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Transport properties of coupled one-dimensional interacting electron systems with impurities

Takashi Kimura, Kazuhiko Kuroki, and Hideo Aoki
Department of Physics, University of Tokyo, Hongo, Tokyo 113, Japan
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We consider two one-dimensional interacting electron systems that are coupled via interchain tunneling to calculate transport properties in the presence of impurities or a single barrier by using the bosonization formalisms for the non-Luttinger-liquid phases on the phase diagram obtained by Fabrizio [Phys. Rev. B **48**, 15 838 (1993)]. We find for the weak- and strong-interaction phases, where the superconducting correlation dominates in the ground state, that the interchain transfer enhances the conductivity and the Anderson localization is suppressed. This shows that multichain systems can have unique transport properties.

There is a long history of studying transport properties of one-dimensional (1D) systems and applying the results to organic conductors. On the other hand, recent studies of quantum transport in mesoscopic systems have brought to light the features specific to low-dimensional systems that are unexpected for classical systems.[1] A motivation for studying these is the semiconductor nanostructure technology by which quasi-1D quantum wires are fabricated. It is interesting to ask what happens when there are two such chains interacting with electron-electron interactions and/or interchain tunneling. In particular, transport properties of the coupled 1D systems[2] are of crucial interest. In a previous publication[3] we have shown that, in the absence of interchain tunneling, the interchain electron-electron interaction does exert an unexpected enhancement of the conductance of the system even when the interaction is repulsive, which comes from a suppression of the charge-density wave (CDW) fluctuations. Another essential question is what happens when we introduce the interchannel tunneling. Effects of the interchannel tunneling also concern a recent interest in the so-called t-J ladder system, which is a tunneling-coupled 1D chain in the limit of large interaction, that is a model for some cuprates,[4] while we are interested in the weak and intermediate interaction regimes.

For single 1D systems, an interacting electron system is generically expressed as the Luttinger liquid.[5] For the transport, Apel and Rice[6] and Suzumura and Fukuyama[7] have found that the Luttinger liquid in the presence of impurities with finite densities should exhibit Anderson localization or delocalization according as $K_\rho+K_\sigma<3$ or >3. Here K_ρ (K_σ) is the critical exponent for the charge (spin) phase of the Luttinger liquid, where, roughly speaking, K_ρ and K_σ decreases (increases) from unity for repulsive (attractive) interactions.

On the other hand, Kane and Fisher[8] and Furusaki and Nagaosa[9] studied the tunneling through a single barrier in the Luttinger liquid. Their results may be summarized that the barrier potential is renormalized to infinity (resulting in a reflection) for repulsive electron-electron interactions, while the barrier is renormalized to zero (transmission) for attractive interactions. This statement holds rigorously for spin-independent [SU(2) invariant] electron-electron interactions (for which $K_\sigma=1$). We can relate this to the $2k_F$ CDW which has the most important influence on the transport phenomena, since it couples with the $2k_F$ component (backward scattering) of the impurity or barrier potentials (while the forward scattering is absorbed in the Hamiltonian in the clean system[10]). The CDW correlation is enhanced (suppressed) by repulsive (attractive) interactions, i.e., the conduction and the CDW formation are anticorrelated.

For dirty multichain 1D systems, few studies exist for the transport phenomena, which is the purpose of the present study. For the clean coupled chains, there are some works.[11] Penc and Sólyom[12] investigated a coupled double chain. They found that, in terms of the bonding and antibonding bands obtained by diagonalizing the interchain hopping, the system renormalizes into the usual two-component Luttinger-liquid fixed point for repulsive interactions when the bands are inequivalent with different electron-electron interactions and Fermi velocities, while the situation becomes quite different when the bands are equivalent as in lattice systems exemplified by the Hubbard model.

