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Low-temperature transport of excitons in type-II GaAs/AlAs quantum wells

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Low-temperature transport of two-dimensional excitons in type-II GaAs/AlAs double quantum wells (DQW's) with rough interfaces is considered. The limiting cases of zero magnetic field and the magnetic quantum limit are studied. We found that (1) the transport of excitons in DQW's is mainly limited by the disorder at the external interfaces and (2) the transport relaxation time τ depends nonmonotonously on the quantum-well widths for electrons (AlAs QW) and holes (GaAs QW). In the magnetic quantum limit the exciton transport relaxation time decreases with magnetic field strength B approximately as $\tau \sim B^{-1/2}$.

I. INTRODUCTION

Excitons in type-II GaAs/AlAs double quantum wells (DQW's) are indirect both in real and in momentum space and thus characterized by large radiative lifetimes, which exceed those of direct GaAs/$Ga_{1-x}Al_xAs$ QW's, by several orders of magnitude (see, e.g., Ref. 1 and references therein). In particular, these structures are good candidates for observation of exciton condensation, because sufficiently high exciton densities can be attained at moderate excitation powers, when the exciton temperature is not too high.[1] Theory predicts[2,3] that the application of strong perpendicular magnetic fields increases exciton binding energies and suppresses the translational kinetic energy of excitons, thus making it easier to realize critical conditions for the formation of an exciton condensate in quasi-two-dimensional systems. Experimental observation of exciton condensation in type-II GaAs/AlAs DQW's in strong magnetic fields has been reported recently in Ref. 4. For condensation effects the effective lifetimes are relevant, which are determined by both radiative and nonradiative recombination channels. The latter are related to the *transport* of excitons to centers of nonradiative recombination, which exhibits strong dependence on the magnetic field B.[4] Theoretical investigation of transport of excitons in GaAs/AlAs DQW's (also in high magnetic fields) is still absent.

Mobilities of electrons (e.g., Refs. 5, 6 and references therein) and excitons[7-10] in quasi-two-dimensional QW's at low temperatures are limited by the interface roughness (IFR), i.e., by local fluctuations in the QW width, which produce an effective scattering potential. In this paper, we theoretically study the effects of IFR on transport of indirect excitons in GaAs/AlAs DQW's under the assumption that the exciton gas is dilute (i.e., below the regime of exciton condensation). We aim, in this paper, at a qualitative description of excitonic transport in DQW's and study the limiting cases of zero magnetic field and the magnetic quantum limit. In particular, for DQW's we identify "negative interference" effects in scattering of the exciton as a whole in a random potential which, in principle, opens the possibility to optimize the conditions for excitonic transport.

II. THEORY

A. Interface roughness

The interaction of excitons with an external field is described by the potential operator

$$\hat{V} = V_e(\mathbf{r}_e) + V_h(\mathbf{r}_h) \ , \tag{1}$$

which consists of the parts acting on the electron and the hole, respectively. In general, we have $V_e(\mathbf{r}) = -V_h(\mathbf{r})$ — in contrast to the interaction with an electric field.

When the exciton transport is limited by the interface roughness and in narrow DQW's the electric quantum limit for both the electron and the hole is realized, the scattering Hamiltonian can be obtained from Eq. (1), by the expansion of the confinement potentials,

$$\hat{V} = \left(\frac{\partial E_e^{(0)}}{\partial d_e}\right) \Delta_e(\mathbf{r}_e) + \left(\frac{\partial E_h^{(0)}}{\partial d_h}\right) \Delta_h(\mathbf{r}_h) \ , \tag{2}$$

where $E_\gamma^{(0)}$ are the lowest-subband energies and $\Delta_\gamma(\mathbf{r}_\gamma)$ are local fluctuations in the QW widths; $\gamma = e, h$ and $\mathbf{r} = (x, y)$ is the in-plane coordinate (see Fig. 1). The quantities $\partial E_\gamma^{(0)}/\partial d_\gamma$, which play the role of the "coupling constants" in Eq. (2), are shown in Fig. 2. In calculations the X-conduction-band and the Γ-valence-band offsets at the AlAs/GaAs interface are taken to be $\Delta E_X = 198$ meV and $\Delta E_{\rm VB} = 526$ meV, respectively. The electron and hole perpendicular effective masses $m_{ze(h)}$ in AlAs (GaAs) are taken to be $m_{ze} = 1.1$ (1.3) and $m_{zh} = 0.75$ (0.34); for the in-plane electron and hole masses in AlAs and GaAs, we take, respectively, $m_{\parallel e} = 0.19$ and $m_{\parallel h} = 0.18$ (see, e.g., Table I in Ref. 11 and references therein). The effective masses in $Ga_{1-x}Al_xAs$ and the band offsets at the interfaces with $Ga_{1-x}Al_xAs$ are obtained by linear interpolation. The changes of the potential profiles (dashed lines in the inset to Fig. 3), due to the presence of the remote interfaces, are disregarded. Note that the partial derivatives $\partial E_\gamma^{(0)}/\partial d_\gamma$ are directly related with the quantum mechanical "pressure" experienced by particles confined to QWs. Therefore, Eq. (2) implicitly takes into account

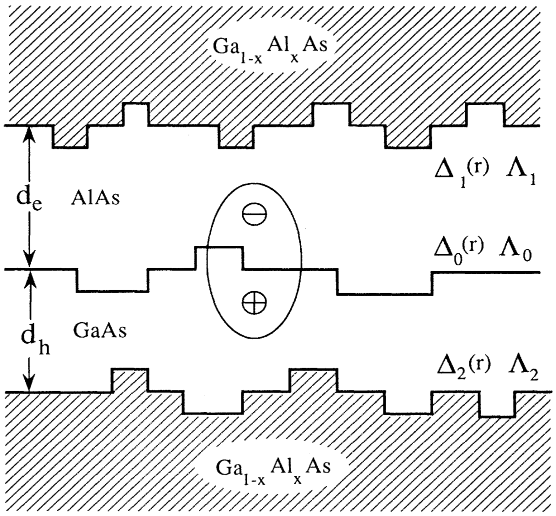

FIG. 1. Real-space (and k-space) indirect excitons in the type-II GaAs/AlAs DQW, with interface roughness (IFR). Local fluctuations in the quantum-well widths are characterized at different interfaces by the amplitudes $\Delta_i(\mathbf{r})$ and the correlation lengths Λ_i.

