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Persistent spin currents induced by the Aharonov-Casher effect in mesoscopic rings

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We investigate the Aharonov-Casher effect in a mesoscopic ring in the presence of a cylindrically symmetric electric field. The Aharonov-Casher phase is obtained as a function of the tilt angle of the applied electric field. The exact energy spectrum of the system is obtained by using a gauge transformation. We demonstrate the persistent spin current induced by the Aharonov-Casher effect at finite temperature.

I. INTRODUCTION

Since the remarkable discovery of the Berry phase,[1] much attention has been paid to geometric phases during the last decade. When the system interacting with the slowly varying environment undergoes adiabatic and cyclic evolution, the quantum state acquires the Berry phase. Without the restriction on the adiabaticity, Aharonov and Anandan[2] (AA) demonstrated that the acquired phase of a quantum state after a cyclic evolution in the projective Hilbert space can be expressed as a sum of the dynamical phase and the geometric phase (called the AA phase). A well-known example of the geometric phase is the Aharonov-Bohm (AB) phase[3] which a charged particle moving around a magnetic flux in a force-free region acquires. In 1984, Aharonov and Casher[4] (AC) discovered the dual of the AB effect: a neutral particle with a magnetic moment encircling a charged line accumulates the AC phase. The AC effect has been experimentally verified by Cimmino *et al.* using the thermal neutron,[5] and by Sangster *et al.* for the atomic system.[6] The AC effect is a consequence of the coupling between the spin and the $SU(2)_{\mathrm{spin}}$ gauge field,[7,8] just as the AB effect results from the coupling of the charge and the $U(1)_{\mathrm{em}}$ gauge field. The similarities and differences between the AB and AC effects were pointed out by Goldhaber.[9]

The well-known manifestation of the AB effect is the persistent charge current in a mesoscopic ring threaded by a magnetic flux.[10] The persistent charge currents in mesoscopic rings were experimentally measured by Lévy *et al.*,[11] Chandrasekhar *et al.*,[12] and Mailly *et al.*[13] Loss, Goldbart, and Balatsky[14] found out that the persistent currents can be induced in a mesoscopic ring by the Berry phase due to the coupling of the spin and orbital motions of an electron in a manner similar to the AB effect.

There are two kinds of couplings between the spin and orbital degrees of freedom of an electron. One is the Zeeman term which is the source of the scalar AC effect and the other is the spin-orbit term related to the vector AC effect.[15] In the presence of the inhomogeneous static magnetic field, the Zeeman term couples the spin and orbital motions of an electron in a mesoscopic ring. Loss, Goldbart, and Balatsky[14] showed that the Berry phase due to the Zeeman interaction results in the persistent currents at finite temperature. Stern[16] proposed that the time-dependent Berry phase due to the Zeeman interaction induces a motive force in a ring on the analogy of the Faraday law of the AB effect. Gao and Qian[17] calculated the AA phase of an electron in a mesoscopic ring in the cylindrically symmetric magnetic field.

The effect of spin-orbit scattering in a disordered mesoscopic system was studied by Meir, Gefen, and Entin-Wohlman. Meir, Gefen, and Entin-Wohlman[18] derived that the spin-orbit scattering modifies the bare spectrum depending on the flux Φ_{AB} to the form depending on $\Phi_{\mathrm{AB}} \pm \delta$, where δ is due to the random spin-orbit electric field. Mathur and Stone[19] showed that the effect of spin-orbit interaction is the manifestation of the AC effect. Aronov and Lyanda-Geller[20] explored the effect of the spin-orbit Berry phase on the transport properties of the mesoscopic rings, and studied a spin-dependent motive force due to the time-dependent Berry phase. Balatsky and Altshuler[21] demonstrated the persistent spin current due to the spin-orbit coupling as a manifestation of the AC effect at zero temperature, including the possibility of the spin-dependent motive force induced by the time-dependent AC phase. Choi[22] showed the spontaneous spin current via the AC effect at zero temperature. Recently, Qian and Su[23] showed the existence of the AA phase in the AC effect.