The ground state of the two equivalent chains (coupled Hubbard chains) in the presence of interchain hopping, $t_\perp$, has been investigated in detail by Fabrizio[13] with a perturbational renormalization-group (RG) approach. Remarkably, the system is found to depart from the Luttinger liquid to flow into strong interchain-coupling phases as soon as the interchain tunneling is turned on, although the system eventually reduces to one- or two-component Luttinger liquid for large enough tunneling amplitude. This, in his words, comes from the "flavor anisotropy," i.e., the single-electron states for two fermion flavors (two chains) splitting into bonding and antibonding states. Within the non-Luttinger-liquid region, Fabrizio identified three phases: (I) weak, (II) intermediate, and (III) strong interaction phases. Phases I and III are dominated by the Cooper pairing correlation, while phase II has a dominant out-of-phase CDW correlation. In phases I and II $t_\perp$ is relevant (i.e., does not renormalize to zero). Even when the two chains have different Fermi velocities, the difference is renormalized to zero in the non-Luttinger-liquid region after the RG process.

We ask ourselves here how these non-Luttinger-liquid phases of two coupled Tomonaga-Luttinger (Hubbard) chains behave in terms of the transport properties in the presence of impurities. We shall show that effects peculiar to this region do appear in transport, in which, surprisingly, the interchain coupling enhances the conductivity and the Ander-

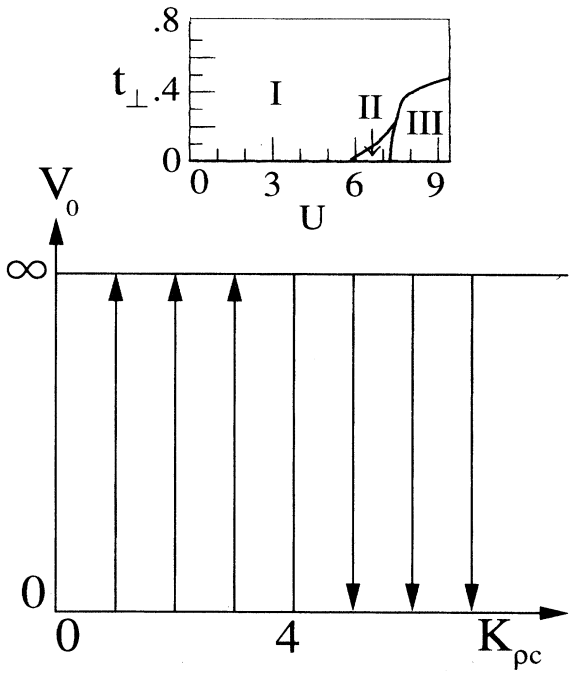

FIG. 1. Renormalization flow for the strength (V_0) of the potential barrier and the critical exponent [$K_{\rho c}=\sqrt{(1-g_0)/(1+g_0)}$] for the total charge phase $\phi_{\rho c}$. The result is for phase II on the phase diagram, depicted as an inset for the interchain hopping ($t_\perp$) and the Hubbard U after Fabrizio (Ref. 13).

son localization does not occur in the weak interaction regime. In the intermediate phase (II), by contrast, a single impurity (potential barrier) becomes relevant to make the transmission probability through the potential damp as $T\rightarrow 0$ with an even larger power than in a single chain.

We start from the observation that the key factor for transport properties is the CDW correlation, where in a double chain we have to discriminate total and in-chain CDW's. We shall show that, peculiarly enough, both the in-chain and total CDW correlations are always suppressed (or has a "charge gap") with an exponential decay with distance or time, except for phase II where the in-chain CDW correlation has a slow (power-law) decay. The model we consider is two Hubbard chains with the intrachain hopping $t_\parallel=1$, the interchain hopping $t_\perp$, and the Hubbard interaction $U>0$. We treat the model with the bosonization technique (the g-ology). Following Fabrizio[13] we start from the fixed-point Hamiltonian in phases I,II,III (inset of Fig. 1),

$$H=H_{\rho c}+H_{\rho s}+H_{\sigma c}+H_{\sigma s}+H', \tag{1}$$

$$H_{\nu l}=v_{\nu l}\int dx\ [(\nabla\phi_{\nu l})^2+(\nabla\theta_{\nu l})^2]. \tag{2}$$