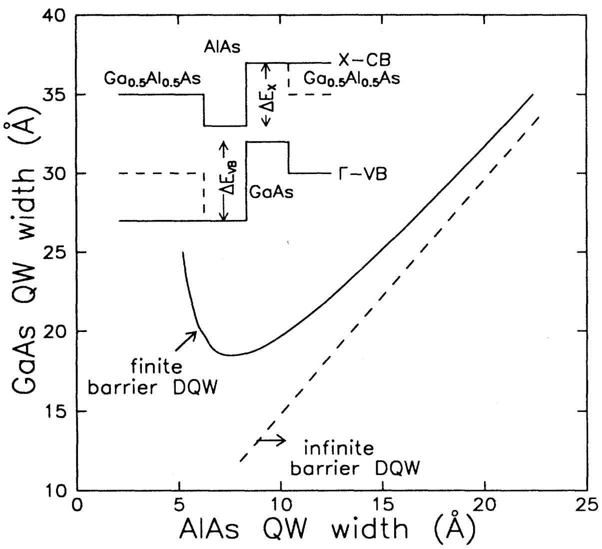

FIG. 3. The line $d_h = f(d_e)$ at which the condition (22) is fulfilled and the IFR at the internal interface does not limit the exciton transport. The broken line is for the infinite barrier DQW, when $\partial E_\alpha^{(0)}/\partial d_\alpha = -\pi^2\hbar^2/m_{z\alpha}d_\alpha^3$, and $d_h = (m_{ze}/m_{zh})^{1/3}\, d_e$ [see Fig. 2 and Eq. (2)]. The inset shows the potential profiles (solid lines) of the quantum wells for electrons and holes used in the calculations of $\partial E_\alpha^{(0)}/\partial d_\alpha$. The changes of the potential profiles (dashed lines in the inset) due to the presence of the remote $Ga_xAl_{1-x}As$/AlAs and GaAs/$Ga_xAl_{1-x}As$ interfaces are disregarded in the calculations.

the effects of the penetration of particles into the barriers and of the effective mass discontinuities at the interfaces.

$\Delta_\gamma(\mathbf{r}_\gamma)$ contain contributions, due to fluctuations $\Delta_0(\mathbf{r})$ on the internal interface, as well as those on the external interfaces $\Delta_1(\mathbf{r})$, $\Delta_2(\mathbf{r})$ (see Fig. 1),

$$\Delta_e(\mathbf{r}_e) = \Delta_1(\mathbf{r}_e) + \Delta_0(\mathbf{r}_e)\ , \qquad (3)$$

$$\Delta_h(\mathbf{r}_h) = \Delta_2(\mathbf{r}_h) - \Delta_0(\mathbf{r}_h)\ . \qquad (4)$$

We assume that the local fluctuations $\Delta_i(\mathbf{r})$ on different interfaces are statistically independent, while the fluctuations on the same interface are described by Gaussian autocorrelation functions, i.e.,

$$\langle\langle \Delta_i(\mathbf{r})\, \Delta_j(\mathbf{r}')\rangle\rangle = \delta_{ij}\, \Delta_i^2\, \exp\left(-\frac{(\mathbf{r}-\mathbf{r}')^2}{2\Lambda_i^2}\right)\ . \qquad (5)$$

In (5), Δ_i are the amplitudes of the fluctuations and Λ_i are the correlations lengths of the disorder. It is important to realize that, due to the fluctuations on the internal interface, the values $\Delta_e(\mathbf{r}_e)$ and $\Delta_h(\mathbf{r}_h)$ are not independent, and the corresponding autocorrelation function

$$\langle\langle \Delta_e(\mathbf{r})\, \Delta_h(\mathbf{r}')\rangle\rangle = -\Delta_0^2\, \exp\left(-\frac{(\mathbf{r}-\mathbf{r}')^2}{2\Lambda_0^2}\right)\ , \qquad (6)$$

for DQW's is *negative.* This is related to the fact the fluctuation of the internal interface of the DQW simultaneously produces a local increase in d_e and a local decrease in d_h (see Fig. 1).

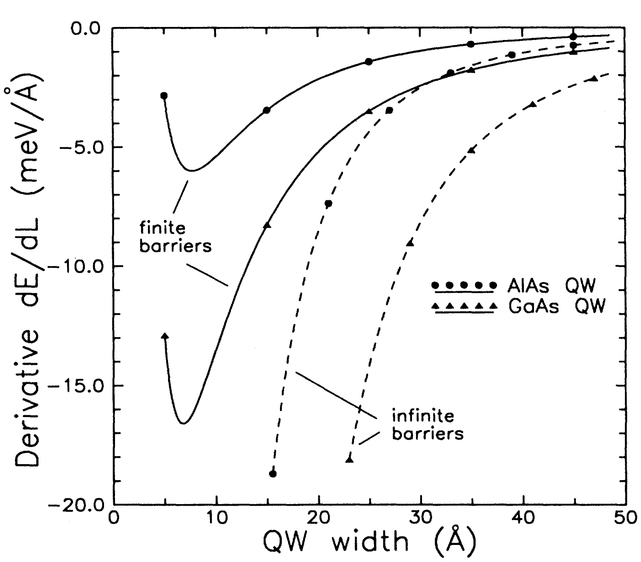

FIG. 2. Partial derivatives of the lowest-subband energies in GaAs/AlAs DQW, entering Eq. (2). See text and the inset to Fig. 3 for the details of calculations.

B. Exciton scattering matrix elements

1. *Zero magnetic field*

The matrix element for the scattering of an exciton between the states with the center of mass momentum $\mathbf{K}$ and with the quantum number n of the internal motion to a state $\mathbf{K}', n'$ in an external field $\hat{V}$ reads

$$\langle n'\,\mathbf{K}'|\hat{V}|n\,\mathbf{K}\rangle = \frac{1}{S^2}\tilde{V}_e(\Delta\mathbf{K})\sum_{\mathbf{k}}\varphi^*_{n'}(\mathbf{k}+\alpha_h\Delta\mathbf{K})\varphi_n(\mathbf{k}) + \frac{1}{S^2}\tilde{V}_h(\Delta\mathbf{K})\sum_{\mathbf{k}}\varphi^*_{n'}(\mathbf{k}-\alpha_e\Delta\mathbf{K})\varphi_n(\mathbf{k}). \tag{7}$$

Here $\tilde{V}_\gamma(\mathbf{k})$ ($\gamma = e,h$) are the two-dimensional (2D) Fourier transforms of the potentials $V_\gamma(\mathbf{r})$ from (1), $\alpha_{e(h)} = m_{\|e(h)}/M_x$, $M_x = m_{\|e}+m_{\|h}$ is the in-plane exciton mass and S, the area of the system and $\Delta\mathbf{K} = \mathbf{K}'-\mathbf{K}$ is the transferred in-plane momentum. $\varphi_n(\mathbf{k})$ is the Fourier transform of the wave function of the relative motion.

