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Precise nonequilibrium distribution function for a one-dimensional electron gas

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A precise solution of the stationary one-dimensional Fokker-Planck-type equation is obtained in quadratures. The interaction of a one-dimensional electron gas with longitudinal-acoustic phonons and charged impurities is considered in detail. The nonequilibrium distribution functions of the gas are found and investigated in a wide range of electric fields and other parameters for the degenerate as well as the nondegenerate gas. The structure peculiarities of the distribution function are discussed. Numerical calculations of the field dependence of the drift velocity and the average kinetic energy are performed.

I. INTRODUCTION

Systems of reduced dimensionality such as quantum wires are the subject of numerous investigations, in purely theoretical as well as applied aspects.[1–9]

The motion of particles in quantum wires is confined in two directions to the space of the order of the de Broglie length. The result is the quantization of energy. The quantization reveals itself most when the distance between the discrete levels is greater than or comparable with the temperature. Thereby particles move, at most, in one dimension because the thermal transitions between these levels are rare enough.

We consider here the case when only the lowest level acts in an infinitely thin quantum wire. The motion of a particle along the wire is described in classical approximation. The latter is allowed when the mean momentum and energy transmitted at collisions are small in comparison with the average momentum and energy of particles.

There is a special reason to choose just that model for the theoretical investigation. Although, in general, kinetic equations cannot be solved precisely, one can obtain a precise analytical solution of the stationary Fokker-Planck-type equation for the one-dimensional macroscopically uniform electron gas. In this case, the kinetic equation becomes an ordinary differential equation containing only the derivatives by the velocity v_x. We suppose below the scattering system to be equilibrium so that there is no necessity to solve the system of coupled kinetic equations for electron and scattering systems. Finally, if we use the equilibrium dielectric screening function $\epsilon(\omega,k_x)$, then the considered Fokker-Planck-type equation can be solved in quadratures: Then, we obtain a challenging opportunity to see the precise nonequilibrium distribution function. The obtained nonequilibrium distribution function is used farther to calculate some macroscopical characteristics of the nonequilibrium electron gas. As a scattering system, we consider longitudinal-acoustic phonons and charged impurities.

II. THE FOKKER-PLANCK-TYPE EQUATION AND ITS SOLUTION

The quantum kinetic equation for the one-dimensional system was derived from the first principles in Ref. 5. For the stationary case, it has the form [see Eq. (51) in Ref. 5]

$$\frac{eE_x}{\hbar}\frac{\partial f_{k_x}}{\partial k_x}=Stf_{k_x}\ , \tag{1}$$

where E_x is an applied uniform electric field. Here we have used the plane-wave representation, k_x is the wave vector. The collisional integral for the equilibrium scattering system is constructed in the following way:

$$\begin{aligned}Stf_{k_x}=(e^2/\hbar L)\sum_{q_x}\langle\delta\bar{\varphi}_S^2\rangle_{\omega,q_x}\{f_{k_x-q_x}-f_{k_x}&\\ -[f_{k_x}(1-f_{k_x-q_x})&\\ +f_{k_x-q_x}(1-f_{k_x})]&\\ \times\tanh(\hbar\tilde{\omega}/2T_S)\}\ .&\end{aligned} \tag{2}$$

Here L is the length of the system, T_S is the temperature of the scattering system, $\tilde{\omega}=\hbar k_x(k_x-q_x/2)/m, f_{k_x}$ is the one-particle density matrix (distribution function), $\langle\delta\bar{\varphi}_S^2\rangle_{\omega,q_x}$ is the correlator of scattering potentials screened by both electrons and the external system:

$$\langle\delta\bar{\varphi}_S^2\rangle_{\omega,q_x}=[1+\Delta\epsilon(\omega,q_x)/\epsilon_S(\omega,q_x)]^{-2}\langle\delta\varphi_S^2\rangle_{\omega,q_x}\ , \tag{3}$$

where $\langle\delta\varphi_S^2\rangle_{\omega,q_x}$ is the correlator of scattering potentials screened by the external system only. $\epsilon_S(\omega,q_x)$ is the dielectric function of the lattice (see Ref. 10). The value $\Delta\epsilon_S(\omega,q_x)$ is the dielectric function of the one-dimensional electron gas. It is considered in detail in Sec. III.

Equation (1) has no precise solution. Consider the clas-

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sical approach, where transferred momentum q_x is supposed to be small in comparison with the typical value of momentum k_x. Expanding in Eq. (2) all the values on the powers of q_x and restricting the expansion by the terms of the second order, one obtains the classical kinetic equation of Fokker-Planck type:

$$\frac{e}{m}E_x\frac{\partial f}{\partial v_x}=\frac{d}{dv_x}\left\{\frac{mv_x}{T_S}D(v_x)f(v_x)[1-f(v_x)]\right\}+\frac{d}{dv_x}\left\{D(v_x)\frac{df(v_x)}{dv_x}\right\} . \tag{4}$$

Here $v_x=\hbar k_x/m$, where m is the effective mass, and

$$D(v_x)=\frac{e^2}{4\pi m^2}\int_{-\infty}^{\infty}k_x^2dk_x\int_{-\infty}^{\infty}d\omega\,\delta(\omega-k_xv_x)\times\left|1+\frac{\Delta\epsilon(\omega,k_x)}{\epsilon_S(\omega,k_x)}\right|^{-2}\times\langle\delta\varphi_S^2\rangle_{\omega,k_x} . \tag{5}$$

Note that Eq. (4) can be derived from the first classical principles using Klimontovich's method of microscopical phase operator (see Ref. 11) as well. This procedure was performed in Ref. 12.

