Quasiparticle spectra and the calculation of thermodynamics for a two-dimensional Fermi liquid

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If the dynamical quasiparticle spectrum of a Fermi liquid (FL), $\epsilon_p^{\rm dy}$, given by the poles of the single-particle propagator, is used in the expression for the entropy of a noninteracting Fermi gas, one finds that the result is different from that calculated directly from the thermodynamic potential. In particular, the coefficient of the $T^3 \ln T$ terms, the leading correction to the linear T dependence for a three-dimensional (3D) FL, is overestimated when calculated from $\epsilon_p^{\rm dy}$. The thermodynamic properties may be calculated correctly from a second quasiparticle spectrum, the statistical quasiparticle spectrum, $\epsilon_p^{\rm st}$. In this paper I calculate the corrections to leading linear energy and temperature dependences for a 2D FL. I compare the difference between the two spectra in the 2D and 3D cases. Although these differences are qualitatively the same, they are quantitatively larger in the 2D case.

I. INTRODUCTION

Calculations of the properties of many-body systems using perturbation theory involve terms with denominators given by the differences between energy levels for intermediate states. In a finite-size system with a discrete spectrum and an energy-level spacing $\sim \frac{1}{L}$, where L is the size of the system,[1] there are terms in which the energy denominators can vanish. Contributions to calculated quantities of these terms are smaller than the contributions where the energy denominators do not vanish by a factor of volume^{-1} and so are negligible for large systems.[2] As the volume of the system goes to infinity, the energy-level spacing approaches zero and the spectrum becomes quasicontinuous. The energy denominators can become arbitrarily small and are referred to as "vanishing" energy denominators. There is no volume^{-1} factor associated with these terms; they are responsible for the leading corrections to low temperature and energy behavior of the many-fermion systems. The presence of vanishing energy denominators introduces the need for a regularization procedure in order to calculate such terms. Regularization schemes for the calculation of thermodynamic properties have been investigated for a system of fermions with random impurities by Balian and DeDominicis[3] and by Luttinger and Liu.[4] They showed that the quasiparticle spectrum that enters the calculation of thermodynamic properties, the statistical quasiparticle spectrum, $e_p^{\rm st}$, is different from the the dynamical quasiparticle spectrum, $e_p^{\rm dy}$, calculated from the poles of the single-particle propagator.

Luttinger and Ward[5] derived an expression for the thermodynamic potential of an interacting Fermi liquid (FL) in three dimensions (3D), which consists of the difference between two terms. One term is given by the fully renormalized single-particle propagator and self-energy. The other is the sum of all skeleton diagrams, closed-linked diagrams without self-energy insertions, whose lines are fully renormalized propagators. Starting from this expression, Carneiro and Pethick[6] showed how the regularization of contributions to the entropy for a 3D FL, S, leads to extra terms beyond the result given by the single-particle propagator and self-energy alone, $S^{\rm dy}$. The difference comes from the fact that the closure line in the self-energy contribution has a Matsubara frequency and so does not have to be regularized while the corresponding line in the temperature derivative of the skeleton diagram does. This difference is clearly associated with the vanishing energy denominators in perturbation theory, which arise from real scattering processes involving at least two vanishing energy denominators. As the temperature approaches zero, the phase space for scattering falls and this difference, $S - S^{\rm dy}$, comes from terms with two vanishing energy denominators. These processes lead to $T^3 \ln T$ contributions to the entropy of 3D FL.

In their work Carneiro and Pethick[6] considered fully renormalized propagators, which went beyond the earlier calculations of these terms based on perturbation theory.[7–10] In these earlier perturbation theory calculations, the interaction between free fermions was described by a contact interaction. These calculations had shown that associated with the $T^3 \ln T$ terms in the entropy there is a $\xi_p^3 \ln|\xi_p|$ term in the real part of the single-particle self-energy, $\Sigma(p,\xi_p)$ where ξ_p is the difference between the quasiparticle energy and the chemical potential. Pethick and Carneiro[11] also calculated these effects within the Landau theory of Fermi liquids.[12] In this work they discussed the difference between $\epsilon_p^{\rm st}$ and $\epsilon_p^{\rm dy}$ in terms of the T matrix and the reactance matrix. This treatment parallelled the work based on scattering theory by Fukuda and Newton[1] and DeWitt[13] who were the first to discuss the relation between phase shifts and energy-level shifts as the volume of the system goes to infinity. More recently Coffey and Bedell[14] have shown that in 2D the leading corrections to linear temperature dependence in the entropy go as T^2 and have corresponding terms in $\Sigma(p,\xi_p)$ which go as $\xi_p|\xi_p|$.

2D FL's have recently been of general interest as mod-

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els for the normal state properties of the high-T_c cuprate superconductors. The generic temperature dependences of FL's have not been observed in transport and thermodynamic properties. This may be partly due to the high value of the superconducting transition temperature in these materials. In this case, the temperature dependence of quantities in the normal state are measured at such high temperatures that the temperature dependence is dominated by the corrections to the leading low-temperature dependences. Another possible explanation is that the ground state of a system of fermions in 2D is not a FL but is similar to a 1D Luttinger liquid in which the weight in the quasiparticle pole vanishes at the Fermi surface.[15]