Considering essentially narrow type-II DQW's ($d_e <$ 40 Å, $d_h <$ 35 Å, see Fig. 1), we shall make the strictly 2D approximation for the exciton internal motion. For the ground $1s$ state of the strictly 2D exciton, neglecting the scattering to excited states of the internal motion [i.e., taking $n' = n = 1s$ in (7)], we have

$$\langle 1s\,\mathbf{K}'|\hat{V}|1s\,\mathbf{K}\rangle = \frac{1}{S}\tilde{V}_e(\Delta\mathbf{K})\left[1+\tfrac{1}{16}(\alpha_h\Delta K a^*_X)^2\right]^{-3/2} + \frac{1}{S}\tilde{V}_h(\Delta\mathbf{K})\left[1+\tfrac{1}{16}(\alpha_e\Delta K a^*_X)^2\right]^{-3/2}, \tag{8}$$

which does not depend on the wave functions of the z motion. In Eq. (8), $a^*_X = \varepsilon\hbar^2/\mu_x e^2$ is the exciton Bohr radius and $\mu_x = m_{\|e}m_{\|h}/M_x$ is the reduced in-plane exciton mass.

2. *The magnetic quantum limit*

When the perpendicular magnetic field B is sufficiently strong, both the electron and the hole forming the exciton can be characterized by Landau level indices n, m and the mixing between different Landau levels can be neglected. This is valid when $\ell_B = (\hbar c/eB)^{1/2} \ll a^*_{Be(h)} = \epsilon\hbar^2/m_{\|e(h)}e^2$. In that limit, the wave functions of the bound e-h pair (i.e., a magnetoexciton) is mainly determined by the magnetic field[12] and the corresponding magnetoexciton dispersion is given by

$$\varepsilon_{nm}(K) = \int\frac{d^2q}{(2\pi)^2}\exp(i\mathbf{q}\cdot\mathbf{K}\ell_B^2 - \tfrac{1}{2}q^2\ell_B^2)\,\tilde{U}(q)L_n\left[\tfrac{1}{2}q^2\ell_B^2\right]L_m\left[\tfrac{1}{2}q^2\ell_B^2\right]F_{eh}(q), \tag{9}$$

where $\tilde{U}(q) = -2\pi e^2/\epsilon q$ is the 2D Fourier transform of the Coulomb e-h interaction, $L^s_n(x)$ are generalized Laguerre polynomials $[L^0_n(x) \equiv L_n(x)]$ and

$$F_{eh}(q) = \int_{-\infty}^{\infty}dz_e\int_{-\infty}^{\infty}dz_h\,\exp(-q|z_e-z_h|)\,\zeta^2_e(z_e)\,\zeta^2_h(z_h) \tag{10}$$

is the form factor of the wave functions of the lowest electric subbands. As above for $B = 0$, considering essentially narrow type-II DQW's, we shall make the strictly 2D approximation in (9), namely, we put $F_{eh}(q) = 1$. Then, for the ground state of interest with $n = m = 0$ the dispersion (9) can be calculated analytically,[12]

$$\varepsilon(K) = -E_0\exp\left(-\frac{K^2\ell_B^2}{4}\right)I_0\left(\frac{K^2\ell_B^2}{4}\right), \tag{11}$$

where $E_0 = \sqrt{\pi/2}\,e^2/\epsilon\ell_B \sim B^{1/2}$ is the binding energy of the magnetoexciton of zero momentum and $I_n(x)$ are modified Bessel functions; we omit from now on the indices $n = m = 0$ pertaining to the exciton in the zero Landau levels. At small momenta $K^2\ell_B^2/4 \ll 1$, the dispersion (11) is parabolic with the effective magnetoexciton mass $\tilde{M}_x = 2\hbar^2/E_0\ell_B^2 \sim B^{1/2}$, which does not depend on the single-particle effective masses and is entirely due to the Coulomb e-h interaction; it can be shown that $\tilde{M}_x \sim (a^*_{Be(h)}/\ell_B)M_x \gg M_x$, when $\ell_B \ll a^*_{Be(h)}$.[12]

In interactions with external fields, we shall treat the magnetoexciton as a neutral quasiparticle whose motion does not depend on B, but its characteristics (such as the mass $\tilde{M}_x$, the dipole momentum $\mathbf{d} = e\,\mathbf{K}\times\hat{e}_z\,\ell_B^2$ at $\mathbf{K}\neq 0$, etc.) are determined by the magnetic field B.[12,13] The scattering between two magnetoexciton states with momenta $\mathbf{K},\mathbf{K}'$ ($\Delta\mathbf{K} \equiv \mathbf{K}'-\mathbf{K}$) is described by the matrix element,[14]

$$\langle n'm'\,\mathbf{K}'|\hat{V}|n\,m\,\mathbf{K}\rangle = \frac{1}{S}\tilde{V}_e(\Delta\mathbf{K})\exp\left(\frac{i}{2}[\mathbf{K}'\times\mathbf{K}]_z\,\ell_B^2\right)\times s_{n'n}(K_x,K_y)\,\delta_{m,m'} + \frac{1}{S}\tilde{V}_h(\Delta\mathbf{K})\times\exp\left(-\frac{i}{2}[\mathbf{K}'\times\mathbf{K}]_z\,\ell_B^2\right)\times s_{m'm}(K_x,-K_y)\,\delta_{n,n'}, \tag{12}$$

where (e.g., for $n' > n$)

$$s_{n'n}(K_x,K_y) = \left(\frac{n}{n'}\right)^{1/2}\left[\tfrac{1}{\sqrt{2}}(iK_x-K_y)\ell_B\right]^{n'-n}\times\exp\left(-\tfrac{1}{4}\Delta K^2\ell_B^2\right)L_n^{n'-n}\left[\tfrac{1}{2}q^2\ell_B^2\right]. \tag{13}$$