As is shown in Refs. 10 and 12, the dielectric function $\epsilon_S(\omega,q_x)$ for a three-dimensional system with the dielectric constant ϵ_L can be represented through the Bessel function. In the classical approach

$$1/\epsilon_S(\omega,q_x)\to 1/\epsilon_S(q_x)=2\ln\left[\frac{\sqrt{8}}{l|q_x|}\right]\Big/\epsilon_L , \tag{6}$$

where l is the width of the quantum well; at this only point we need to depart from the approximation of the infinitely thin quantum wire for regularization.

The screening function of the one-dimensional electron gas (1DEG) is expressed through the electron distribution function $f(v_x)$ in the following manner:

$$\Delta\epsilon(\omega,k_x)=\frac{e^2k_x}{\pi\hbar}\int\frac{[df(v_x)/dv_x]dv_x}{\omega-k_xv_x+i0} . \tag{7}$$

Here $f(v_x)$ is normalized by the condition

$$\frac{m}{\pi\hbar}\int_{-\infty}^{\infty}f(v_x)dv_x=n^{(1)} , \tag{8}$$

where $n^{(1)}$ is the linear concentration of particles.

The equilibrium function (here F is the Fermi energy)

$$f_0(v_x)=\left[1+\exp\left[\frac{(mv_x^2/2)-F}{T_S}\right]\right]^{-1} \tag{9}$$

can be rewritten in the form

$$f_0(v_x)=\frac{f_0(0)\exp[-mv_x^2/(2T_S)]}{1-f_0(0)\{1-\exp[-mv_x^2/(2T_S)]\}} . \tag{10}$$

Integrating Eq. (4) over the velocity from v_x to infinity [with the condition $f(v_x\to\infty)=0$], we obtain the equation of the first order:

$$\frac{e}{m}E_xf(v_x)=D(v_x)\left[\frac{df(v_x)}{dv_x}+\frac{mv_x}{T_S}f(v_x)[1-f(v_x)]\right] . \tag{11}$$

The solution of this equation has an evident symmetry $f(v_x,E_x)=f(-v_x,-E_x)$. Therefore, it is sufficient to find the solution of Eq. (11) at $v_x\geq 0$.

We introduce the dimensionless velocity $w=v_x/v_T\equiv v_x\sqrt{m/2T_S}$ and the field function

$$\gamma(w)=-\gamma(-w)\equiv-\frac{e}{|e|}E_x\int_0^w\frac{dw'}{E_0(w')} , \tag{12}$$

where

$$E_0(w)=\frac{mD(wv_T)}{|e|v_T}=\frac{|e|}{4\pi mv_T}\int_{-\infty}^{\infty}k_x^2dk_x\left|1+\frac{\Delta\epsilon(k_xv_Tw,k_x)}{\epsilon_S(k_x)}\right|^{-2}\times\langle\delta\varphi_S^2\rangle_{k_xv_Tw,k_x} . \tag{13}$$

Now the formal solution of Eq. (11) takes the form $(w\geq 0)$

$$f(v_x)\equiv f(v_Tw)=\frac{f(0)\exp[-w^2+\gamma(w)]}{1-2f(0)\int_0^w u\,du\exp[-u^2+\gamma(u)]} . \tag{14}$$

Farther we shall write simply $f(w)$ instead of $f(v_Tw)$. The constant of integration $f(0)$ depends on the field E_x and is found from Eq. (8). At $E_x=0$, the expression (14) goes to (10). Equation (14) shows that the form of the nonequilibrium distribution function $f(w)$ is determined totally by the form of the field function $\gamma(w)$. Note that the function $\gamma(w)$ depends in turn on the distribution function $f(w)$ through Eqs. (12), (13), and (7). If we use in the electron screening function (7) the equilibrium distribution function (9), then we can consider the form (14) as a precise solution of the kinetic Eq. (4) in quadratures. (Farther, we shall work just in this model.)

As it results either from the form (14) or from Eq. (11), the inclination of the nonequilibrium distribution function at $w=0$ is given by

$$[f'(w)]_{w=0}=\frac{eE_x}{|e|E_0(0)}f(0) . \tag{15}$$

For the elastic scattering [see, for example, Eq. (30)] $\langle\delta\varphi_S^2\rangle_{\omega,k_x}\propto\delta(\omega)$ hence the inclination of the nonequilibrium distribution function at $w=0$ is equal to zero (see below, Figs. 1, 2, and 6).