The principle focus of the most recent work is to find out if the FL ground state is stable in 2D or if the ground state is closer to that suggested by Anderson.[15] The aim of much of this work is to find signatures of the breakdown of perturbation theory other than the conventional charge-density wave (CDW), spin-density wave (SDW), or BCS instabilities. Serene and Hess[16] have investigated the leading frequency dependence of the self-energy and the leading temperature dependence of the thermodynamic potential with the Hubbard model. Including both particle-particle and particle-hole channels they found results consistent with a finite quasiparticle weight at the Fermi surface and a T^2 dependence for the thermodynamic potential as expected for a Fermi liquid. Other investigations have concentrated on the low doping regime in which $\Sigma(\mathbf{k}, E)$ is approximated by an expansion in the particle-particle channel.[17,18] This approximation becomes exact in the limit of very low density. Engelbrecht and Randeria[17] discovered that as a result of the two-dimensionality a two-particle bound state appears below the band with a repulsive interaction. This is a collective effect absent from 3D. However, it does not appear to influence the leading corrections to the low energy or temperature dependences of $\Sigma(\mathbf{k}, E)$ and the difference between 2D and 3D can be traced to the difference in phase space. No evidence has been found for the breakdown of perturbation theory and arguments have been presented showing that in general there is no breakdown for systems with dimensionality greater than one.[20] However, Anderson questions whether perturbation theory is valid to begin with.[21] Here I assume that the corrections to a free Fermi gas can be calculated in perturbation theory.

In this paper, I calculate the corrections to the leading term in the statistical and dynamical quasiparticle spectra for a 2D FL in pertubation theory using the same short-range interaction used previously in the investigation of 3D FL's,[7–10] and of 2D FL's.[16–18] I show that the difference between these two spectra in 2D depends on all powers of the scattering amplitude but the ratio of the coefficients of the leading temperature energy corrections is independent of the strength of the interaction within the approximation used here. This is in contrast to 3D where the difference is proportional to scattering amplitudes to the third power and the ratio approaches three only in the limit of strong interaction. I find that the temperature dependence of the quasiparticle spectrum contributes a larger fraction of the corrections to linear temperature dependence of the entropy in 2D than in 3D. The outline of the paper is as follows. In Sec. II, I introduce the paramagnon model and calculate the corrections to the linear ξ_p dependence of $\Sigma(p, \xi_p)$ at low temperatures close to the Fermi surface. In Sec. III, I calculate the thermodynamic potential for a 2D FL. In Sec. IV I compare the contribution to the entropy from the dynamical quasiparticle spectrum, $\epsilon_p^{\rm dy}$, with the entropy calculated from the statistical quasiaprticle spectrum, $\epsilon_p^{\rm st}$.

II. MODEL

First I calculate the leading corrections to the dynamical quasiparticle spectrum using a short-range interaction between fermions. This model was originally used to investigate the corrections to the linear temperature dependence of the specific heat for a 3D FL by Engelsberg and co-workers:[7,8]

$$H = \sum_{\mathbf{p},\sigma} \xi_{\mathbf{p}} c^\dagger_{\mathbf{p},\sigma} c_{\mathbf{p},\sigma} + \sum_{\mathbf{p},\mathbf{q},\sigma,\sigma'} I(\mathbf{q}) c^\dagger_{\mathbf{p},\sigma} c_{\mathbf{p}',\sigma'} c^\dagger_{\mathbf{p}'-\mathbf{q},\sigma'} c_{\mathbf{p}+\mathbf{q},\sigma}. \quad (1)$$

I is the the strength of the interaction and multiplied by the density of states of one spin is the paramagnon parameter, $\bar{I}$. The interaction is cutoff at $|\mathbf{q}| = q_c$. I will consider a doping regime in which the leading corrections to the FL behavior at low energy and temperature comes primarily from particle-hole pairs excitations. $\xi_{\mathbf{p}}$ is the Fermion dispersion, which I take to be $\frac{p^2 - p_F^2}{2m_0}$, where m_0 is the mass of the Fermion. The expression for the self-energy due to repeated scattering of particle-hole pairs is

$$\Sigma(p, \imath E_n) = -T \sum_{\mathbf{q},\sigma',\omega_l} G_0(\mathbf{p} - \mathbf{q}, \imath E_n - \imath\omega_l) V^{\rm eff}(\mathbf{q}, \omega_l), \quad (2)$$

where $\omega_l = 2\pi k_B T l$ are Bose Matsubara frequencies and T is the temperature. $G_0(\mathbf{p}, E)$ is the unperturbed Green function. $V^{\rm eff}(\mathbf{q}, \omega)$ is given by

$$V^{\rm eff}(\mathbf{q}, \omega) = \sum_{\mathbf{q}} \left[\nu_s \frac{V_s^2 \chi(\mathbf{q}, \omega)}{1 - V_s \chi(\mathbf{q}, \omega)} + \nu_a \frac{V_a^2 \chi(\mathbf{q}, \omega)}{1 - V_a \chi(\mathbf{q}, \omega)} \right], \quad (3)$$

$V_s = I$, $V_a = -I$, $\nu_s = 1/2$, and $\nu_a = 3/2$, and

$$\chi(\mathbf{q}, \omega) = \sum_{\mathbf{p}} \frac{f_{\mathbf{p}} - f_{\mathbf{p}-\mathbf{q}}}{\omega - (\xi_{\mathbf{p}-\mathbf{q}} - \xi_{\mathbf{p}})}, \quad (4)$$

where $f_{\mathbf{p}}$ are Fermi-Dirac distributions. The form of $\chi_{\rm 2D}(q, \omega)$ is shown in Figs. 1 and 2 for $q = 0.01 p_F$, $q = p_F$, and $q = 2.1 p_F$ at zero temperature.[24] The sharp structure in $\chi(q, \omega)$ in the long-wavelength limit leads to the strong temperature dependence shown in Figs. 3 and 4.