For the case of interest, when $n' = m' = n = m = 0$ (and omitting these indices), we have

$$\langle\mathbf{K}'|\hat{V}|\mathbf{K}\rangle = \frac{1}{S}\tilde{V}_e(\Delta\mathbf{K})\exp\left(\frac{i}{2}[\mathbf{K}'\times\mathbf{K}]_z\,\ell_B^2 - \frac{\Delta K^2\ell_B^2}{4}\right) + \frac{1}{S}\tilde{V}_h(\Delta\mathbf{K})\exp\left(-\frac{i}{2}[\mathbf{K}'\times\mathbf{K}]_z\,\ell_B^2 - \frac{\Delta K^2\ell_B^2}{4}\right). \tag{14}$$

Note that the matrix elements (14) [apart from the usual dependence on $\tilde{V}_\gamma(\Delta\mathbf{K})$] also contain (i) the factor $\exp(-\Delta K^2\,\ell_B^2/4)$ corresponding to the projection onto Landau levels, and (ii) the phase factors $\exp(\pm\frac{i}{2}[\mathbf{K}'\times\mathbf{K}]_z\,\ell_B^2)$, which depend on the relative orientation of the vectors $\mathbf{K},\mathbf{K}'$, and differ for the electron and the hole contributions by complex conjugation (corresponding to time reversal $t\to -t$).

C. Exciton transport relaxation time

We treat the transport momentum relaxation time of excitons $\tau(K)$ in the Born approximation,

$$\tau^{-1}(K) = \frac{2\pi}{\hbar} \sum_{\mathbf{K}'} \langle\langle \, |\langle \mathbf{K}'|\hat{V}|\mathbf{K}\rangle|^2 \, \rangle\rangle \, (1 - \cos\Theta_{\mathbf{KK}'}) \times \delta(\varepsilon_K - \varepsilon_{K'}) \, . \quad (15)$$

[The microscopic derivation of the excitonic transport relaxation time and the diffusion constant using the diagram technique will be presented elsewhere;[15] quantum corrections to the transport of excitons in high magnetic fields (i.e., weak localization) are considered in Ref. 16.] The Born approximation is valid for scattering of slow particles in a potential V, with a length scale a when $\hbar^2/ma^2 \gg V$. This condition, when applied to the exciton in a QW with interface roughness, takes the form $\hbar^2/M_x\Lambda^2 \gg \Delta\,\partial E^{(0)}_{e(h)}/\partial d \sim \Delta\,\hbar^2/\pi^2 m_{\|e(h)} d^3$. It implies the following limitation on the correlation length and the amplitude of IFR: $\Lambda^2\Delta \ll d^3\left(m_{ze(h)}/\pi^2 M_x\right)$. Similarly, for the magnetoexciton (when $M_x \to \tilde{M}_x$), we have $\Lambda^2\Delta \ll d^3\left(\ell_B/\pi^2 a^*_{Be(h)}\right)$.

There are three contributions to $\tau^{-1}(K)$ in Eq. (15),

$$\tau^{-1}(K) = \tau_e^{-1}(K) + \tau_h^{-1}(K) + \tau_{eh}^{-1}(K) \, , \quad (16)$$

which follow from the terms $\langle\langle|V_e|^2\rangle\rangle$, $\langle\langle|V_h|^2\rangle\rangle$, describing the single-particle scattering, and from the "*interference*" term $\langle\langle V_e V_h^* + V_e^* V_h\rangle\rangle$, respectively. The latter appears because the exciton is scattered as a unit. The first two contributions in (16) are positive τ_e^{-1}, $\tau_h^{-1} > 0$. The third contribution τ_{eh}^{-1} for DQW's, due to (6) is *negative* (in the magnetic field τ_{eh}^{-1} changes its sign at finite K, see below). The negative contribution τ_{eh}^{-1} partly cancels the positive contributions $\tau_e^{-1} + \tau_h^{-1}$, thus enhancing τ. For *single* quantum wells (SQW's) the contribution τ_{eh}^{-1}, describing the "interference" effects, is positive $\tau_{eh} > 0$. Typically, for SQW's, this can reduce the total relaxation time τ by a factor of 4, compared with the single-particle scattering.[17]

1. Zero magnetic field

For zero magnetic field, from Eqs. (8) and (15), (16), we have

$$\tau_{e(h)}^{-1}(K) = \frac{\pi}{\hbar} \left|\frac{\partial\varepsilon_K}{\partial K^2}\right|^{-1} \left(\frac{\partial E^{(0)}_{e(h)}}{\partial d_{e(h)}}\right)^2 \sum_{i=0,1(2)} \Delta_i^2\Lambda_i^2 \times \int_0^{2\pi} \frac{d\Theta}{2\pi} \frac{\exp\left[-K^2\Lambda_i^2(1-\cos\Theta)\right]}{\left[1+\frac{1}{8}\alpha^2_{h(e)}K^2 a_X^{*2}(1-\cos\Theta)\right]^3} \, , \quad (17)$$

and

$$\tau_{eh}^{-1}(K) = -\frac{2\pi}{\hbar} \left|\frac{\partial\varepsilon_K}{\partial K^2}\right|^{-1} \left(\frac{\partial E_e^{(0)}}{\partial d_e}\right) \left(\frac{\partial E_h^{(0)}}{\partial d_h}\right) \times \Delta_0^2\Lambda_0^2 \int_0^{2\pi} \frac{d\Theta}{2\pi} \frac{\exp\left[-K^2\Lambda_0^2(1-\cos\Theta)\right]}{\left\{[1+\frac{1}{8}\alpha_h^2 K^2 a_X^{*2}(1-\cos\Theta)][1+\frac{1}{8}\alpha_e^2 K^2 a_X^{*2}(1-\cos\Theta)]\right\}^{3/2}} \, . \quad (18)$$

Here, $\varepsilon_K = \hbar^2K^2/2M_x$ is the parabolic dispersion of the exciton and $\left|\partial\varepsilon_K/\partial K^2\right|^{-1} = 2M_x/\hbar^2$ (up to a constant) is the exciton density of states determined by the exciton mass. It can be shown from Eqs. (17), (18) that both $\tau_{e(h)}^{-1}$ and τ_{eh}^{-1} are finite at $K=0$ and fall off algebraically at large $K \gg a_X^*, \Lambda_i$. For arbitrary K, $\tau_{e(h)}^{-1}$ and τ_{eh}^{-1} have to be determined numerically. Similar expressions have been considered for SQW's in Ref. 18 (with the erroneous sign for τ_{eh}^{-1}).