Here the index ν specifies the charge (ρ) or spin (σ) degrees of freedom, while the index l specifies the symmetric (pseudocharge, c) or antisymmetric (pseudospin, s) combinations of the densities of the bonding and antibonding bands, which emerge when the interchain hopping term is diagonalized. $\phi_{\nu l}$'s are the density operators for the phase for the νl degrees of freedom, while $\theta_{\nu l}$'s are the corresponding dual operators. The coefficients $v_{\nu l}$ are given by $v_{\rho c}=v_F\sqrt{1-g_0^2}$, $v_{\rho s}=v_{\sigma c}=v_{\sigma s}=v_F\sqrt{1-(g/2)^2}\equiv v$. Here $g_0=U/2\pi v_F$ is the initial coupling constant of two Hubbard chain, v_F is the Fermi velocity, while g is the (region dependent) renormalized strong coupling constant. Hereafter we take $\hbar=k_B=1$.

The last term, H', in the Hamiltonian couples different fields. The coupling, which occurs among three fields in a cyclic manner, changes according to the phase as

$$H'=\int dx[\cos(\phi_{\sigma c})\cos(\phi_{\sigma s})+\cos(\phi_{\sigma s})\cos(\theta_{\rho s}) -\cos(\theta_{\rho s})\cos(\phi_{\sigma c})] \tag{3}$$

in phase I,

$$H'=-\int dx[\cos(\phi_{\sigma c})\cos(\theta_{\sigma s})+\cos(\theta_{\sigma s})\cos(\theta_{\rho s}) +\cos(\theta_{\rho s})\cos(\phi_{\sigma c})] \tag{4}$$

in phase II, and

$$H'=\int dx[\cos(\phi_{\sigma c})\cos(\theta_{\sigma s})+\cos(\theta_{\sigma s})\cos(\phi_{\rho s}) -\cos(\phi_{\rho s})\cos(\phi_{\sigma c})] \tag{5}$$

in phase III, where the sign for each term is explicitly shown. Here H' is measured in units of $v_F g/\pi\alpha^2$, where α is a short-range cutoff, while ϕ's and θ's are measured in units of $\sqrt{2K}$ with $K=\sqrt{(2-g)/(2+g)}$. Since g flows towards a strong coupling, long-range orders should exist in the phases occurring in the cosine terms while the correlations of the dual fields decay exponentially at large distances.

Let us pay attention to the CDW correlations, which are relevant to transport. The *total* CDW operator (sum of the CDW's for two chains) is defined as

$$O^{\text{total}}_{\text{CDW}}(x)=\sum_{i\sigma}\psi^\dagger_{i\sigma R}(x)\psi_{i\sigma L}(x)+\text{H.c.},$$

where $\psi_{R(L)}$ is the annihilation operator of the right (left) going electrons. This CDW operator is constructed from $\phi_{\rho c}$, $\phi_{\rho s}$, $\phi_{\sigma c}$, and $\phi_{\sigma s}$ in the phase representation. At least one of the operators ($\theta_{\rho c}$, etc.) dual to these appear in H' and should have a long-range order in all the phases (I,II,III), so that the total CDW correlation should always decay exponentially.

By contrast, the *in-chain* CDW operator, O^i_{CDW} ($=\Sigma_\sigma\psi^\dagger_{i\sigma R}\psi_{i\sigma L}+\text{H.c.}$, $i=1,2$), is constructed from the total CDW operator and other terms involving $\phi_{\rho c}$, $\theta_{\rho s}$, $\phi_{\sigma c}$, and $\theta_{\sigma s}$. Since the total CDW term is suppressed, we can concentrate on the other terms which represent the out-of-phase CDW ($=\Sigma_\sigma\psi^\dagger_{0\sigma R}\psi_{\pi\sigma L}+\text{H.c.}$) in terms of the bonding (0) and antibonding (π) bands. In phases I and III, one of the fields dual to these has a long-range order, and thus the CDW correlation is suppressed. However in phase II, $\theta_{\rho s}$, $\phi_{\sigma c}$, and $\theta_{\sigma s}$ all have a long-range order, so that the resulting CDW operator is given by

