\beta r^4/s^6$ (which we will denote by h). The nonzero matrix elements of h in the nearly degenerate subspace are found to be of the form $\langle m_a+X,m_b+Y,m_c+Z|h|m_a,m_b,m_c\rangle$ where $\{X,Y,Z\}$ is any permutation of $\{-2,0,2\}$. For example, $|3,1,1\rangle$ mixes with $|1,3,1\rangle$ and $|1,1,3\rangle$ yielding three new mixed (i.e., entangled) states. These entangled states then compete with each other to become ground states for finite s [see Fig. 3(iii b)]. The entangled states are the $P=3$ dot analogs of the interdot correlated, crystallike states for $P=2$.

As the number of dots P increases, the gain in energy of the entangled (i.e., interdot correlated) states at finite s actually *increases* as compared to the $s\to\infty$ limit. This increase is nonlinear with P. Note that we can think of the earlier cases of $P=2$ and $P=3$ as a "diatomic molecule" and "triatomic molecule," respectively. Consider the "polyatomic molecule" with P identical atoms (dots) all in the $m=1$ state. The state $m=3$, for example, can be created on any one of these dots yielding P degenerate tight-binding combinations or molecular states in the limit $s\to\infty$. The degeneracy of these

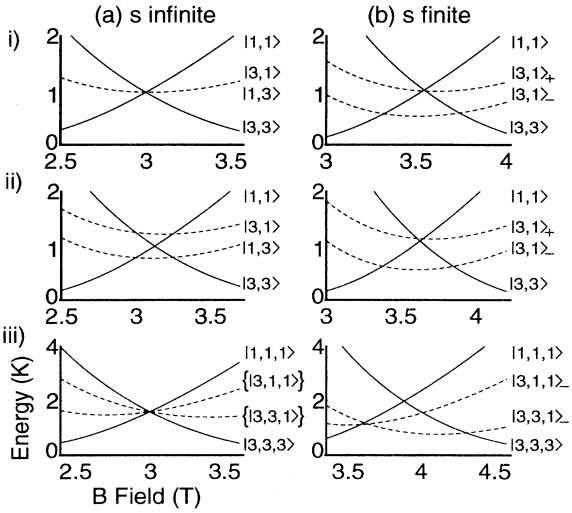

FIG. 3. Energies (in Kelvin relative to an arbitrary zero) of low-lying states as a function of B for three different systems. (i) $P=2$ dots of equal sizes. Parts (a) and (b) are magnifications from Figs. 2(a) and 2(b), respectively. (ii) $P=2$ dots of unequal size. (iii) $P=3$ dots of equal size; in (a) each dashed line is triply degenerate while in (b) only the lowest two of the six entangled states are shown.

states is broken for finite s by the coupling between adjacent dots. In contrast to an actual tight-binding molecule where it is the single-body tunneling term that couples atoms, here the coupling is via the two-body interaction V_{inter}. The states represent "Frenkel excitons"; the exciton (i.e., $m=3$ state) on a given dot can transfer its energy resonantly to all members of the chain. For large P, the analysis is simplified considerably by introducing periodic boundary conditions. This removes "end effects," introducing translational symmetry into the problem and allowing application of Bloch's theorem. Consider a system of P dots with a lattice structure; if $\omega_i=\omega$ for $i=1,\ldots,P$ then this is a "monatomic crystal." If alternating ω's differ, we have a "diatomic crystal." The entangled states are now traveling wave excitations and yield an exciton band structure.

In summary, we have presented details of entangled ground states arising in colinear, multiple dot structures. The tendency for formation of such states *increases* with the number of dots.

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[1] For example, P. Maksym and T. Chakraborty, Phys. Rev. Lett. **65**, 108 (1992).
[2] M. Wagner *et al.*, Phys. Rev. B **45**, 1951 (1992).
[3] P. Hawrylak, Phys. Rev. Lett. **71**, 3347 (1993).
[4] R.C. Ashoori *et al.*, Phys. Rev. Lett. **71**, 613 (1993).
[5] K. Kempa *et al.*, Phys. Rev. B **43**, 9343 (1991).
[6] J. Dempsey *et al.*, Phys. Rev. B **42**, 11 708 (1990).
[7] T. Chakraborty *et al.*, Phys. Rev. B **43**, 14 289 (1991).
[8] C.A. Stafford and S. Das Sarma, Phys. Rev. Lett. **72**, 3590 (1994).

[9] C. Lent and P.D. Tougaw, J. Appl. Phys. **74**, 6227 (1993).
[10] Possible applications have been suggested in the emerging field of quantum computation. See, for example, J.P. Paz and G. Mahler, Phys. Rev. Lett. **71**, 3235 (1993).
[11] Such weakly coupled arrays of confined few-electron systems are not uncommon in various naturally occurring biological systems.
[12] W.G. Teich *et al.*, Phys. Rev. B **37**, 8111 (1988).
[13] S. He *et al.*, Phys. Rev. B **43**, 9339 (1991).
[14] M. Tewordt *et al.*, Phys. Rev. B **49**, 8071 (1994).
[15] L. Quiroga *et al.*, Solid State Commun. **86**, 775 (1993).
[16] J.M. Kinaret *et al.*, Phys. Rev. B **46**, 4681 (1992).
[17] Entangled states cannot be written as a direct product ($|m_1, m_2\rangle$).