2. Magnetic quantum limit

In the magnetic quantum limit, the contributions to τ^{-1} can be calculated analytically:

$$\tau_{e(h)}^{-1}(K) = \frac{\pi}{\hbar} \left|\frac{\partial\varepsilon_K}{\partial K^2}\right|^{-1} \left(\frac{\partial E^{(0)}_{e(h)}}{\partial d_{e(h)}}\right)^2 \sum_{i=0,1(2)} \Delta_i^2\Lambda_i^2 \times \exp(-\beta_i K^2\ell_B^2)\left[I_0(\beta_i K^2\ell_B^2) - I_1(\beta_i K^2\ell_B^2)\right] \quad (19)$$

and

$$\tau_{eh}^{-1}(K) = -\frac{2\pi}{\hbar} \left|\frac{\partial\varepsilon_K}{\partial K^2}\right|^{-1} \left(\frac{\partial E_e^{(0)}}{\partial d_e}\right) \left(\frac{\partial E_h^{(0)}}{\partial d_h}\right) \times \Delta_0^2\Lambda_0^2 \exp(-\beta_0 K^2\ell_B^2)\left[I_0(\sqrt{\beta_0^2-1}\,K^2\ell_B^2) - \frac{\beta_0}{\sqrt{\beta_0^2-1}} I_1(\sqrt{\beta_0^2-1}\,K^2\ell_B^2)\right] , \quad (20)$$

where $\beta_i \equiv 1 + \Lambda_i^2/\ell_B^2$. In Eq. (19), τ_e^{-1}, τ_h^{-1} are positive for all K and have finite values for both $K\ell_B \ll 1$ and $K\ell_B \gg 1$. The latter behavior is due to the K dependence of the quantity $|\partial\varepsilon_K/\partial K^2|^{-1} \equiv 2\tilde{M}_x(K)/\hbar^2$, which determines the exciton density of states and can be also considered as the K-dependent magnetoexciton mass $\tilde{M}_x(K)$, which increases with K and behaves as $\tilde{M}_x(K) \sim K^3$ for $K\ell_B \gg 1$. For a parabolic dispersion (when $|\partial\varepsilon_K/\partial K^2|^{-1} = \text{const}$), one would have $\tau_{e(h)} \sim K^3$ for $K\ell_B \gg 1$. The third contribution τ_{eh}^{-1} is *negative* for DQW's at small K. Notably, due to the presence of the oscillating phase factors in (14), in the magnetic field τ_{eh}^{-1}

changes its sign at intermediate $K \sim (\ell_B^2 + \Lambda_0^2)^{-1/2}$ and falls off exponentially at large $K^2(\ell_B^2 + \Lambda_0^2) \gg 1$.

3. Negative interference: consequences

When the disorder on the external interfaces is absent [i.e., $\Delta_1(\mathbf{r}) = \Delta_2(\mathbf{r}) = 0$], it follows from Eqs. (16–20) that for both $B = 0$ and in the magnetic quantum limit $\tau(K)$ at small momenta diverges as

$$\tau(K) \sim K^{-4} \, , \; K \to 0 \, , \tag{21}$$

when the condition

$$\frac{\partial E_e^{(0)}}{\partial d_e} = \frac{\partial E_h^{(0)}}{\partial d_h} \tag{22}$$

is fulfilled. Physically, such a behavior is due to the fact that for the DQW, when (22) is fulfilled, local small deviations in the QW's widths $\Delta_e(\mathbf{r}) = -\Delta_h(\mathbf{r})$ do not produce a change in the total energy of the exciton of zero momentum [see Eq. (2)]. More formally, when $\Delta_1(\mathbf{r}) = \Delta_2(\mathbf{r}) = 0$, Hamiltonian (2) consists of potentials $V_e(\mathbf{r}) = -V_h(\mathbf{r})$, which differ by sign for the electron and the hole. Thus, for the exciton the interaction is strongly suppressed. Another example of such a situation is presented below in Sec. II C 5, where we consider briefly the effects of scattering by ionized impurities on the transport of excitons.

The position of the line $d_h = f(d_e)$ determined by condition (22) is shown in Fig. 3. In the electric quantum limit, it is practically independent on the perpendicular magnetic field $B_\perp$, because the in-plane motion determined by $B_\perp$ and the perpendicular motion of the particles relevant for (22) are uncoupled. However, both a *parallel* strong magnetic field $B_\parallel$ and a perpendicular electric field $\mathbf{E}_\perp$ would change the wave function of the perpendicular motion and, hence, influence the position of the line $d_h = f(d_e)$. Therefore, $B_\parallel$ or $\mathbf{E}_\perp$ can strongly affect the role of the IFR at the internal AlAs/GaAs interface.

4. IFR: numerical results and discussion

In order to study if some features, due to the singular behavior (21), can survive for rough outer interfaces, we numerically calculate the exciton transport relaxation time τ, using the formulas (16–20) for different parameters of the disorder. For simplicity, we consider the case when (i) the amplitudes of the fluctuations Δ_i from Eq. (5) are the same on all interfaces $\Delta_i \equiv \Delta = 3$ Å, i.e., correspond to one-monolayer fluctuations, and (ii) the correlation lengths at the external interfaces coincide with each other $\Lambda_1 = \Lambda_2$.

The calculated transport relaxation time τ at $K = 0$, as a function of the quantum-well widths d_e, d_h is plotted in Fig. 4 for different parameters of the disorder and $B =$ 20 T. The value $\tau(K = 0)$ can serve for excitons as a representative one at low temperatures. It is seen from Fig. 4(a) that in the presence of a small amount of the disorder at the external interfaces [i.e., when Λ_1 is small], $\tau(K = 0)$ turns out to be comparatively large at and near