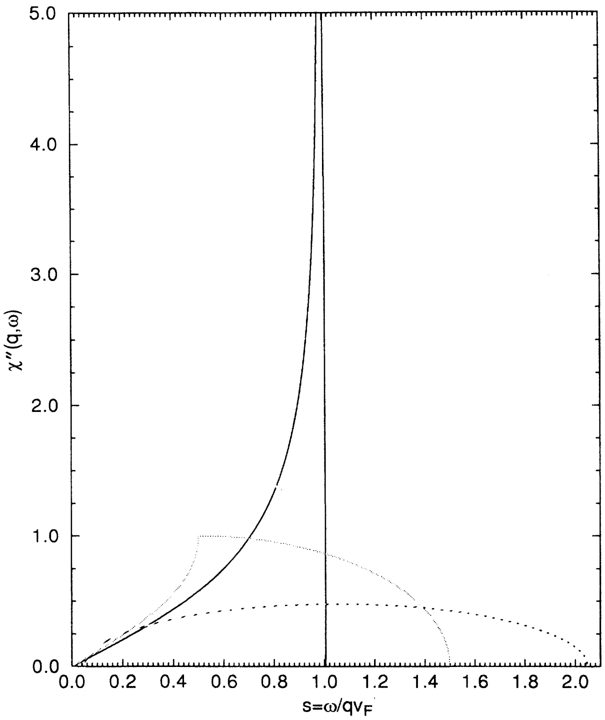

FIG. 1. Imaginary part of $\chi_{2D}(q,\omega)$ at zero temperature plotted as a function of $s = \frac{\omega}{qv_F}$ for $q = 0.01p_F$ (full line), $q = p_F$ (dashed line), and $q = 2.1p_F$ (dot-dashed line).

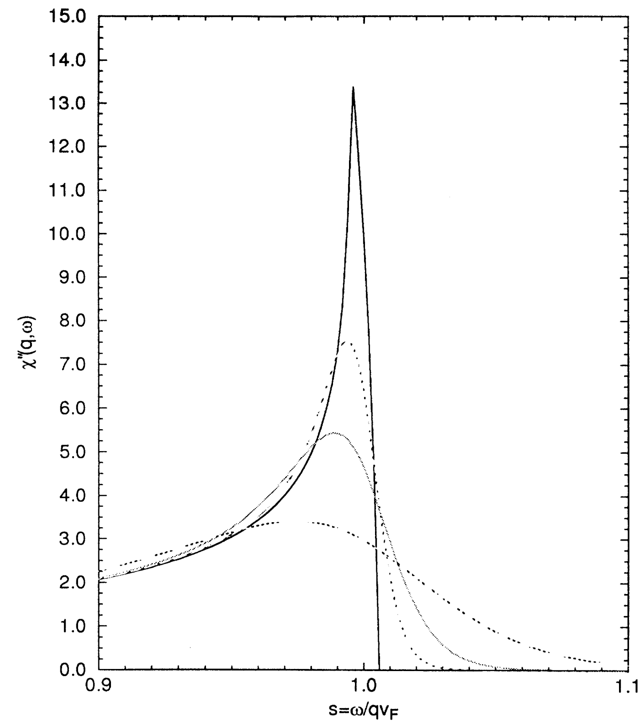

FIG. 3. Temperature dependence of the imaginary part of $\chi_{2D}(q,\omega)$ as a function of $s = \frac{\omega}{qv_F}$ for zero temperature (full line), $T = 0.01E_F$ (dot-dash line), $T = 0.02E_F$ (dash line), and $T = 0.05E_F$ (long dash line) with $q = 0.01p_F$.

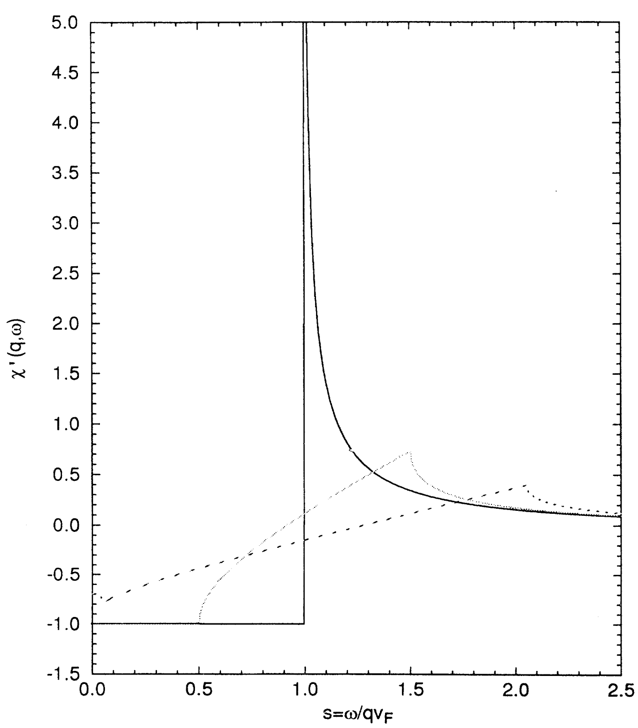

FIG. 2. Real part of $\chi_{2D}(q,\omega)$ at zero temperature plotted as a function of $s = \frac{\omega}{qv_F}$ for $q = 0.01p_F$ (full line), $q = p_F$ (dashed line), and $q = 2.1p_F$ (dot line).