For the nondegenerate gas, the form (14) simplifies to

$$f(w)=f(0)\exp[-w^2+\gamma(w)] . \tag{16}$$

The precise expression (16) allows us to write at once the relaxation time $\tau(v_x)$. Indeed, at usual τ approximation (see, for example, Ref. 13) one introduces the relaxation time in the following manner:

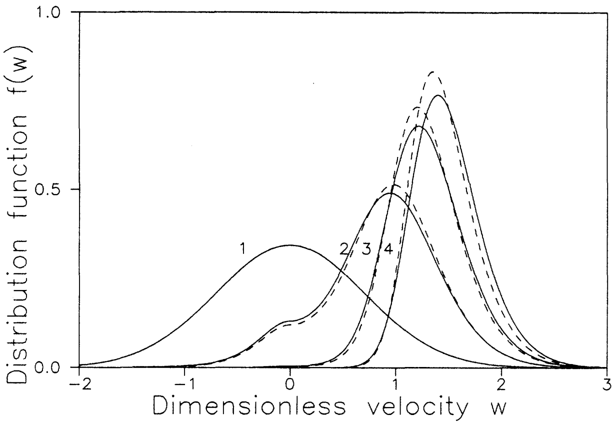

FIG. 1. The distribution function of the nondegenerate gas at different values of electric field ϵ at $\lambda=1$. (1) $\epsilon=0$; (2) $\epsilon=0.02$; (3) $\epsilon=0.05$; (4) $\epsilon=0.1$. The solid line represents exact screening, the dashed line represents simplified screening.

$$\frac{1}{\tau(v_x)} = \left| \frac{eE_x}{m} \frac{df_0/dv_x}{f_0(v_x)-f(v_x)} \right| . \tag{17}$$

Using the same combination with the function (16), we have

$$\tau(wv_T) = \left| \frac{mv_T}{2ew}\int_0^w \frac{du}{E_0(u)} \right| . \tag{18}$$

To obtain the field function $\gamma(w)$, one should first find the explicit form of the equilibrium electron screening function $\Delta\epsilon_0(\omega,k_x)$ and the correlator $\langle \delta\varphi_S^2 \rangle_{\omega,k_x}$ [see (12) and (13)].

III. THE DIELECTRIC FUNCTION OF THE EQUILIBRIUM 1DEG AND THE CORRELATOR OF SCATTERING POTENTIALS

The calculation of the function $\Delta\epsilon_0(\omega,k_x)$ is performed according to the formula (7) with the equilibrium distribution function $f_0(v_x)$. For the nondegenerate gas, the normalized function is

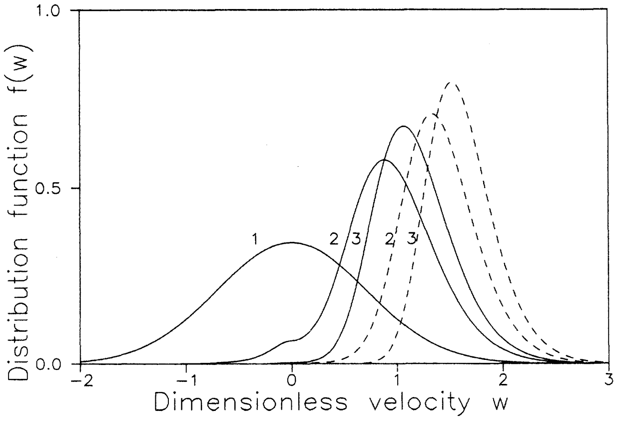

FIG. 2. The distribution function of the nondegenerate gas at different electric fields ϵ and $\lambda=5$ (solid line) and $\lambda=0.5$ (dashed line). (1) $\epsilon=0$; (2) $\epsilon=0.05$; (3) $\epsilon=0.1$.

$$f_0(w)=\frac{\sqrt{\pi}\hbar n^{(1)}}{mv_T}\exp(-w^2) . \tag{19}$$

Substituting (19) in Eq. (7) we obtain

$$\Delta\epsilon_0(k_x v_T w, k_x) = \frac{e^2 n^{(1)}}{T_S}[1+F(w)] , \tag{20}$$

where

$$F(w)=\frac{w}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{\exp(-z^2)}{z-w-i0}dz . \tag{21}$$

For the highly degenerate electron gas, we use the step function

$$f_0(w)=\vartheta(w_F-|w|)=\begin{cases}1 & \text{for } w_F>|w| \\ 0 & \text{for } w_F<|w| .\end{cases} \tag{22}$$

Here $w_F=v_F/v_T=\sqrt{F/T_S}=\hbar\pi n^{(1)}/\sqrt{8mT_S}$.

As distinct from the precise quantum-mechanical expression, which has a logarithmic form (see, for example, Ref. 10) in the classical approach ($|q_x| \ll k_F$), the dielectric function of the degenerate gas has the very simple form

$$\mathrm{Re}\Delta\epsilon_0(k_x v_T w, k_x) = \frac{4me^2}{\pi^2\hbar^2 n^{(1)}} . \tag{23}$$

[The imaginary part of the function $\Delta\epsilon_0$ actually is nearly δ type and gives no noticeable contribution to the $E_0(w)$.]