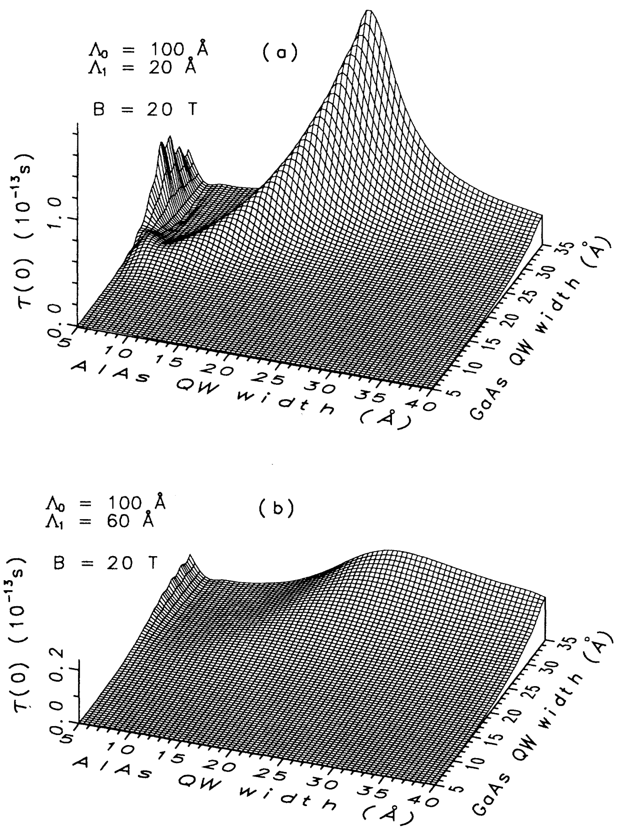

FIG. 4. Transport relaxation time $\tau(K = 0)$, as a function of the DQW widths d_e, d_h at $B = 20$ T for $\Delta = 3$ Å and for two different correlation lengths: (a) $\Lambda_0 = 100$ Å, $\Lambda_1 = \Lambda_2 = 20$ Å and (b) $\Lambda_0 = 100$ Å, $\Lambda_1 = \Lambda_2 = 60$ Å.

the line $d_h = f(d_h)$ determined by (22). This means that some remainders of the singular behavior (21) are still present. Only when Λ_1 is large [Fig. 4(b)], these features are almost completely washed out. At zero magnetic field the behavior of $\tau(K = 0)$ is qualitatively the same (for quantitative differences see in Figs. 5 – 8 and below).

The dependence of τ on momentum K and on exciton kinetic energy ε_K (measured in temperature units) at $B = 0$ and in high magnetic fields, is shown in Figs. 5, 6, respectively. Note that for the considered fields $B \leq 30$ T, we have only $\ell_B \leq a^*_{Be(h)}$. Thus, the high magnetic field approximation is only qualitatively correct. The obtained values of the transport relaxation time τ are very low. This is also reflected by the exciton mean free paths $\ell(K) = \tau(K)v_K$, which are shown in Figs. 7, 8. Here, $v_K = \partial\varepsilon_K/\hbar\partial K$ is the exciton center-of-mass velocity. At small exciton momenta K (which correspond to low energies and thus determine the low-temperature transport), the mean free paths are very small $\ell(K) \leq$ 10–15 Å and typically are smaller than the correlation length of the disorder Λ. Strictly speaking, in such a situation, the perturbation theory which starts from the plane waves is no longer valid. Physically, however, the result that $\ell(K) < \Lambda$ at small K means that the long-wavelength excitons are trapped in islands of potential wells of aver-

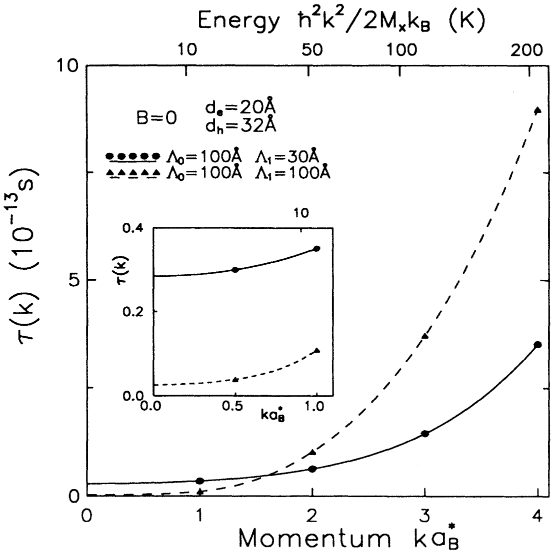

FIG. 5. Transport relaxation time τ at $B = 0$ as a function of the momentum K [measured in units of a_B^{*-1}, where $a_B^* = 98$ Å is the electron Bohr radius in GaAs] for DQW with $d_e = 20$ Å, $d_h = 30$ Å. Parameters of IFR are $\Delta = 3$ Å, $\Lambda_0 = 100$ Å, and $\Lambda_1 \equiv \Lambda_2 = 30$ Å (solid line) and $\Lambda_0 = \Lambda_1 = 100$ Å (dashed line). The upper axis shows the exciton kinetic energy measured in temperature units. The inset shows the small K region relevant at low temperatures.

age dimension Λ, i.e., are strongly localized. Estimated exciton mobilities $\mu \equiv e\tau/M_x$, corresponding to Fig. 9, range from 140 cm^2/V s at $T = 5$ K to 450 cm^2/V s at $T = 50$ K. These values are comparable with the exciton mobilities observed for narrow 4-nm $GaAs/Ga_{1-x}Al_xAs$

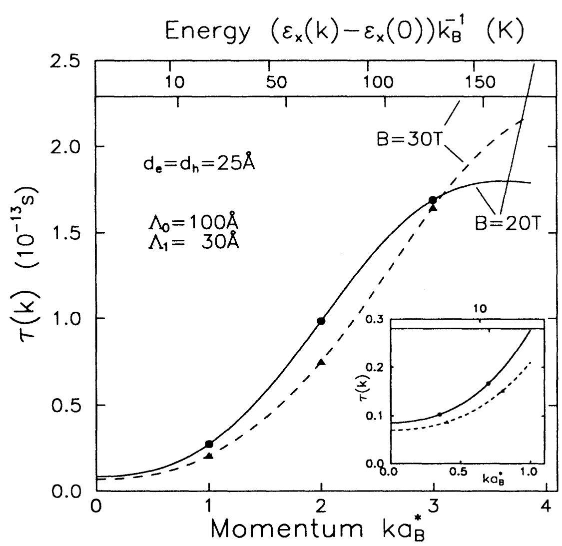

FIG. 6. Transport relaxation time τ, as a function of the exciton momentum K at $B = 20$ T (solid line), and $B = 30$ T (dashed line) for DQW with $d_e = d_h = 25$ Å. The parameters of the IFR are $\Delta = 3$ Å, $\Lambda_0 = 100$ Å, and $\Lambda_1 = \Lambda_2 = 30$ Å. The inset shows the small K region relevant at low temperatures.

