Theory of the interface exciton in a strong magnetic field

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A simple theoretical model for the interface exciton in a strong magnetic field is developed. We propose that in the case of a strong magnetic field, a bound state for the interface exciton exists for fields greater than some critical value $\mathbf{B}_0$. It is shown that contrary to the case with no field, the interface exciton in a strong magnetic field is localized near the junction plane and can have a large binding energy. Using the developed model, we made a theoretical study of the explanation by Clark *et al.* of the nature of the B peak observed in their recent photoluminescence experiment [Clark *et al.*, Physica B **201**, 301 (1994)], and we obtained good agreement with their results.

As proposed by Clark *et al.*,[1] the B line in the photoluminescence of a GaAs single heterojunction in a strong magnetic field can be interpreted as the recombination of an exciton lightly bound in the z direction [the direction perpendicular to the interface (x,y) plane] which retains the character of a single-pair excitation, and which does not directly involve the electrons in the correlated two-dimensional (2D) electron system. The theory of this interface exciton was given by Balslev,[2] who showed that its effective Bohr radius is a few Bohr radii of the bulk (3D) exciton, and that it is strongly polarized, since the oscillator strength is rather small due to the small overlap of the electron and hole wave function in the z direction. The structure of interface excitons is in a way similar to those formed in quantum wells in an external electric field.[3] Balslev's model of the interface exciton provided a good explanation of the result that the luminescence line vanishes at temperatures about 10–20 K. Unfortunately the model is too complicated, so it can be treated by numerical calculation only.

In order to examine the proposal by Clark *et al.*, in a previous paper[4] we presented a simple model for the interface exciton with no magnetic field present, which can be used to give analytical expressions for its main parameters such as binding energies, effective Bohr radius, wave functions, and oscillator strengths. We concluded that for an explanation of the experimental results we need a theory for the stabilization of the interface exciton in the presence of a strong magnetic field. For that purpose, in this paper we develop a theory for the interface exciton in a strong magnetic field.

We assume that due to the conduction-band-gap potential Φ_c, the electrons are confined in the junction plane (x,y), i.e., we have 2D electrons with their wave functions in the z direction given as $\varphi^e(z)\sim\delta(z)$ and with effective mass m_e, and 3D holes with effective mass m_h. We consider a strong magnetic field $\mathbf{B}$ applied in the z direction. The motion of the center of mass of the electron-hole pair in the (x,y) plane can be separated as usual, and we obtain the Schrödinger equation for the relative motion of the electron and hole in the form

$$\left[\frac{\mathbf{p}^2}{2\mu}+\frac{ie}{2\mu c}\gamma(\mathbf{B}\cdot\mathbf{r}\times\mathbf{p})-\frac{e^2}{8\mu c^2}B^2r^2+\frac{p_z^2}{2m_h}+\Phi(z)-\frac{e^2}{\varepsilon_0}\frac{1}{\sqrt{(\mathbf{r}+\mathbf{r}_0)^2+z^2}}-E\right]\psi(\mathbf{r})\varphi(z)=0\ , \qquad (1)$$

where $\lambda=\sqrt{\hbar c/eB}$ is the magnetic length, $\mathbf{r}_0=(\lambda^2/B)\mathbf{B}\times\mathbf{P}$; $\mathbf{p}$ and $\mathbf{r}$ are the operators of 2D momentum and coordinates of the in-plane relative motion, respectively; z is the out-of-plane coordinate of the hole, ε_0 is the dielectric constant of the medium; μ is the reduced mass of the 2D electron-hole pair; $\mathbf{P}$ is the exciton total momentum operator; $M_0=m_e+m_h$ is the exciton mass at zero magnetic field; and $\gamma=(m_h-m_e)/M_0$. Here we denote $\Phi(z)$ as the valence edge potential, which was the solution of a Poisson equation and strongly dependent on the charge-carrier concentration. We note that the band-edge potential Φ pushes away the hole from the junction plane, and resists the formation of a bound state. In contrast the Coulomb potential attracts the hole to the (x,y) plane and favors binding. For the case of large Φ, and in the absence of a magnetic field, a bound state of the interface exciton has a very small binding energy or does not exist.[2,4]

Balslev[2] and Appelbaum[5] previously described the determination of $\Phi(z)$ in some detail. It was shown that measuring Φ from U_c, where U_c is the average Coulomb potential in the absence of the edge potential and magnetic field, we can perform a simplifying ansatz

$$\Phi(z)=\Phi_0\exp\left[-\frac{z}{l}\right]-U_c\ , \qquad (2)$$

where Φ_0 and l are the parameters of our model, and l means the depletion length of the hole.