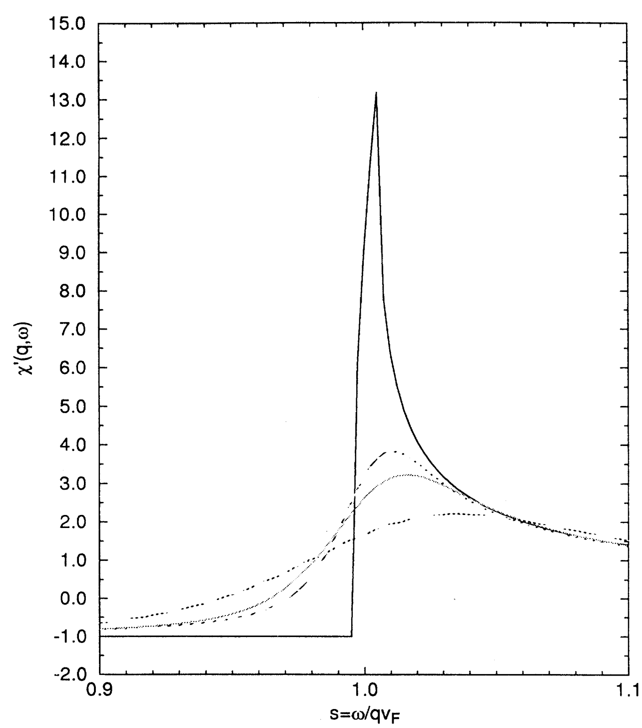

FIG. 4. Temperature dependence of the real part of $\chi_{2D}(q,\omega)$ as a function of $s = \frac{\omega}{qv_F}$ for zero temperature (full line), $T = 0.01E_F$ (dot-dash line), $T = 0.02E_F$ (dash line), and $T = 0.05E_F$ (long dash line) with $q = 0.01p_F$.

The real part of $\Sigma(p,\xi_p)$, $\mathrm{Re}\Sigma(p,\xi_p)$, is

$$\mathrm{Re}\Sigma(p,\xi_p) = -\sum_{\mathbf{q}}[1-2f_{\mathbf{p}-\mathbf{q}}]\mathrm{Re}V^{\mathrm{eff}}(\mathbf{q},\xi_p-\xi_{\mathbf{p}-\mathbf{q}}) + \sum_{\mathbf{q}}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}[1+2n_B(\omega)] \times \mathrm{Im}V^{\mathrm{eff}}(\mathbf{q},\omega)\mathrm{Re}G_0(\mathbf{p}-\mathbf{q},\xi_p-\omega), \tag{5}$$

where $n_B(\omega)$ is the Bose Einstein distribution function. The dynamical quasiparticle spectrum is given by the poles of the single-particle propagator. To the present order of approximation $\epsilon_p^{\mathrm{dy}} = \xi_p + \mathrm{Re}\Sigma(p,\xi_p)$.

In the particle-hole expansion, the leading corrections to the linear dependence on ξ_p and E in $\Sigma(p,E)$ come from the long-wavelength limit of the effective interaction, $V^{\mathrm{eff}}(q,\omega)$. In the long-wavelength limit and low energies $\xi_{\mathbf{p}} = v_F(p-p_F)$, $v_F = p_F/m_0$ is the Fermi velocity, and the q^2 are dropped in $\xi_{\mathbf{p}-\mathbf{q}}$. $\chi(q,\omega)$ becomes a function of $s = \frac{\omega}{qv_F}$ only. In the long-wavelength limit, $\chi(q,\omega)$ becomes

$$\chi_{2\mathrm{D}}(q,\omega) = \chi'_{2\mathrm{D}}(s) + \imath\chi''_{2\mathrm{D}}(s) = N(0)\left[-1+\imath\frac{s}{\sqrt{1-s^2}}\right]\Theta(1-|s|), \tag{6}$$

$$\chi_{3\mathrm{D}}(q,\omega) = \chi'_{3\mathrm{D}}(s) + \imath\chi''_{3\mathrm{D}}(s) = N(0)\left[-\left(1-\frac{s}{2}\ln\left|\frac{1+s}{1-s}\right|\right)+\imath\frac{\pi s}{2}\right] \times\Theta(1-|s|), \tag{7}$$

where $N(0)$ is the density of states at the Fermi surface for a 2D or 3D parabolic band. The difference in phase space leads to different results for 2D and 3D FL's. $\mathrm{Re}V^{\mathrm{eff}}(s)$ is an even function of s and, expanding in powers of s^2, one finds for 3D that the well-known $\xi_p^3\ln|\xi_p|$ corrections to the leading ξ_p term in $\mathrm{Re}\Sigma(p,\xi_p)$ come from the s^2 term in the expansion but that all orders of s^2 contribute to the ξ_p^3 term. In 2D on the other hand, all powers of s^2 contribute to $\xi_p|\xi_p|$, which is the leading correction to the linear ξ_p term in the self-energy. The corrections to the linear term ξ_p term in the self-energy are

$$\mathrm{Re}\delta\Sigma_{2\mathrm{D}}(p,\xi_p) = -B_{2\mathrm{D}}^{\mathrm{dy}}\xi_p\left[\frac{|\xi_p|}{8\pi E_F}+\frac{\ln 2T}{\pi E_F}\right] + O(\xi_p^3,\xi_pT^2), \tag{8}$$

where

$$B_{2\mathrm{D}}^{\mathrm{dy}} = -\sum_\lambda \nu_\lambda\int_0^1 d\alpha\frac{V^{\mathrm{eff}}(\alpha)-V^{\mathrm{eff}}(0)}{\alpha^2\sqrt{1-\alpha^2}} = -\sum_\lambda \nu_\lambda\int_0^1\frac{d\alpha}{\sqrt{1-\alpha^2}}\frac{A_\lambda^3}{\{(1-\alpha^2)+[A_\lambda\alpha]^2\}}. \tag{9}$$