In this paper we study the AC phase and the persistent spin current in a mesoscopic ring in the presence of a cylindrically symmetric electric field, based on the $SU(2)_{\mathrm{spin}}$ gauge theory. This paper is organized as follows. In Sec. II we present a system of noninteracting electrons confined to a mesoscopic ring embedded in the cylindrically symmetric electric field. Based on the $SU(2)_{\mathrm{spin}}$ gauge theory, we calculate the AC phase as a function of the strength and tilt angle of the applied electric field. We demonstrate the AC phase as a sum of the AA and dynamical phases of the spin evolution and discuss the adiabatic limit. In Sec. III we obtain the exact energy spectrum in connection with the $SU(2)_{\mathrm{spin}}$ gauge transformation, and then calculate the persistent spin current at finite temperature. A summary of the results is given in Sec. IV. In the Appendix, we derive the exact energy spectrum by diagonalizing the Hamiltonian.

II. AC PHASE AND $SU(2)_{\rm SPIN}$ GAUGE FIELD

Consider noninteracting electrons confined to a mesoscopic ring of radius r embedded in a cylindrically symmetric electric field $\mathbf{E}(\phi) = E(\cos\chi\hat{\mathbf{r}} - \sin\chi\hat{\mathbf{z}})$ as depicted in Fig. 1. The tilt angle χ is the one with respect to the plane on which the ring lies. Our model bears some resemblance to the mesoscopic ring in a cylindrically symmetric magnetic field considered by Loss, Goldbart, and Balatsky.[14]

We approach this problem based on the gauge theory as follows. The nonrelativistic Hamiltonian with $U(1)_{\rm em} \times SU(2)_{\rm spin}$ gauge symmetry of an electron with a charge $e < 0$ in an external electromagnetic field[7,8] is given by

$$i\hbar\frac{\partial}{\partial t}\Psi = \left[\frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A} - \frac{\mu}{c}\mathbf{a}\right)^2 + eA^0 + \mu a^0\right]\Psi\,, \quad (1)$$

where Ψ is a two component spinor and $\mu = g\mu_B/2$ is a magnetic moment of an electron in the ring. g is a gyromagnetic ratio and $\mu_B = e\hbar/(2mc)$. Here $A^\mu = (A^0, \mathbf{A})$ is a $U(1)_{\rm em}$ electromagnetic four vector potential, and $a^\mu = (-\boldsymbol{\sigma}\cdot\mathbf{B}, \boldsymbol{\sigma}\times\frac{\mathbf{E}}{2})$ is an $SU(2)_{\rm spin}$ vector potential that represents a Zeeman and a spin-orbit coupling. σ^a with $a = 1,2,3$ are Pauli matrices. Suppose the wave function $\Psi(x^\mu)$ is parallel transported along a curve C from an initial point x_0^μ to a final point x^μ in the presence of $U(1)_{\rm em}\times SU(2)_{\rm spin}$ gauge fields. After the parallel transport, the wave function $\Psi(x^\mu)$ is related to wave function $\Psi(x_0^\mu)$ by parallel transporter (gauge transformation) $\Omega(C)$ (Refs. 7 and 24)

$$\Psi(x^\mu) = \Omega(C)\Psi(x_0^\mu) \quad (2a)$$

$$= \exp\left(-i\frac{e}{\hbar c}\int_C A_\mu dx^\mu\right) \times P\exp\left(-i\frac{\mu}{\hbar c}\int_C a_\mu dx^\mu\right)\,\Psi(x_0^\mu)\,, \quad (2b)$$

where P is a path ordering operator. The first factor of Eq. (2b) gives the AB phase and the second one the AC phase. The parallel displacement equation of the electron under the $U(1)_{\rm em}\times SU(2)_{\rm spin}$ gauge fields reads

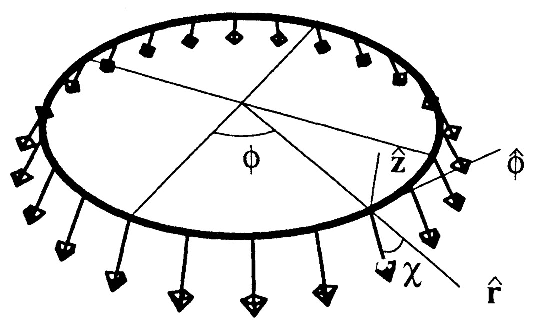

FIG. 1. The mesoscopic ring of radius r in a cylindrically symmetric electric field with tilt angle χ. The electron picks up an AC phase $\Phi_{\rm AC}^{(\pm)}$, while encircling the ring.