The correlator of scattering potentials $\langle \varphi^2 \rangle_{\omega,\mathbf{k}}$ for the three-dimensional case is introduced by the formula

$$\langle [\varphi(\omega,\mathbf{k}),\varphi(\omega',\mathbf{k}')]_+ \rangle = (2\pi)^4 \langle \varphi^2 \rangle_{\omega,\mathbf{k}} \delta(\omega+\omega')\delta(\mathbf{k}+\mathbf{k}') , \tag{24}$$

where $\mathbf{k}$ is a three-dimensional vector and there is the correlator of Fourier components of potentials $\varphi(\mathbf{r},t)$ on the left side. It is adopted that $[A,B]_+=\frac{1}{2}(AB+BA)$. The value $\delta\varphi$ entering Eq. (5) is defined as $\delta\varphi=\varphi-\langle\varphi\rangle$. For the infinitely thin quantum wire (see Refs. 10 and 12),

$$\langle \delta\varphi^2 \rangle_{\omega,k_x} = \frac{1}{(2\pi)^2}\int \langle \delta\varphi^2 \rangle_{\omega,\mathbf{k}} dk_y dk_z . \tag{25}$$

For equilibrium longitudinal-acoustic phonons with the dispersion law $\omega=sk$ (here s is the sound velocity, $k=\sqrt{k_x^2+k_y^2+k_z^2}$) the correlator takes the form (see Ref. 12)

$$\langle \delta\varphi_{\mathrm{ac}}^2 \rangle_{\omega,\mathbf{k}} = \left[\frac{E_d}{e}\right]^2 \frac{\pi\hbar\omega}{2\rho s^2}\coth\left[\frac{\hbar\omega}{2T_S}\right] \times [\delta(\omega-sk)+\delta(\omega+sk)] . \tag{26}$$

Here E_d is the constant of the deformation potential and ρ is the crystal density. Substituting (26) into Eq. (25) we obtain finally

$$\langle \delta\varphi_{\mathrm{ac}}^2 \rangle_{\omega,k_x} = \frac{E_d^2 \hbar \omega^2}{4e^2 \rho s^4} \coth \left[\frac{\hbar|\omega|}{2T_S} \right] \vartheta(\omega^2 - s^2 k_x^2) \; . \tag{27}$$

According to Eq. (13) the value $\langle \delta\varphi_{\mathrm{ac}}^2 \rangle_{k_x v_T w, k_x}$ enters the field $E_0(w)$. This results immediately that the velocities $|w| \le s/v_T$, i.e., $|v_x| \le s$ give no contribution in the integral (12). Indeed, in the classical approach the energy-momentum conservation law cannot be satisfied at $|v_x| \le s$ for the 1DEG being scattered by three-dimensional acoustical phonons.

As the other scattering system, we consider impurities (with the charge e_I) disposed macroscopically homogeneously in three-dimensional space with the concentration $n_I^{(3)}$. For this case, we have for a single impurity

$$\varphi_I(\omega, \mathbf{k}) = 2\pi\delta(\omega) \frac{4\pi e_I}{\epsilon_L \mathbf{k}^2} \; . \tag{28}$$

The value averaged over the positions of impurities is

$$\langle \delta\varphi_I^2 \rangle_{\omega,\mathbf{k}} = \frac{32\pi^3 e_I^2 n_I^{(3)}}{\epsilon_L^2 \mathbf{k}^4} \delta(\omega) \; . \tag{29}$$

Substituting (29) into (25) we obtain

$$\langle \delta\varphi_I^2 \rangle_{\omega,k_x} = \frac{8\pi^2 e_I^2 n_I^{(3)}}{\epsilon_L^2 k_x^2} \delta(\omega) \; . \tag{30}$$

IV. THE CALCULATION OF THE FIELD $E_0(w)$ AND THE FIELD FUNCTION $\gamma(w)$

Regarding the mentioned two scattering systems, we can represent the characteristic field $E_0(w)$ as a sum

$$\begin{aligned} E_0(w) &= E_0^{\mathrm{ac}}(w) + E_0^I(w) \\ &= \frac{|e|}{2\pi m v_T \eta(w)} \int_0^{K_1} k_x^2 dk_x [\langle \delta\varphi_{\mathrm{ac}}^2 \rangle_{k_x v_T w, k_x} \\ &\qquad + \langle \delta\varphi_I^2 \rangle_{k_x v_T w, k_x}] \; . \end{aligned} \tag{31}$$

Here, in accordance with the classical approach, we limit the integration with the value $K_1 = k_T = m v_T/\hbar$ for the nondegenerate gas and $K_1 = k_F = m v_F/\hbar = \pi n^{(1)}/2$ for the degenerate gas, and use the forms

$$\eta(w) = \left| 1 + \frac{2e^2 n^{(1)}}{\epsilon_L T_S} [1 + F(w)] \ln \left[\frac{\sqrt{8}}{k_T l} \right] \right|^2 \tag{32}$$

for the nondegenerate gas, and

$$\eta(w) = \left| 1 + \frac{8e^2 m}{\pi^2 \hbar^2 n^{(1)} \epsilon_L} \ln \left[\frac{\sqrt{8}}{k_F l} \right] \right|^2 \tag{33}$$

for the degenerate gas. Performing in (31) the integration with the functions (27) and (30), we obtain

$$E_0^I(w) = \frac{E_1}{|w| \eta(|w|)} \; , \tag{34}$$

$$E_0^{\mathrm{ac}}(w) \approx E_2 w^2 \vartheta(|w| - w_1)(p + |w|^{-1})/\eta(|w|) \; , \tag{35}$$

where $p = k_F/k_T$; $w_1 = s/v_T$;

$$E_1 = \frac{\pi |e| e_I^2 n_I}{\epsilon_L^2 T_S} \; , \tag{36}$$

$$E_2 = \left[\frac{E_d}{e} \right]^2 \frac{|e| k_T^4 T_S}{8\pi \rho m s^4} \; . \tag{37}$$