$$O^i_{\mathrm{CDW}}=(-1)^{i+1}\frac{2}{\pi\alpha}\cos(\sqrt{K_{\rho c}/2}\,\phi_{\rho c}),$$

where $K_{\rho c}=\sqrt{(1-g_0)/(1+g_0)}$ is the critical exponent of the total charge phase $\phi_{\rho c}$.

Having analyzed the total and in-chain CDW correlations, we can now turn to our original aim of looking into the transport properties. We assume that the impurity (or impurities) are distributed at different position(s) in the two chains with an electrode connected to the chains at either end. In phases I and III, we can see that the impurity backward scattering should be completely suppressed at low temperatures, since all the CDW correlations decay exponentially at large distances. Thus the Anderson localization is curiously absent in the system with a finite impurity density and the resistivity will exponentially vanish as the temperature goes to zero.

Let us estimate the conductance of the system with a single barrier. Since the barrier potential is irrelevant, we can calculate the conductance G in the low-temperature limit using the formula[8,14]

$$G=\lim_{\omega\to 0}\frac{1}{\omega L}\int d\tau\int dx e^{i\omega\tau}\langle T_\tau J(\tau)J(0)\rangle$$
$$=\frac{2e^2}{\pi}K_{\rho c},\qquad(6)$$

where $J=(e/\pi)\nabla_\tau(\theta_1+\theta_2)=(e/\pi)\sqrt{2K_{\rho c}}\nabla_\tau\phi_{\rho c}$ is the current operator of the total system, $\theta_{1(2)}$ the charge phase in the chain 1 (2), τ the imaginary time, and L the system size. The impurity-dependent terms are exponentially suppressed at finite temperatures. One may readily check that the conductance of the double chain is just twice the value for the *clean* single chain with the same Hubbard U, although the barrier potential is relevant for the dirty single chain with $U>0$.

In phase II we consider the case in which the single barrier only exists on chain 1 in accordance with the assumption that the disorder in the two chains are independent. We start from the action,

$$S=-\frac{1}{4\pi}\int_0^\beta d\tau\int dx\left\{\frac{1}{v_{\rho c}}[\nabla_\tau\phi_{\rho c}(x,\tau)]^2+v_{\rho c}[\nabla_x\phi_{\rho c}(x,\tau)]^2\right\}-\frac{2V_0}{\pi\alpha}\int_0^\beta d\tau\times\cos\left[\sqrt{\frac{K_{\rho c}}{2}}\,\phi_{\rho c}(x=0,\tau)\right],\qquad(7)$$

where V_0 is the strength of the barrier located at $x=0$. We assume $V_0>0$ but the result is independent of the sign of V_0. Following Refs. 8 and 9, we can integrate out the continuum phase field leaving the phase field at the impurity site $[\phi_0\equiv\phi_{\rho c}(x=0)]$ to have an effective action for a particle subject to a Caldeira-Leggett dissipation.[15] The result is given by

$$S_{\mathrm{eff}}=\frac{1}{2\pi\beta}\sum_{\omega_n}|\phi_0(\omega_n)|^2-\frac{2V_0}{\pi\alpha}\int_0^\beta d\tau\cos\left[\sqrt{\frac{K_{\rho c}}{2}}\,\phi_0(\tau)\right],\qquad(8)$$

where $\beta=1/T$. Integrating the fast modes of θ_0, we then evaluate the effective action for the slow modes in the weak barrier case. The resulting RG equation becomes

$$dV_0/d\ln\Lambda=(\tfrac{1}{4}K_{\rho c}-1)V_0(\Lambda),$$

where $\Lambda\sim v_F/\alpha$ is the high-frequency cutoff. The renomalization flow (Fig. 1) shows that the barrier potential is irrelevant if $K_{\rho c}>4$ ($g_0<-15/17$), or relevant otherwise.