The temperature dependence of $\chi_{2\mathrm{D}}(s)$ is limited to values of s close to one. Furthermore, the singularity in s comes from the use of bare propagators in the particle-hole propagator, $\chi_{2\mathrm{D}}(s)$. If the single-particle propagator lines in $\chi_{2\mathrm{D}}(s)$ were dressed so that lifetime effects due to scattering with other quasiparticles were included, $\mathrm{Im}\Sigma(p,E)\neq 0$, the sharp feature in $\chi_{2\mathrm{D}}(s)$ at zero temperature would be smoothed out and the strong temperature dependence would be considerably reduced. Consequently the temperature dependence of $\chi_{2\mathrm{D}}(s)$ will be neglected, although I will return to this point below. In 3D, Pethick and Carneiro[11] found

$$\mathrm{Re}\delta\Sigma_{3\mathrm{D}}(p,\xi_p) = \frac{B_{3\mathrm{D}}^{\mathrm{dy}}}{24}\xi_p\left[\left(\frac{\xi_p}{E_F}\right)^2+\left(\frac{\pi k_BT}{E_F}\right)^2\right] \times\ln\left|\frac{\max[\xi_p,T]}{q_cv_F\zeta}\right| + O(T^3,\xi_p^3), \tag{10}$$

where

$$B_{3\mathrm{D}}^{\mathrm{dy}} = \sum_{\lambda=s,a}\nu_\lambda A_\lambda^2\left[1-\frac{\pi^2}{4}A_\lambda\right]. \tag{11}$$

$A_s = \frac{\bar{I}}{1+\bar{I}}$, $A_a = \frac{-\bar{I}}{1-\bar{I}}$, and $\bar{I}$ is the paramagnon parameter. A_s and A_a are the scattering amplitudes in the symmetric (density) and antisymmetric (spin) channels. The coefficient in the log term, $q_cv_F\zeta$, depends on the details of the q dependence of the effective interaction determining the region of temperature over which the $T^3\ln T$ terms characterize the corrections to the linear temperature dependence in the entropy.[25] The temperature dependence of the self-energy increases the coefficient of the leading correction in both 2D and 3D appreciably.

These results may also be obtained by first calculating the imaginary part of the self-energy as a function of energy and then using the Kramers-Kronig relation to calculate the real part. Ignoring the momentum dependence, the energy dependence in 2D $\mathrm{Im}\Sigma(p,\epsilon)\sim\epsilon^2\ln|\epsilon|$.[26–28] Calculating $\Sigma(\mathbf{k},E)$ using an expansion in the particle-particle channel, Fukuyama and Hasegawa[19] found the same functional dependence on the energy. However, in these calculations of some of the $\epsilon^2\ln|\epsilon|$ terms come from particle-particle pairs with a net momentum $\sim 2k_F$ as opposed to long-wavelength particle-hole pairs in the present work. Moriya[22] pointed out that because $\chi''(q,\omega)$ was finite for small ω as $q\to 2k_F$, there are no $T^3\ln T$ terms from particle-hole pairs with $q\sim 2p_F$. As can be seen from Fig. 1, $\chi''(q,\omega)$ is finite as $q\to 2k_F$ for all ω in 2D and so the same result applies to 2D. There are no contributions to the $\xi_p|\xi_p|$ terms from finite q dependence in 2D.

III. THERMODYNAMIC PROPERTIES

The shift in the thermodynamic potential is calculated by coupling constant integration using the same approximation used in the calculation of the self-energy. This gives

$$\Delta\Omega(T,\mu) = k_B T \sum_{\mathbf{q},\omega_n,\lambda=s,a} \nu_\lambda \Big[\ln[1 - V_\lambda \chi(\mathbf{q},\omega_n)] + V_\lambda \chi(\mathbf{q},\omega_n) + \frac{1}{4} V_\lambda^2 \chi^2(\mathbf{q},\omega_n) \Big] = \Delta\Omega_{\text{qp}} + \Delta\Omega_{\text{coll.modes}} \quad (12)$$

where ω_n are Matsubara frequencies. The last term in the large brackets of Eq. (12) is present to account for the double counting of terms quadratic in V_λ, which appear both in the sum of the ring diagrams and ladder diagrams. $\Delta\Omega(\mu,T)$ is broken into a contribution from quasiparticles, $\Delta\Omega_{\text{qp}}(\mu,T)$ and a contribution from collective modes given by poles of the T matrix, $\Delta\Omega_{\text{coll.modes}}(T,\mu,V)$. First, we consider $\Delta\Omega_{\text{qp}}(\mu,T)$, which is

$$\Delta\Omega_{\text{qp}}(\mu,T) = \sum_{|\mathbf{q}|<q_c} \int_0^\infty \frac{d\omega}{\pi} \coth\left(\frac{\omega}{2T}\right) [F^{\text{st}}(q,\omega) + 2I^2 \chi'(\mathbf{q},\omega)\chi''(\mathbf{q},\omega)] , \quad (13)$$

where

$$F^{\text{st}}(q,\omega) = \sum_{\lambda=s,a} \nu_\lambda \left[\tan^{-1}\left(\frac{-V_\lambda \chi''(\mathbf{q},\omega)}{1 - V_\lambda \chi'(\mathbf{q},\omega)} \right) + V_\lambda \chi''(\mathbf{q},\omega_n) \right]. \quad (14)$$