We also introduce the dimensionless electric field

$$\epsilon = -eE_x/(2|e|E_1) \tag{38}$$

and the ratio of characteristic fields

$$\lambda = E_2/E_1 \; . \tag{39}$$

Then the field function $\gamma(w)$ can be written as

$$\gamma(w) = 2\epsilon \int_0^w \frac{u\eta(u)du}{1 + \lambda u^2 (1 + pu) \vartheta(u^2 - w_1^2)} \; . \tag{40}$$

This function reflects the details of the electron screening. The details of the screening are especially significant for the nondegenerate gas. For the highly degenerate gas, we use the constant value (33) as $\eta(u)$.

To perform the analytical integration in (40), we set in (32) approximately

$$\mathrm{Re}F(w) = -1 + \exp(-2w^2) \; . \tag{41}$$

Then for the nondegenerate gas ($p \ll 1$), we have the form (16) with

$$\begin{aligned} \gamma(w \ge 0) = \epsilon \Bigg\{ & \frac{a^2}{4} \left[1 + \pi + \frac{4}{a} \right] + \vartheta(w_1 - w)\kappa(w) + \Bigg[\ln \left| \frac{g(w,\lambda)}{g(w_1,\lambda)} \right| + 2a \left[1 - \frac{\pi a}{2\lambda} \right] R(w, w_1, \lambda) + a^2 R(w\sqrt{2}, w_1\sqrt{2}, \lambda/2) \\ & + \frac{\pi a^2}{2} (e^{-2w_1^2} - e^{-2w^2}) + \lambda\kappa(w_1) \Bigg] \lambda^{-1} \vartheta(w - w_1) \Bigg\} \; , \end{aligned} \tag{42}$$

where

$$\kappa(w) = w^2 - a(1 + \pi a/4 + w^2 \pi a/2) e^{-2w^2} - \frac{a^2}{4} e^{-4w^2} \; ,$$

$$R(w, w_1, \lambda) = e^{2/\lambda} \{ \mathrm{Ei}[g(w,\lambda)] - \mathrm{Ei}[g(w_1,\lambda)] \} \; ,$$

$$a = \frac{2e^2 n^{(1)}}{\epsilon_L T_S} \ln \left[\frac{\sqrt{8}}{k_T l} \right], \quad g(w,\lambda) = -2w^2 + 2\lambda^{-1} \; .$$

Here Ei is the integral-exponent function. Remember that $\gamma(w) = -\gamma(-w)$ and $f(-w,\epsilon) = f(w,-\epsilon)$.

For the highly degenerate gas ($p \gg 1$)

$$\gamma(w>0)=\epsilon\{w^2+\vartheta(w-w_1)[w_1^2-w^2+\mu^{-2}\sigma(w,\mu)]\}\ , \tag{43}$$

where $\mu=(\lambda p)^{1/3}$ and

$$\sigma(w,\mu)=\tfrac{1}{6}\ln\left[\left(\frac{1+\mu w_1}{1+\mu w}\right)^2\frac{1-\mu w+\mu^2w^2}{1-\mu w_1+\mu^2w_1^2}\right] + \frac{1}{\sqrt{3}}\left[\arctan\frac{2\mu w-1}{\sqrt{3}}-\arctan\frac{2\mu w_1-1}{\sqrt{3}}\right]. \tag{44}$$

The functions (42) and (43) give the approximate analytical forms of the distribution function for the limit cases $k_T/k_F \ll 1$ and $k_T/k_F \gg 1$, respectively. However, in numerical calculations of figures presented below, we use the precise solutions of the kinetic equation in quadratures (14).

V. THE NUMERICAL CALCULATION OF THE DISTRIBUTION FUNCTION AND MACROSCOPIC CHARACTERISTICS OF THE ONE-DIMENSIONAL GAS. DISCUSSION OF RESULTS

Below we represent the graphs of the distribution function obtained by means of the numerical calculation of the formulas (14) and (40).

In what follows, we use the parameters pertinent to GaAs: $m=6\times10^{-29}$ g, $\rho=5.3$ g/cm^3, $s=4\times10^5$ cm/s, $E_d=7.8$ eV, and $\epsilon_L=13$. The electron gas can be considered as nondegenerate at $v_F \ll v_T$, or $\sqrt{2T_S/m} \gg \hbar\pi n^{(1)}/2m$. The condition of high degeneration is straightly opposite. At $T_S=4$ K, the gas is nondegenerate at $n^{(1)} \ll 10^5$ cm^{-1} and highly degenerate at $n^{(1)} \gg 10^5$ cm^{-1}. For the given temperature and charged impurities concentration $n_I^{(3)}=10^{15}$ cm^{-3}, the characteristic fields become, respectively, $E_1 \approx 10^3$ V/cm, and $E_2 \approx 4.3$ V/cm. Note [see (37)] that $E_1 \propto T_S^{-1}$ and $E_2 \propto T_S^3$. For the given values of parameters $E_1=E_2=280$ V/cm at $T_S \approx 16$ K.