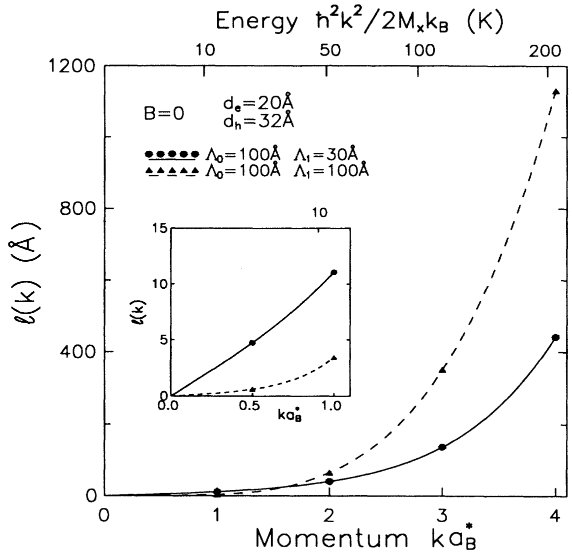

FIG. 7. Mean free path $\ell = \tau_K v_K$, as a function of the exciton momentum K at $B = 0$. Other parameters are as in Fig. 5.

QW's at $B = 0$.[7] An increase of exciton mobility requires crystal growth techniques, which reduce the correlation length of the surface roughness, for instance by intentional omission of growth interruption during heteroepitaxy.

The dependence of $\tau(K = 0)$ on the magnetic field B is shown in Fig. 9. In the magnetic quantum limit $\tau(K = 0)$ decreases monotonously with B approximately as $B^{-1/2}$. [This behavior is mainly due to the increase in the effective magnetoexciton mass $\tilde{M}_x \sim B^{1/2}$, i.e., the factor $|\partial\varepsilon_K/\partial K^2|^{-1}$ in (19), (20). To a lesser extent τ depends on B through the magnetoexciton scattering matrix elements (14).] Qualita-

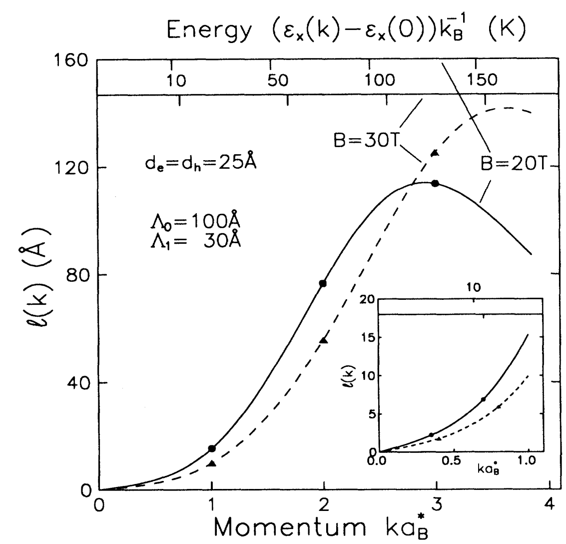

FIG. 8. Mean free path $\ell = \tau_K v_K$ as a function of the exciton momentum K at $B = 20$ T (solid line) and $B = 30$ T (dashed line). Other parameters are as in Fig. 6.

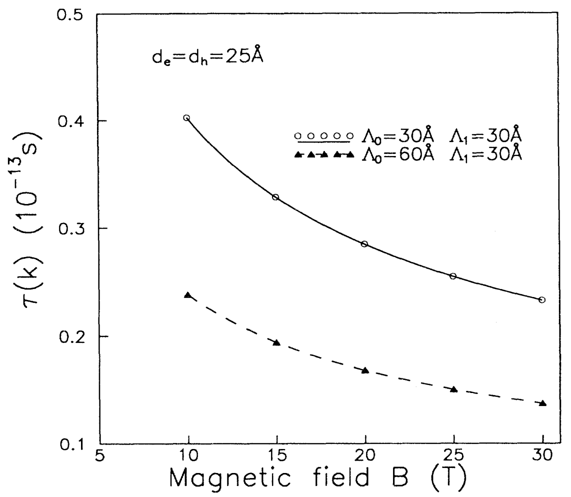

FIG. 9. The exciton transport relaxation time $\tau(K=0)$ versus the magnetic field B, for the DQW, with $d_e=d_h=25$ Å for two different parameters of the IFR $\Lambda_0=\Lambda_1=30$ Å (solid line) and $\Lambda_0=60$ Å, $\Lambda_1=30$ Å (dashed line).

tively, this prediction of the theory is consistent with the monotonous decrease with B of the excitonic transport, which was observed experimentally in 50 Å/55 Å/50 Å GaAs/$Ga_{1-x}Al_xAs$/GaAs DQW's and in strained 60 Å/60 Å/60 Å $In_{1-x}Ga_xAs$/GaAs/$In_{1-x}Ga_xAs$ DQW's.[19] Experimentally, the situation is different for type-II GaAs/AlAs DQW's, where strongly nonmonotonous effect of B was found.[4] This might be due to the different regime, which is realized in essentially narrow GaAs/AlAs DQW's (where excitons can be more strongly localized) and the effect of the magnetic field on the excitonic transport in the regime of strong localization can be different. Another possible origin of strongly nonmonotonous behavior of transport with B could be the precursors to the excitonic condensation, i.e., the coherence effects which influence the excitonic magnetotransport.[4] Clearly, both effects are beyond the scope of the present theory.

5. Ionized impurities

Here, we present the results for the transport properties of 2D magnetoexcitons in the presence of ionized impurities. We consider the situation when ionized Coulomb impurities are distributed at random in the same 2D plane where the excitons move. Even in this case, when the scattering is much more stronger than for remote donors or acceptors, the scattering of neutral excitons by ionized impurities turns out to be rather ineffective in comparison with IFR. It is shown that it is even more suppressed by a strong magnetic field B.