On the other hand, in the strong-barrier limit we can use the dilute instanton gas approximation to obtain the so-called duality mapping[8,9] where we relate $K_{\rho c}/4\to 4/K_{\rho c}$ and $V_0/\pi\alpha\to t$. Here t is the instanton fugacity, i.e., the matrix element for the tunneling correspinding to $\sqrt{K_{\rho c}/2}\,\theta_0=0\to 2\pi$ (which means $\theta_1+\theta_2=0\to 4\pi$ for the original phase, so that this corresponds to the tunneling of four electrons). The resulting action indicates that the boundary at which the barrier becomes relevant is the same as that of the weak-barrier limit.

Thus the critical value of g_0 at which the barrier becomes relevant shifts from the single-chain value of $g_0=0$ ($K_\rho=1$) to an attractive side ($g_0=-15/17$) for two chains, which may be traced back to the fact that three ($\theta_{\rho s}$ and the two spin phases) out of four phases are frozen into a constant and exert no contribution to the RG processes.

Remembering that we have started from a repulsive $g_0>0$, we find that the barrier potential is always relevant in phase II. Since even a single barrier is relevant, the Anderson localization occurs as usual for a finite impurity density at low temperatures in this case. The relevance of the barrier potential indicates that we must use the perturbational approach for the tunneling matrix element to calculate the conductance. In this approach, the conductance G can be calculated using Fermi's golden rule for the tunneling events to obtain

$$G=De^2\left(\frac{t}{\Lambda}\right)^2\left(\frac{\pi T}{\Lambda}\right)^{2(4/K_{\rho c}-1)},\qquad(9)$$

where $D=16\pi^{3/2}\Gamma(4/K_{\rho c})/\Gamma(4/K_{\rho c}+\frac{1}{2})$. We can see that the shift of the boundary between the relevance and the irrelevance of the barrier potential makes the temperature dependence of the conductance sharper than that of the single chain, where the conductance becomes temperature independent for $g=0$. In other words, the presence of two chains makes the tunneling probability more suppressed (with a power-law dependence on T) than in a single chain, in a sharp contrast to the $t_\perp$-*assisted* conduction in phases I and III. The above argument comes from the fact that the relevant operator is the in-chain CDW when the impurities are independently distributed in two chains. If we unrealistically put the impurities on both chains at the same x coordinates, the impurities would become accidentally irrelevant because the total CDW correlation, which now couples to the disorder, is suppressed.

Thus the message here is that double chains with the interchain tunneling have unusual transport properties, such as the absence of the Anderson localization, that are distinct from either single chains or 2D systems. This is a manifestation of the property of the double chain where the ground state can be non-Luttinger liquid. Technically, the RG procedure may degrade for the strong-coupling phase III, but the result for phase I still stands.

This contrasts with the usual single-chain Luttinger liquid, where the delocalization needs considerably large interactions.

An intriguing observation is the following. Fabrizio has looked at the superconducting correlation in each phase, which shows that superconducting correlations dominates in phases I and III while a CDW correlation dominates in phase II. Since the pairing-dominated phases just coincide with the phases where the localization is absent, one may be tempted to conjecture that the superconducting correlation washes out the localization, but such an argument would be too simple. We can in fact find a counterexample in the clean single-channel Luttinger liquid, where the superconducting correlation diverges but the Anderson localization persists (although a single impurity becomes irrelevant) when we increase the electron-electron attraction to have $2<K_\rho+K_\sigma<3$. Thus the anticipated anticorrelation between the superconducting correlation and localization cannot be always the case. Nevertheless, we note that the superconducting correlation remains dominant for dirty systems in phases I and III, although the conduction may not be regarded as a supercurrent in 1D where long-range orders are inhibited. These effects should be observable experimentally in 1D chains or quantum wires.

We are much indebted to Professor M. Fabrizio for illuminating discussions.

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