The values mentioned above are used in the graphs that follow. For the nondegenerate gas, we take the concentration $n^{(1)}=5\times10^4$ cm^{-1}, correspondingly the parameter $p=0.3$. For the degenerate gas, $n^{(1)}=3.27\times10^5$ cm^{-1} and the corresponding $p=2$.

In Fig. 1, the solid lines correspond to the calculation of the distribution function (16) and (40) with the exact screening function (32). The dashed lines correspond to the simplified treating of screening with the approximation (41). Figure 2 shows how the form of the distribution function depends on the electric field ϵ and on the parameter λ [see Eq. (39)]. Note the zero inclination at the point $w=0$ (see discussion in Sec. II). At the weak electric field, the form of the distribution function is rather complex. A somewhat similar complex form of the distribution function was obtained in Ref. 14 by numerical calculations of kinetic equation for the three-dimensional case. The dependences of the average velocity (drift velocity) on the field, practically the current-voltage characteristics of the nondegenerate gas, are shown in Fig. 3. Here and below $\langle w^n \rangle = \int_{-\infty}^{\infty} w^n f(w)dw / \int_{-\infty}^{\infty} f(w)dw$. At high fields all the characteristics are sublinear: It is typical of the phonon scattering.

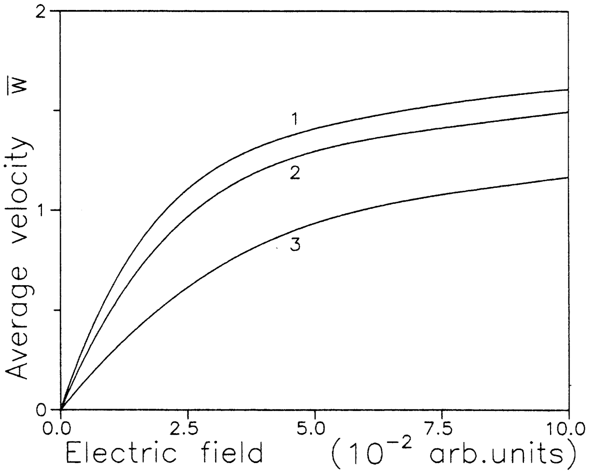

FIG. 3. Dimensionless drift velocity of the nondegenerate gas vs electric field. (1) $\lambda=0.5$; (2) $\lambda=1$; (3) $\lambda=5$.

Consider two other macroscopic characteristics of the electron gas. The first is the total kinetic energy of the gas. The second is the difference between the former and the energy of the drift of the gas as a whole, i.e., it is the energy of the intrinsic disorderly motion. The first value, normalized on the kinetic energy of the equilibrium gas, we denote as Q_1 and the second normalized by the same way as Q_2. So

$$Q_1(\epsilon)=\frac{\langle w^2\rangle}{\langle w^2\rangle_{\epsilon=0}},\quad Q_2(\epsilon)=\frac{\langle w^2\rangle-\langle w\rangle^2}{\langle w^2\rangle_{\epsilon=0}}\ . \tag{45}$$

Note that while the total energy Q_1 monotonously rises

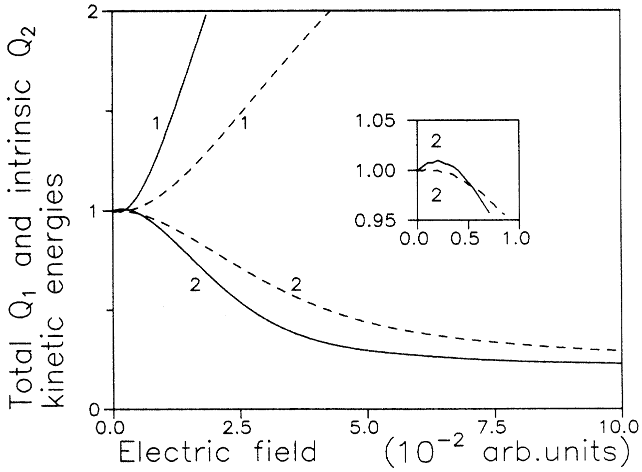

FIG. 4. Normalized kinetic and intrinsic energies Q_1 and Q_2 of the nondegenerate gas vs electric field ϵ at $\lambda=1$ (solid lines) and $\lambda=5$ (dashed lines). (1) $Q_1(\epsilon)$; (2) $Q_2(\epsilon)$.