We shall show later in this paper that a magnetic field produces binding of the interface exciton; i.e., for any Φ, we can determine some value of magnetic field $\mathbf{B}_0$, such that if $\mathbf{B}>\mathbf{B}_0$ then the bound state of the exciton is stabilized.

We consider that $\Phi_0 > U_c$, so at zero field $\mathbf{B}=0$ the bound state of the interface exciton cannot exist, and we shall study two limiting cases: (i) l is large, $l \gg a_x$; and (ii) l is small, $l \ll a_x$, where a_x is the Bohr radius of the bulk exciton.

For case (i), the edge potential Φ varies slowly with z, so that we can replace it by some constant value Φ^* such as

$$\Phi^* = \int dz\, \Phi(z)|\varphi(z)|^2 - U_c \approx \Phi_0 - U_c \ . \tag{3}$$

Then the theory can be developed in analogy with that for the hydrogen atom in a magnetic field.[6,7] For the case of weak magnetic fields, such that the cyclotron energy $\hbar\omega_c = \hbar eB/\mu c$ is much smaller than the effective bulk-exciton Rydberg R_x, i.e., $\hbar\omega_c \ll R_x$, we can use perturbation theory and obtain the well-known quadratic Zeeman effect[6] with the exciton energy shift ΔE_x caused by a magnetic field given as

$$\Delta E_x = \left[\frac{n^*}{2}\right]^4 \frac{\hbar^2\omega_c^2}{R_h} \ , \tag{4}$$

where $R_h = m_h e^4/2\hbar^2\epsilon_0^2$ is the hole effective Rydberg, n^* is the effective quantum number, and $n^* \approx 0.15$ for the ground state.

In the case of medium field such that $\hbar\omega_c \sim R_x$, the situation is rather complicated. We can use the trial wave function in the form

$$\psi(\mathbf{r})\varphi(z) = (2^{3/2}\pi^{3/2}a^3 a_x^3\epsilon)^{-1/2} \times \exp\left[\frac{-1}{4a^2a_x^2}\left[x^2+y^2+\frac{z^2}{\epsilon^2}\right]\right] , \tag{5}$$

with two variational parameters a and ϵ. A straightforward computation for the trial value of energy gives

$$E = \hbar\omega_c + \Phi^* - R_x\left[\frac{1}{2a^2}\left[1+\frac{\epsilon^2}{2}\frac{\mu}{m_h}\right] + \frac{a^2}{2}\left[\frac{\hbar\omega_c}{R_x}\right]^2 - \frac{1}{a}\left[\frac{2}{\pi}\right]^{1/2}\frac{\epsilon}{\sqrt{1-\epsilon^2}}\ln\left[\frac{1+\sqrt{1-\epsilon^2}}{1-\sqrt{1-\epsilon^2}}\right]\right] . \tag{6}$$

Minimizing the expectation value of the energy with respect to a and ϵ, we obtain the set of two equations $\partial E/\partial a = 0$ and $\partial E/\partial\epsilon = 0$, and resulting equations should be solved by numerical calculation only.

For fields so strong that $\hbar\omega_c \gg R_x$, however, the situation again becomes simple if we use perturbation theory. In the zeroth approximation in the Coulomb interaction, the in-plane relative motion wave functions are independent of γ and are given by

$$\psi_{N,M}(\mathbf{r}) = \left[\frac{N!}{2^{|M|+1}(N+M)!\pi}\right]^{1/2} e^{-iM\varphi}\left[\frac{r}{\lambda}\right]^{|M|} \times L_N^{|M|}\left[\frac{r^2}{2\lambda^2}\right]\exp\left[-\frac{r^2}{4\lambda^2}\right] , \tag{7}$$

where L_N^M are the Laguerre polynomials. However, their corresponding energies turn out to be strongly dependent on γ:

$$E_{N,M} = \omega_c[N + \tfrac{1}{2}(|M| - \gamma M + 1)] \ . \tag{8}$$

We note here that the 2D effective exciton mass is infinite in the zeroth approximation.