Calculating the contribution to the entropy due to interactions $\Delta S_{\text{qp}}(T) = -\left[\frac{\partial \Delta\Omega_{\text{qp}}}{\partial T}\right]_\mu$ one finds

$$\Delta S_{\text{qp}} = -2 \sum_{|\mathbf{q}|<q_c} \int_0^\infty \frac{d\omega}{\pi} \frac{\partial n_B(\omega)}{\partial T} [F^{\text{st}}(q,\omega) + 2I^2 \chi'(\mathbf{q},\omega)\chi''(\mathbf{q},\omega)] - 2 \sum_{|\mathbf{q}|<q_c} \int_0^\infty \frac{d\omega}{2\pi} (1 + 2n_B(\omega)) \frac{\partial}{\partial T} [F^{\text{st}}(q,\omega) + 2I^2 \chi'(\mathbf{q},\omega)\chi''(\mathbf{q},\omega)]. \quad (15)$$

The temperature dependence of the term in brackets vanishes in the long-wavelength limit and so does not contribute to the leading corrections to the linear temperature dependence. This term is the counterpart of the contribution to $\text{Re}\Sigma(p,\xi_p)$, which contained the Bose factors.

The double-counting terms may also be dropped because the leading corrections come solely from long-wavelength effects. In this case, the $\mathbf{q}$'s of interest in the ring diagrams are small. On the other hand, the net momenta flowing in the ladders are small but the $\mathbf{q}$ exchanged at each interaction along the ladder is not in general small. As a result there is little overlap between the relevant values of $\mathbf{q}$ in the two channels. In effect, there is no double counting in the calculation of these leading corrections and I drop the contribution to the shift in the entropy from the double-counting terms.

The term in the effective interaction linear in $\chi''(q,\omega)$ contribute linear terms, cubic terms, and higher-order terms in temperature, but no T^2 terms so that from the point of view of the leading correction the effective interaction is

$$F^{\text{st}}(s) = \sum_{\lambda=s,a} \nu_\lambda \left[\tan^{-1}\left(\frac{-V_\lambda \chi''(s)}{1 - V_\lambda \chi'(s)} \right) + \frac{V_\lambda \chi''(s)}{1 - V_\lambda \chi'(s)} \right]. \quad (16)$$

Making the wavelength approximation, the effective interaction is a function only of odd powers of s. Expanding in effective interaction in powers of s, one finds in 3D that the $T^3 \ln T$ terms in the entropy come from the s^3 term in the expansion, which depends on A_λ^2 and A_λ^3. The corresponding T^2 terms in 2D come from all terms in the expansion. The leading corrections to the linear temperature dependence of the entropy are $\Gamma_{2\text{D}} T^2 + O(T^3)$ in 2D and $\Gamma_{3\text{D}} T^3 \ln T + O(T^3)$ in 3D where

$$\Gamma_{2\text{D}} = -\frac{3\zeta(3)n}{\pi T_F^2} B_{2\text{D}}^{\text{st}}, \quad (17)$$

$$\Gamma_{3\text{D}} = \frac{\pi^4}{20} \frac{n}{T_F^3} B_{3\text{D}}^{\text{st}},$$

and

$$B_{2\text{D}}^{\text{st}} = \sum_\lambda \nu_\lambda \int_0^1 ds \left[\frac{\frac{A_\lambda s}{\sqrt{1-s^2}} + \tan^{-1}\left(\frac{-A_\lambda s}{\sqrt{1-s^2}}\right)}{s^3} \right] = \frac{B_{2\text{D}}^{\text{dy}}}{2}, \quad (18)$$

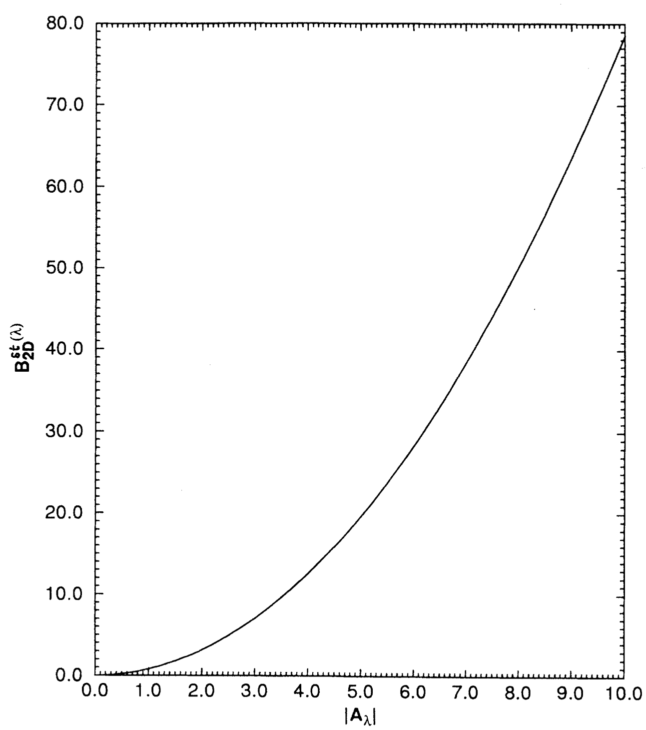

FIG. 5. The contribution to $B_{2\text{D}}^{\text{st}}$ which comes from the channel where $A_\lambda = \frac{|\bar{I}|}{1-|\bar{I}|}$ as a function of A_λ.