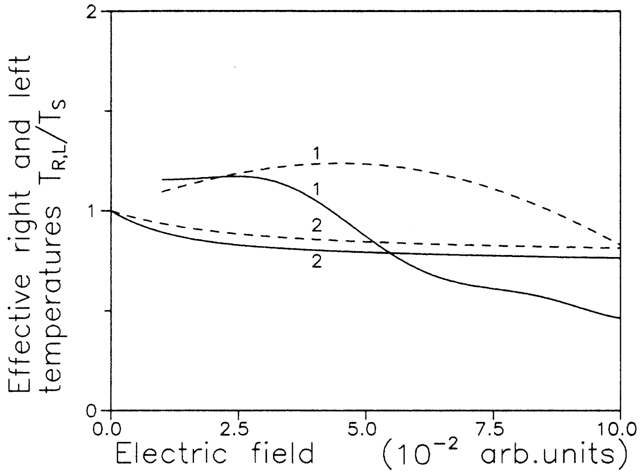

FIG. 5. Field dependence of the right and left effective temperatures of the nondegenerate gas. (1) T_L/T_S; (2) T_R/T_S. $\lambda=1$ (solid lines), $\lambda=5$ (dashed lines).

with the increase of the field ϵ, the intrinsic energy Q_2 is the nonmonotonous function of the field (see inset in Fig. 4). At high fields one can say about the "cooling" of the electron gas (see also Ref. 15). One can notice in Figs. 1 and 2 that at sufficiently strong electric fields the distribution function acquires a form rather similar to the Gaussian function. The more detailed observation discerns here the different curvatures on the right side of the function and on the left one. Therefore, it seems to the point to approximate the distribution function with the following forms:

$$f(w)=\begin{cases} A_R\exp\left[-\dfrac{(w-\langle w\rangle)^2 T_S}{T_R}\right] & \text{on the right side} \\ A_L\exp\left[-\dfrac{(w-\langle w\rangle)^2 T_S}{T_L}\right] & \text{on the left side ,} \end{cases} \tag{46}$$

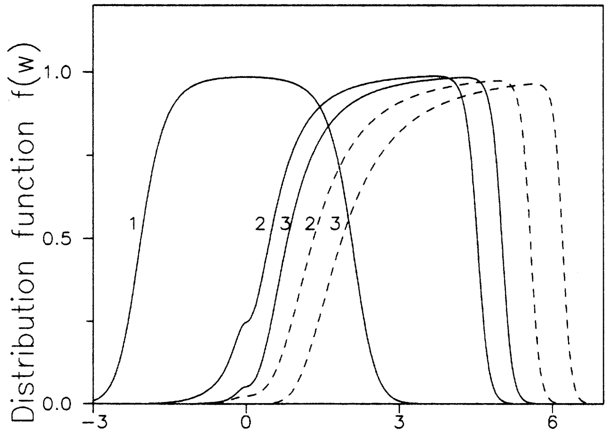

FIG. 6. Distribution functions of the degenerate gas. (1) $\epsilon=0$; (2) $\epsilon=0.05$; (3) $\epsilon=0.1$. $\lambda=5$ (solid lines), $\lambda=1$ (dashed lines).

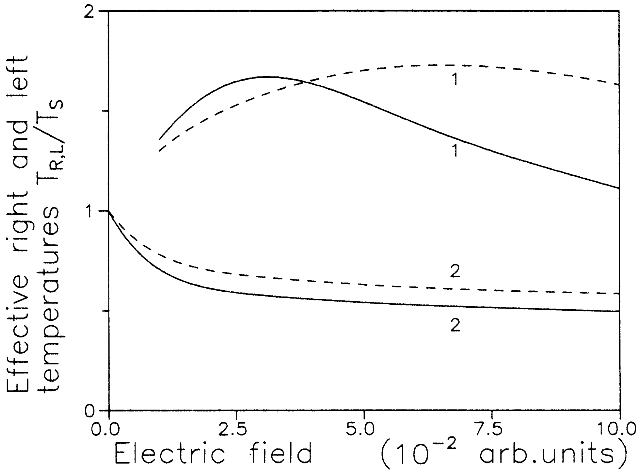

FIG. 7. Field dependence of the right and left effective temperatures of the degenerate gas. (1) T_L/T_S; (2)T_R/T_S. $\lambda=1$ (solid lines), $\lambda=5$ (dashed lines).

where $A_{R,L}$ and $T_{R,L}$ are the parameters to pick out. By this manner we actually introduce two effective temperatures T_R and T_L. The graphs of the right and left effective temperatures versus the electric field ϵ at two different λ are plotted in Fig. 5. We draw the curves for T_L beginning from the nonzero field ϵ because of the complex non-Gaussian form of the left side of the distribution function at small fields. The graph $T_R(\epsilon)$ shows the monotonous cooling. Along with that, the field dependence of T_L changes from the heating to the cooling at high fields.

The results obtained show that the one-temperature model of the distribution function and the use on this basis of the terms "heating" and "cooling" can be meaningless. Really, if in the obvious manner we used the model of Gaussian with a single effective temperature T, we would obtain $T/T_S=Q_2$; in accordance with Fig. 4, it would be the cooling at $\epsilon>5\times10^{-3}$. On the other hand, if an effective temperature were introduced through the

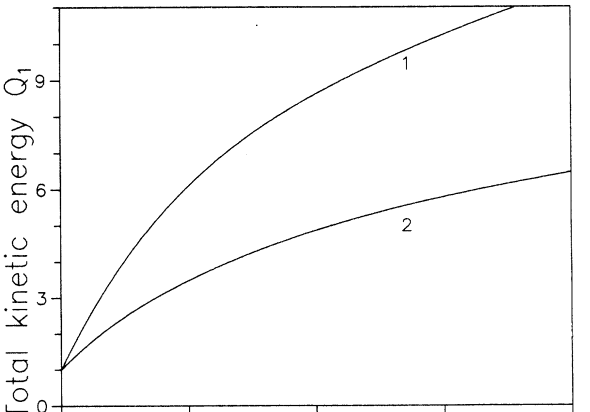

FIG. 8. Normalized kinetic energy Q_1 of the degenerate gas vs electric field ϵ. (1) $\lambda=1$; (2) $\lambda=5$.