For the concentration of the ionized impurities, $c(\mathbf{r})$, we will assume the Gaussian statistics and the autocorrelation function,

$$\langle\langle c(\mathbf{r})\, c(\mathbf{r}')\rangle\rangle = n_{\rm imp}\,\delta(\mathbf{r}-\mathbf{r}') \;, \tag{23}$$

where $n_{\rm imp}$ is the areal concentration of impurities (cm^{-2}). Since for the Coulomb interactions $V_e(\mathbf{r}) = -V_h(\mathbf{r})$, for the averaged squared scattering matrix elements from Eqs. (15), (14), we obtain

$$\langle\langle\, |\langle \mathbf{K}'|\hat{V}|\mathbf{K}\rangle|^2\,\rangle\rangle = \frac{1}{S}\,4\,n_{\rm imp}\,\tilde{v}^2(\Delta K)\exp\left(-\tfrac{1}{2}\Delta K^2\ell_B^2\right) \times \sin^2\left(\tfrac{1}{2}[\mathbf{K}'\times\mathbf{K}]_z\,\ell_B^2\right) \;, \tag{24}$$

where $\tilde{v}(q)$ is the Fourier transform of the electron (hole)-impurity potential of interaction. For $\tilde{v}(q)$, we will take the unscreened Coulomb interaction $\tilde{v}^2(q) = (2\pi e^2/\epsilon q)^2$. Then the inverse transport relaxation time of excitons is obtained from (24) and (15) in the form

$$\tau^{-1}(K) = \left(\frac{2\pi e^2}{\epsilon K}\right)^2 \left|\frac{\partial\varepsilon_K}{\partial K^2}\right|^{-1}\frac{n_{\rm imp}}{2\hbar}\int_0^{2\pi}\frac{d\Theta}{2\pi} \times\exp\left[-K^2\ell_B^2\,(1-\cos\Theta)\right] \times\left[1-\cos(K^2\ell_B^2\sin\Theta)\right] \;. \tag{25}$$

Using the value of the integral

$$\int_0^{2\pi}\frac{d\Theta}{2\pi}\,\exp\left(a\cos\Theta\right)\cos(a\sin\Theta) = 1 \;, \tag{26}$$

we obtain for $\tau^{-1}(K)$ the analytic expression,

$$\tau^{-1}(K) = \left(\frac{2\pi e^2}{\epsilon K}\right)^2 \left|\frac{\partial\varepsilon_K}{\partial K^2}\right|^{-1}\frac{n_{\rm imp}}{2\hbar} \times\exp\left(-K^2\ell_B^2\right)\left[I_0(K^2\ell_B^2)-1\right] \;. \tag{27}$$

There is only one length scale in (27) which determines the K dependence, namely, the magnetic length ℓ_B. The asymptotic behavior of $\tau^{-1}(K)$ at small and at large $K\ell_B$ is given by, respectively,

$$\tau^{-1}(K) = \begin{cases} 2E_0\hbar^{-1}\nu_{\rm imp}\,K^2\ell_B^2 \sim B^{-3/2}\;, & K^2\ell_B^2 \ll 1 \\ 2E_0\hbar^{-1}\nu_{\rm imp} \sim B^{-1/2}\;, & K^2\ell_B^2 \gg 1\;. \end{cases} \tag{28}$$

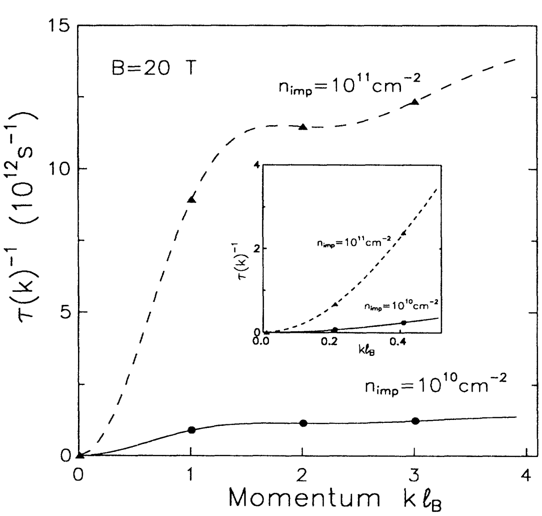

FIG. 10. The inverse transport relaxation time τ^{-1} Eq. (27) determined by scattering at ionized impurities, as a function of the exciton momentum for $B=20$ T. The solid line is for the impurity concentration $n_{\rm imp}=10^{10}$ cm^{-2} and the dashed line is for $n_{\rm imp}=10^{11}$ cm^{-2}.

In (28) the dimensionless parameter, $\nu_{\rm imp} \equiv 2\pi\ell_B^2 n_{\rm imp} \sim B^{-1}$ (the impurity "filling factor") is introduced. According to the asymptotics (28), the transport relaxation time $\tau(K)$ diverges at small K as K^{-2}. This reflects the fact that in the limit $K \to 0$ the magnetoexciton behaves as a neutral particle which does not "feel" charged impurities.[20]

Note that the divergence of $\tau(K) \sim K^{-2}$ at small K is in this case less singular than in the case of IFR (21), due to the behavior $\tilde{v}^2(K) \sim K^{-2}$ of the unscreened Coulomb potential. The functional dependence $\tau(K)^{-1}$ given by Eq. (27) is shown in Fig. 10 for two different impurity concentrations. The effect of a reduction of the interaction between the magnetoexciton and the field of charged impurities at small K outweighs the increase in $\tilde{M}_x \sim B^{1/2}$, and τ is an increasing function of B: $\tau(K) \sim B^{3/2}$. It can be shown that the exciton diffusion constant D in this case practically does not depend on temperature.[15]

III. CONCLUSION

In conclusion, we have considered transport of excitons in type-II GaAs/AlAs DQW's limited at low temperatures by interface roughness. The "interference" effects in exciton scattering in a random potential play an important role in determining the transport of excitons. For DQW's, these effects are negative and reduce the influence of IFR on the internal AlAs/GaAs interface, thus enhancing the transport relaxation time τ. [In contrast, for SQW's "interference" effects are positive and reduce τ.] Thus, the conditions for excitonic transport can be optimized by adjusting GaAs and AlAs QW widths. In principle, a situation can be realized when the transport of excitons in DQW's is mainly limited by the disorder on the external AlAs/$-$GaAs/$Ga_{1-x}Al_xAs$ interfaces.

The obtained values of the exciton transport relaxation time τ and mean free paths are generally very low. We interpret this as an indication that excitons in essentially narrow type-II GaAs/AlAs DQW's are localized in potential fluctuations caused by IFR, which is consistent with the experimental findings.[10] Therefore, at low temperatures transport of excitons should be described as thermally-activated transfer (see, e.g., Ref. 21) between islandlike structures formed by IFR.

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