$$B_{3D}^{st} = \sum_\lambda \nu_\lambda A_\lambda^2 \left[1 - \frac{\pi^2}{12} A_\lambda\right], \tag{19}$$

and $\zeta(3) \simeq 1.202$ is the Riemann zeta function.[23] The variation of B_{2D}^{st} is dominated by the contribution of the channel in which $|A_\lambda| = \frac{I}{1-I}$ can increase as the Stoner instability is approached. In Fig. 5 I plot this contribution, $B_{2D}^{st}(\lambda)$ as a function of A_λ.

The contribution to the entropy in 2D from the collective mode determined from the poles of the T matrix goes as T^2, because the collective mode dispersion $\propto q$ and so has the same temperature dependence as the leading temperature dependence from interactions. The term that has been calculated here would be identified in measurements of the specific heat by subtracting off the contribution to the T^2 dependence determined from the measured collective and phonon dispersion curves. This is unlike the case in 3D where collective modes and phonons give a T^3 contribution and so contribute to the cutoff of the $T^3 \ln T$ term rather than its coefficient. Next I consider the contribution to the entropy from ϵ_p^{dy} and compare this spectrum with the statistical quasiparticle spectrum, ϵ_p^{st}.

IV. QUASIPARTICLE SPECTRA AND THERMODYNAMICS

Putting the dynamical quasipaticle spectrum, $\epsilon_p^{dy} = \xi_p + \mathrm{Re}\Sigma(p,\xi_p)$ into the expression for the entropy for a noninteracting Fermi gas and expanding to linear order in the self-energy, one finds for the leading correction to the linear temperature dependence

$$\Delta S^{dy} = \sum_{\mathbf{p}} \frac{\xi_p \Sigma(p,\xi_p)}{T^2} \frac{\partial f(\epsilon)}{\partial \epsilon}\bigg|_{\epsilon=\xi_p} = \sum_{\mathbf{p}} \Sigma(p,\xi_p) \frac{\partial f(\xi_p)}{\partial T}. \tag{20}$$

For the 2D case, one finds $\Delta S_{2D}^{dy} = \Gamma_{2D}^{dy} T^2$ and for the 3D case $\Delta S_{3D}^{dy} = \Gamma_{3D}^{dy} T^3 \ln T$. Γ_{2D}^{dy} and Γ_{3D}^{dy} are given by

$$\Gamma_{2D}^{dy} = -\frac{3}{\pi}\zeta(3)\left[\frac{3}{8} + \frac{\pi^2 \ln 2}{9\zeta(3)}\right]\frac{n}{T_F^2} B_{2D}^{dy} = -\frac{3}{\pi}\zeta(3)[1.007]\frac{n}{T_F^2} B_{2D}^{dy}, \tag{21}$$

$$\Gamma_{3D}^{dy} = \frac{\pi^4}{20}\left[\frac{7}{12} + \frac{5}{12}\right]\left(\frac{n}{T_F^3}\right) B_{3D}^{dy} = \frac{\pi^4}{20}\left(\frac{n}{T_F^3}\right) B_{3D}^{dy}. \tag{22}$$

The first terms in the square brackets in the expressions for Γ_{2D}^{dy} and Γ_{3D}^{dy} come from the zero-temperature self-energy and the second terms from the leading temperature dependence. Neglecting the temperature dependence in 2D would underestimate the magnitude of the coefficient by $\simeq \frac{3}{8}$ in 2D and by $\frac{7}{12}$ in 3D. The importance of taking the temperature dependence of the quasiparticle spectrum into account was first pointed out by Brenig *et al.*[10] for the 3D case and the effect is similar in 2D although somewhat bigger.

After making this correction for the use of the zero-temperature spectrum, there is still a difference between the entropy as calculated from the thermodynamic potential and the dynamical quasiparticle spectrum. This is due to the difference in the statistical and dynamical quasiparticle spectra. The expression for the dynamical quasiparticle spectrum, determined by $\Sigma(p,\xi_p)$, is given in Eq. (2). The corresponding part of the statistical quasiparticle spectrum can be found by rewritting the shift in the thermodynamic potential in terms of fermions rather than with a Bose distribution. Multiplying the Bose factor by $\chi''(q,\omega)$ and dividing $F^{st}(q,\omega)$ by $\chi''(q,\omega)$, the integral over ω of the Bose distribution times $\chi''(q,\omega)$ is turned into a sum over a fermion momentum of Fermi-Dirac distributions with a new effective interaction between the fermions

$$\Delta\Omega(\mu,T) = \sum_{\mathbf{q}} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \chi''(q,\omega) \tilde{F}(q,\omega)[2n_B(\omega)+1)] = \sum_{\mathbf{q},\mathbf{p}} f_{\mathbf{p}-\mathbf{q}}(1-f_{\mathbf{p}})\tilde{F}^{st}(q,\xi_p - \xi_{\mathbf{p}-\mathbf{q}}) + \sum_{\mathbf{q},\mathbf{p}} f_{\mathbf{p}-\mathbf{q}}(1-f_{\mathbf{p}})\tilde{F}^{st}(q,\xi_p - \xi_{\mathbf{p}-\mathbf{q}}), \tag{23}$$