For out-of-plane motion, the problem is then reduced to solving a 1D Schrödinger equation of the form

$$\left[\frac{p_z^2}{2m_h} + W_{N,M}(z)\right]\varphi(z) = E_{1x}\varphi(z) , \tag{9}$$

where

$$W_{N,M} = \Phi^* - \frac{e^2}{\varepsilon_0}\int d\mathbf{r}\,|\psi_{N,M}(\mathbf{r})|^2 \frac{1}{\sqrt{(\mathbf{r}+\mathbf{r}_0)^2+z^2}} \ . \tag{10}$$

For the orbital case $\mathbf{P} \approx \mathbf{0}$, and if we confine ourselves to an exciton in the zeroth Landau band, then

$$W(z) = W_{0,0} = \Phi^* - \frac{e^2\sqrt{2}}{\varepsilon_0\lambda}\exp\left[\frac{z^2}{2\lambda^2}\right]\mathrm{erfc}\left[\frac{z}{\lambda\sqrt{2}}\right] . \tag{11}$$

From this equation we find that for any values Φ_0 and l of the actual potential Φ, there exists some value of magnetic length λ_0 such that if $\lambda < \lambda_0$ (or $\mathbf{B} > \mathbf{B}_0$) then the total effective potential W has a negative part $W < 0$, i.e., we have the stabilizing effect of the applied strong magnetic field for the interface exciton. It is of course not possible to solve Eq. (8) for motion in the z direction with the form of potential $W(z)$ taken like Eq. (10). It may, however, be solved approximately by using the methods developed in Refs. 6 and 7 for hydrogenic atoms in a strong magnetic field. Then the energy of the interface exciton equals

$$E_x = \hbar\omega_c + \Phi^* - E_{1x} \ , \tag{12}$$

with the energy of the 1D exciton given as

$$E_{1x} = R_h\eta^2 \ , \tag{13}$$

where η is the solution of the equation

$$\eta = 2\ln\left[\frac{\mu a_x}{\sqrt{2}m_h\lambda\eta}\right] - \chi \ , \tag{14}$$

where χ is the Euler's constant.

With increasing magnetic field $\mathbf{B}$, the effective potential W becomes deeper and narrower near the point $z=0$; thus, contrary to the case when the field is absent, the interface exciton in a strong magnetic field is localized near the junction plane, and can have a large binding energy.

Here we obtain the effect of stabilizing the interface exciton by a strong magnetic field.

In the limit of extremely high field $\mathbf{B}\to\infty$, the interface exciton has the behavior of a 2D exciton in a strong magnetic field, with the energy

$$E_x(P)=\hbar\omega_c+\Phi^*-\left[\frac{\pi}{2}\right]^{1/2}\frac{e^2}{\varepsilon_0\lambda}+\frac{P^2}{2M}, \tag{15}$$

where the effective mass of the 2D exciton at low momenta M can be obtained by calculating the matrix elements of the Coulomb interaction in the first approximation, or

$$M=\frac{2\varepsilon_0}{e^2}\left[\frac{2\pi e}{\hbar c}B\right]^{1/2}. \tag{16}$$

It tends to infinity in the limit $\mathbf{B}\to\infty$. We can conclude that in extremely high magnetic field the energy of the interface exciton is independent of the effective masses of the electron and hole, and that it has a universal character for all semiconductors.

For case (ii), with small $l \ll a_x$, the picture is more complicated because the edge potential now varies rapidly with z. If $l \ll a_x \ll \lambda$, the contribution of the magnetic field is small, and the effective potential has a form like that we studied in our previous paper.[4] We can use the same model for the interface exciton, and take the magnetic field as perturbation, and obtain the Zeeman effect. The interface exciton has stable states a few Bohr radii from the heterointerface with small binding energy. In the case of strong and extremely high magnetic fields, we have a situation corresponding to case (i). The interface exciton is localized near the junction plane, and has a large binding energy. We note here that this is a universal behavior for all semiconductors in the extremely high field. In the case of medium field, where $l\sim\lambda$, or $a_x\sim\lambda$, the problem is very difficult, and it is possible to obtain numerical results only by using the variational methods.

Now we apply our theory of an interface exciton in a magnetic field to parameters relevant to the work of Clark *et al.*[1] For GaAs, take $l=4800$ Å $\gg a_x$, i.e., corresponding to case (i), and note that the results are not sensitive to that large value of l. We chose standard parameters for the bulk exciton as $a_x=137$ Å, $R_x=4$ meV, and $m_e=0.06m_0$. In Fig. 1 we present the negative part of the effective potential $W<0$; this is relevant for $\lambda<\lambda_0$, or $\mathbf{B}>\mathbf{B}_0$. The case of threshold field $\mathbf{B}_0\approx 2$ T for the B line, which was observed in the experiment of Clark *et al.*, corresponds to the value $\Phi^*\approx 0.5R_x$. We also calculated the dependence of the photoluminescence peak on the magnetic field, where the hole mass was taken to be the light one $m_h=0.12m_0$. The results of the numerical calculation are plotted in Fig. 2. We find rather good agreement between the theoretical results and experimental data of Clark *et al.* (1994) (Ref. 1) for the B line. If we took the heavy-hole mass, the curve was too far below the experiment. Two possible reasons for this might be that the light interface exciton is much more strongly coupled to light than the heavy one,[4] and the decrease of the effective mass resulting from intralayer Coulomb interaction.[8]