$$\frac{dx^\mu}{ds}\left(\partial^\mu + i\frac{e}{\hbar c}A^\mu + i\frac{\mu}{\hbar c}a^\mu\right)\Psi(s) = 0\,, \quad (3)$$

where $\Psi(s) = \Psi(x(s))$, and the curve C is represented by $x^\mu(s)$ satisfying $x^\mu(0) = x_0^\mu$, $x^\mu(1) = x^\mu$ with parameter s, $0 \le s \le 1$. Equation (2) is a solution of Eq. (3).

Since we are interested in the AC effect due to the electric field, we put $A^\mu = 0$ and $a^0 = 0$ in Eq. (1). Then in cylindrical coordinates (r, ϕ, z), the ϕ component of the $SU(2)_{\rm spin}$ vector potential becomes

$$a_\phi = \left(\boldsymbol{\sigma}\times\frac{\mathbf{E}}{2}\right)\cdot\hat{\phi} = \frac{E}{2}(\sin\chi\hat{\mathbf{r}} + \cos\chi\hat{\mathbf{z}})\cdot\boldsymbol{\sigma}\,. \quad (4)$$

The Hamiltonian for the electron becomes

$$H = \frac{1}{2mr^2}\left(P_\phi - \frac{\mu r}{c}a_\phi\right)^2 \quad (5a)$$

$$= \frac{\hbar^2}{2mr^2}\left[-i\frac{d}{d\phi} - \frac{\mu E r}{2\hbar c}(\sin\chi\cos\phi\sigma^1 + \sin\chi\sin\phi\sigma^2 + \cos\chi\sigma^3)\right]^2\,, \quad (5b)$$

where $P_\phi = -i\hbar\frac{\partial}{\partial\phi}$ is the z component of the orbital angular momentum. The above Hamiltonian (5) implies that the $SU(2)_{\rm spin}$ vector potential $\mathbf{a}$ of the AC effect plays a similar role to a $U(1)_{\rm em}$ vector potential $\mathbf{A}$ of the AB effect. Thus one may expect that the AC effect results in persistent spin currents as the AB effect leads to persistent charge currents.

The parallel displacement equation (3) in the presence of the $SU(2)_{\rm spin}$ gauge field is

$$i\frac{\partial}{\partial\phi}\Psi = -\frac{\mu r}{\hbar c}a_\phi\,\Psi = -\mathbf{B}_{\rm eff}\cdot\boldsymbol{\sigma}\,\Psi\,, \quad (6)$$

where $\mathbf{B}_{\rm eff} \equiv \frac{\mu E r}{2\hbar c}(\sin\chi\cos\phi, \sin\chi\sin\phi, \cos\chi)$ is an "effective magnetic field." The evolution of a spin state in the presence of the $SU(2)_{\rm spin}$ gauge field is governed by Eq. (6). If ϕ in (6) is replaced by time t, then Eq. (6) is exactly identical to the Schrödinger equation of a spin in a rotating magnetic field, which has been well studied.[1,25] Thus we identify the Schrödinger-type equation in Ref. 23 with a parallel displacement equation of the spin in the $SU(2)_{\rm spin}$ gauge field. The solution of Eq. (6) becomes

$$\Omega(\phi) \equiv P\exp\left[i\frac{\mu E r}{2\hbar c}\int_0^\phi(\sin\chi\cos\phi'\sigma^1 + \sin\chi\sin\phi'\sigma^2 + \cos\chi\sigma^3)d\phi'\right]\,, \quad (7)$$

where path C is represented by angle ϕ. Equation (7) is identical to the time evolution operator for a spin in a rotating magnetic field if the angle ϕ is replaced by time t. The exact solution of Eq. (7) is easily calculated by employing the rotating frame method and given by

$$\Omega(\phi) = \exp\left[-i\frac{\phi}{2}\sigma^3\right] \times \exp\left[i\left(\omega_1\sigma^1 + (\omega_3+1)\sigma^3\right)\frac{\phi}{2}\right], \tag{8}$$

where $\omega_1 \equiv \frac{\mu E r}{\hbar c}\sin\chi$ and $\omega_3 \equiv \frac{\mu E r}{\hbar c}\cos\chi$. This is a purely ϕ dependent gauge transformation.