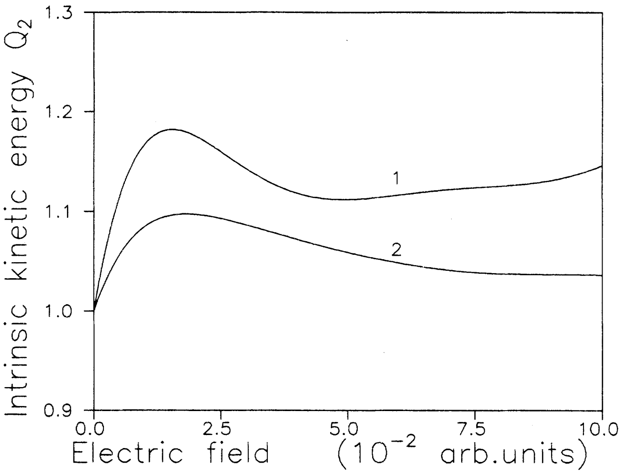

FIG. 9. Normalized intrinsic energy Q_2 of the degenerate gas vs electric field ϵ. (1) $\lambda=1$; (2) $\lambda=5$.

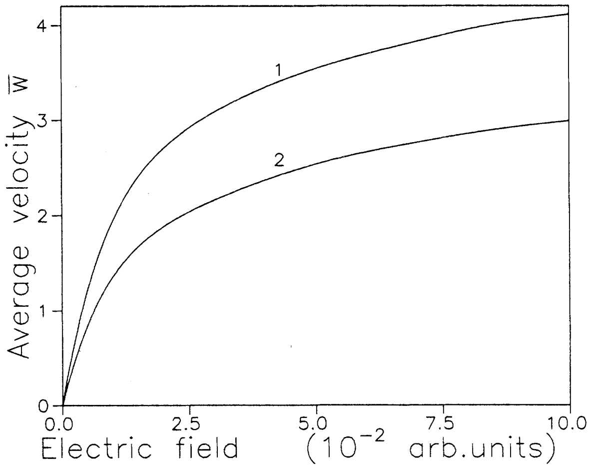

FIG. 10. Dimensionless drift velocity of the degenerate gas vs electric field. (1) $\lambda=1$; (2) $\lambda=5$.

total kinetic energy, i.e., $T/T_S=Q_1$, it would follow the heating from Fig. 4. At the same time, Fig. 5 shows that in the two-temperature model there is no cooling or heating of the electron gas as a whole since the left and the right effective temperatures can depart from the T_S in the opposite directions.

Now we go to the discussion of results related to the degenerate electron gas. Here the distribution function has the form (14) and we use (40) and (33). The following numerical calculations were performed for the same parameters m, ϵ_L, T_S, E_d, λ, s, and $n_I^{(3)}$ as for the nondegenerate gas but at the concentration $n^{(1)}=3.27\times10^5$ cm^{-1}.

Figure 6 shows the distribution function at different electric fields ϵ and different values of the parameter λ [see Eq. (39)]. All the graphs have the zero inclination at $w=0$ and something as a plateau, which shifts with change of ϵ. At high ϵ the graphs get the form similar to the shifted Fermi distribution with different curvatures on the right and left sides though. By the same manner as above, we approximate the distribution function for great ϵ with the forms:

$$f(w)=\begin{cases}\left[\exp\left[\dfrac{(w-\langle w\rangle)^2-w_F^2}{T_R}T_S\right]+1\right]^{-1} & \text{on the right side}\\ \left[\exp\left[\dfrac{(w-\langle w\rangle)^2-w_F^2}{T_L}T_R\right]+1\right]^{-1} & \text{on the left side ,}\end{cases}\qquad(47)$$

where T_R and T_L are the right and left effective temperatures, w_F is the Fermi velocity found from the normalizing condition at $\epsilon=0$. The field dependences of the T_R and T_L are shown in Fig. 7. The dependences of the kinetic energy and the energy of the intrinsic disorderly motion [see Eq. (45)] on the electric field are represented in Figs. 8 and 9; the current-voltage characteristics are represented in Fig. 10. These three latter figures have, in general, the same features as those for nondegenerate gas (see Figs. 3, 4, and 5). For example, Fig. 7 shows the necessity to use the two-temperature model instead of the one-temperature model.

The results obtained show that the investigation of the precise solution of the kinetic equation allows us to find a lot of interesting and even unexpected earlier information concerning the form of the distribution function of the nonequilibrium gas. We regard as nontrivial details the zero inclination in the origin of coordinate related to the elastic scattering and the necessity to describe the form by two effective temperatures. We have considered the classical kinetic equation for particles moving in an infinitely thin wire. In this work, we have used the screening function with the equilibrium distribution function. In the following papers we intend to consider the case of the nonequilibrium screening function and to find the ranges when one can use the equilibrium form. We hope also to consider more general models of kinetic equation. The results of this work will be used as benchmarks for the cases that do not allow precise solutions.

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