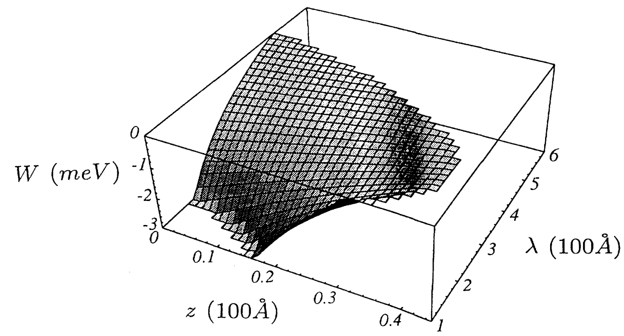

FIG. 1. The negative part (attractive region) of the effective potential W as a function of the magnetic length λ and the relative coordinate z between the hole and electron.

The main behavior observed in the recent experiment[1] on field dependence of the photoluminescence intensity of the B peak can also be understood in the framework of our model. At the field $\mathbf{B}<\mathbf{B}_0$, a bound state cannot exist, hence there is no overlap between electron and hole wave functions in the z direction, and thus the oscillator strength is null, so that no B peak appears in the photoluminescence spectrum. At $\mathbf{B}=\mathbf{B}_0$, the B line begins to be observed, and its intensity increases with the field, because the interface exciton is more localized near the junction plane, and this causes an increase of the overlap of the wave functions and thus increases the oscillator strength. At higher field $\mathbf{B}\gg\mathbf{B}_0$, the interface exciton behaves more like the 2D exciton in a strong magnetic field, so the intensity of the B peak slopes with the field.[1] More detailed variations of the B peak intensity as a function of magnetic field at fractional Landau-level filling factors $\nu=\frac{2}{7},\frac{2}{9},\frac{1}{5},\ldots$, and its splitting into two peaks S and B' from the field $\mathbf{B}_0'\approx 8$ T are related to Wigner solidification and many electron effects of the 2D electron state, and need further theoretical modeling in order to carry out investigations.

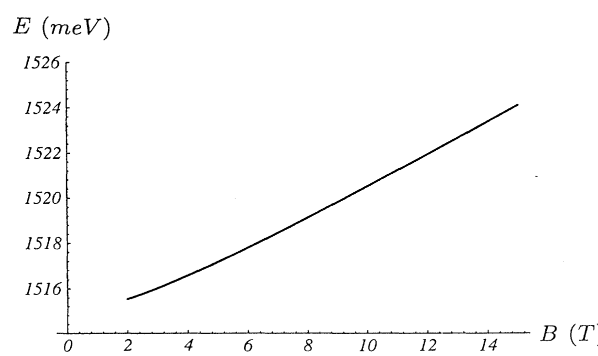

FIG. 2. The calculated photoluminescence peak as a function of magnetic field $\mathbf{B}$. The agreement with the experimental data taken from Ref. 1 for the B line is rather good. The threshold field $\mathbf{B}_0$ is about 2 T. We chose standard parameters for the bulk exciton in GaAs as $a_x=137$ Å, $R_x=4$ meV, $m_e=0.06m_0$, $m_h=0.12m_0$, and $l=4800$ Å.

In summary, we have presented a simple theoretical model of the interface exciton in a strong magnetic field. The effect of stabilizing the interface exciton by a strong magnetic field was shown. Contrary to the case when the field is absent,[2,4] the interface exciton in a strong magnetic field is localized near the junction plane, and can have a large binding energy. This is similar to the effect of the enhancement of the exciton binding energy in quantum wells by a strong magnetic field, which was investigated in the work by Cen, Chen, Bajaj, and Branis.[9] Within our model, we made a theoretical examination of the interpretation of the nature of the B line given by Clark *et al.*[1] in connection with their recent photoluminescence experiment. We obtained good agreement with our model using accepted values of the parameters in our theory.

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