where $\tilde{F}^{st}(q,\omega) = F^{st}(q,\omega)/\chi''(q,\omega)$. Taking a derivative gives the entropy and one finds an expression for ΔS given by

$$\Delta S = \sum_{\mathbf{p}} \frac{\partial f_{\mathbf{p}}}{\partial T} \Delta\epsilon_p^{st}, \tag{24}$$

where

$$\Delta\epsilon_p^{st} = \sum_{\mathbf{q}} (1 - 2f_{\mathbf{p}-\mathbf{q}})\tilde{F}^{st}(q,\xi_p - \xi_{\mathbf{p}-\mathbf{q}}). \tag{25}$$

This is the shift in the statistical quasiparticle spectrum due to interactions and is the counterpart of $\Sigma(p,\xi_p)$ in ϵ_p^{dy}. The difference between ΔS and ΔS^{dy} is seen to come from the two kinds of quasiparticle spectra, $\epsilon_p^{dy} = \xi_p + \mathrm{Re}\Sigma(p,\xi_p)$ and $\epsilon_p^{st} = \xi_p + \Delta\epsilon_p^{st}$. The coefficient of the $\xi_p|\xi_p|$ and the $\xi_p T$ terms in the two spectra are the same except that B_{2D}^{dy} in ϵ_p^{dy} is replaced by B_{2D}^{st} in ϵ_p^{dy} so that the use of the dynamical quasiparticle spectrum in the calculation of the T^2 terms due to quasiparticles in the entropy overestimates this term by a factor of 2 independent of the strength of the interaction for a 2D FL.

The coefficient of the T^2 term in the entropy is slightly different, Γ_{2D} or $\simeq 1.007\Gamma_{2D}$, depending on whether it is calculated from the statistical quasiparticle spectrum or the thermodynamic potential. The origin of this modest difference is the temperature dependence of $\chi''_{2D}(s)$. Going back to Eq. (23) and taking the derivative with respect to temperature of the expression with the Bose distribution, one finds

$$\Delta S = \sum_{\mathbf{q}} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left[2\chi''(q,\omega)\tilde{F}(q,\omega)\frac{\partial n_B(\omega)}{\partial T} + [2n_B(\omega)+1]\tilde{F}(q,\omega)\frac{\partial \chi''(q,\omega)}{\partial T} + \chi''(q,\omega)[2n_B(\omega)+1]\frac{\partial \tilde{F}(q,\omega)}{\partial T} \right]. \quad (26)$$

The first term in this expression is shifted in the entropy as calculated directly from $\Delta\Omega(\mu, T)$. In calculating $\Delta\epsilon_p^{\rm st}$ the effective interaction, $\tilde{F}^{\rm st}(s)$, was assumed to be temperature independent just as the effective interaction was in the calculation of $\mathrm{Re}\Sigma(p,\xi_p)$. The second term involving the temperature dependence of $\chi''(s)$ is seen to be responsible for the difference in the coefficients of the T^2 terms in the entropy. An analogous difference between the calculation of the entropy from $\Delta\Omega(\mu,T)$ and from $\Delta\epsilon_p^{\rm st}$ is present in 3D. In that case the difference appears in the cutoff the $T^3 \ln T$ terms.[25] In both cases the neglect of the temperature dependence of $\chi''(q,\omega)$ leads to a small error.

V. CONCLUSION

Thermodynamic quantities can be calculated directly from the thermodynamic potential or from the statistical quasiparticle spectrum, which is different from the spectrum given by the poles of the single-particle propagator, the dynamical quasiparticle spectrum. I have compared the quasiparticle spectra and entropy for 2D and 3D Fermi liquids. In 2D one finds that the temperature dependence of the quasiparticle spectra is responsible for roughly 5/8 of Γ^{2D}, the coefficient of the leading T^2 correction from the quasiparticle contribution to the entropy. This is larger than the result for 3D Fermi liquids where the leading correction is of the form $T^3 \ln T$.

Use of the dynamical quasiparticle spectrum in 3D leads to an overestimate of the coefficient of the $T^3 \ln T$ terms by a factor of 3 in the case where the scattering amplitude in either the spin or density is large. It is shown here that this effect for 2D is also present but that the overestimate of the coefficient of T^2 terms from quasiparticles is a factor of 2 independent of the interaction strength. The difference between the 2D and 3D cases is that Γ^{2D} depends on the scattering amplitude to all orders whereas Γ^{3D} depends only on the second and third power of the scattering amplitude.

There is a contribution to the T^2 term from the temperature dependence of the imaginary part of the particle-hole propagator in the long-wavelength limit. This temperature dependence comes from values of $s = \frac{\omega}{q v_F} \simeq 1$ and is a very small effect. The difference in phase space between 2D and 3D means that the temperature dependence of $\chi''(s)$ leads to a difference in the T^3 contribution to the entropy in 3D so that it does effect the leading $T^3 \ln T$ correction. As pointed out above, a more sophisticated treatment of the propagator in $\chi(q,\omega)$ would reduce this temperature dependence still further.

In conclusion the relation between $\epsilon_p^{\rm st}$ and $\epsilon_p^{\rm dy}$ for 2D and 3D FL's is similar. The differences can be traced to the difference in phase space for long-wavelength particle-hole pairs and in general they are a bigger in 2D than in 3D.

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