The $SU(2)_{\rm spin}$ vector potential a_ϕ can be gauged away, if a continuous local gauge transformation $U(\phi) = \Omega^{-1}(\phi)$ is made.[24] Then the gauge transformation gives

$$H = \frac{1}{2mr^2}\left(P_\phi - \frac{\mu r}{c}a_\phi\right)^2 \to H' = U(\phi)HU^{-1}(\phi) = \frac{1}{2mr^2}P_\phi^2\,, \tag{9a}$$

$$a_\phi \to a'_\phi = U(\phi)a_\phi U^{-1}(\phi) + i\frac{\hbar c}{\mu r}\,U(\phi)\frac{\partial}{\partial\phi}U^{-1}(\phi) = 0\,, \tag{9b}$$

$$\Psi(\phi) \to \Psi'(\phi) = U(\phi)\Psi(\phi)\,. \tag{9c}$$

Under this gauge transformation, the Hamiltonian (5) is changed to the free Hamiltonian (9a), but the boundary condition of the wave function Eq. (9c) is also changed. In general, the spin state that has been parallel transported around the ring does not return to the initial spin state. However, for the special initial spin state, the spin state after a parallel transport around the ring returns to the initial one apart from the phase factor as $\Psi(2\pi) = \exp[i\Phi_{\rm AC}^{(\pm)}]\,\Psi(0)$.

Let the initial spin state be the eigenstate of $\omega_1\sigma^1 + (\omega_3+1)\sigma^3$. Then the spin state at ϕ is obtained as

$$\Psi^{(\pm)}(\phi) = \Omega(\phi)\Psi^{(\pm)}(0) \tag{10a}$$

$$= e^{i(1-\lambda_\pm)\phi/2}\begin{pmatrix}\cos\frac{\beta_\pm}{2}\\ \pm e^{i\phi}\sin\frac{\beta_\pm}{2}\end{pmatrix}, \tag{10b}$$

where $\lambda_\pm \equiv \pm\sqrt{\omega_1^2 + (\omega_3+1)^2}$ are eigenvalues of $\omega_1\sigma^1 + (\omega_3+1)\sigma^3$, and the angle $\beta_\pm$ are defined by $\tan\beta_+ \equiv \omega_1/(\omega_3+1)$, and $\beta_- = \pi - \beta_+$. After the parallel transport by $\Omega(\phi)$ from $\phi = 0$ to $\phi = 2\pi$, the spin state returns to the initial state apart from the AC phase. The acquired AC phase is from (10b)

$$\Phi_{\rm AC}^{(\pm)}(\chi) = -\pi(1-\lambda_\pm)\,. \tag{11}$$

Thus we obtain the AC phase as a function of the strength and tilt angle χ of the applied electric field.

Since the spin state acquires the AC phase after the cyclic spin evolution, the AC phase can be decomposed into the dynamical phase and the AA phase.[2,23] Following Aharonov and Anandan,[2] the spin state at ϕ can be written

$$\Psi^{(\pm)}(\phi) = e^{i(1-\lambda_\pm)\phi/2}\,\tilde\Psi^{(\pm)}(\phi)\,, \tag{12}$$

where $\tilde\Psi^{(\pm)}(\phi)$ is a periodic function satisfying $\tilde\Psi^{(\pm)}(2\pi) = \tilde\Psi^{(\pm)}(0)$, and given by

$$\tilde\Psi^{(\pm)}(\phi) = \begin{pmatrix}\cos\frac{\beta_\pm}{2}\\ \pm e^{i\phi}\sin\frac{\beta_\pm}{2}\end{pmatrix}. \tag{13}$$

The dynamical phase is calculated as

$$\Phi_{\rm dyn}^{(\pm)} = \int_0^{2\pi} \tilde\Psi^{(\pm)}(\phi)\;\mathbf{B}_{\rm eff}\cdot\boldsymbol{\sigma}\;\tilde\Psi^{(\pm)}(\phi)\,d\phi \tag{14a}$$

$$= \pm\pi\left(\frac{\omega_3^2+\omega_3+\omega_1^2}{\sqrt{\omega_1^2+(\omega_3+1)^2}}\right), \tag{14b}$$

and the AA phase is given by

$$\Phi_{\rm AA}^{(\pm)} = \int_0^{2\pi} \tilde\Psi^{(\pm)}\,i\frac{d}{d\phi}\tilde\Psi^{(\pm)}\,d\phi \tag{15a}$$

$$= -\pi(1-\cos\beta_\pm)\,. \tag{15b}$$

Thus the AC phase can be expressed as

$$\Phi_{\rm AC}^{(\pm)} = \Phi_{\rm dyn}^{(\pm)} + \Phi_{\rm AA}^{(\pm)}\,. \tag{16}$$

We now consider two special cases of χ. First, if $\chi = 0$, then the electric field is in the radial direction, $\mathbf{E} = E\hat{\mathbf{r}}$, corresponding to the case of a charged line. Then $\omega_1 = 0$, $\omega_3 = \frac{\mu E r}{\hbar c}$. Also $\beta_+ = 0$, $\beta_- = \pi$. The initial spin state becomes a spin-up or spin-down state. The parallel transporter becomes simply $\Omega(\phi) = \exp[i\frac{\mu E r}{2\hbar c}\phi\sigma^3]$. The AA phase vanishes. The AC phase is equal to the dynamical phase

$$\Phi_{\rm AC}^{(\pm)} = \Phi_{\rm dyn}^{(\pm)} = \pm\frac{\mu}{\hbar c}(E\pi r)\,, \tag{17}$$

which is equal to the AC phase of the electron moving around the charged line. Next, if $\chi = \pi/2$, then the electric field is $\mathbf{E} = -E\hat{\mathbf{z}}$. The Hamiltonian (5) is identical to Eq. (11) in Ref. 23 within the constant $-m\hbar^2\kappa/2$. $-\frac{\mu E r}{2\hbar c}$ of (5) corresponds to $ma\kappa$ in Ref. 23. The $SU(2)_{\rm spin}$ vector potential becomes $a_\phi = (E/2)\,\hat{\mathbf{r}}\cdot\boldsymbol{\sigma}$ and the "effective magnetic field" $\mathbf{B}_{\rm eff}$ lies on the plane. This situation is similar to that of the planar magnetic field studied by Loss and Goldbart.[14] We have $\omega_1 = \frac{\mu E r}{\hbar c}$, and $\omega_3 = 0$. The AC phase becomes from Eq. (11)

$$\Phi_{\rm AC}^{(\pm)} = -\pi\left(1 \mp \sqrt{\omega_1^2+1}\right), \tag{18}$$

and the AA phase reads

$$\Phi_{\rm AA}^{(\pm)} = -\pi\left(1 \mp \frac{1}{\sqrt{\omega_1^2+1}}\right). \tag{19}$$

The dynamical phase is given by

$$\Phi_{\rm dyn}^{(\pm)} = \pm\pi\frac{\omega_1^2}{\sqrt{\omega_1^2+1}}. \tag{20}$$

Equations (18), (19), and (20) agree with the results obtained by Qian and Su.[23]

Consider the adiabatic approximation of the spin evolution. The condition for the adiabatic limit is $\frac{\mu E r}{\hbar c} \gg 1$. In this case, the initial spin state is an eigenstate of $-\boldsymbol{\sigma}\cdot\mathbf{B}_{\rm eff}(0)$. After adiabatic cyclic evolution with period 2π, the acquired phase of the spin is the approximate value of the AC phase (11)

$$\Phi_{\rm AC}^{(\pm)} \approx -\pi\left(1 \mp \sqrt{\omega_1^2+\omega_3^2} + \cos\chi\right). \tag{21}$$

The dynamical phase of the adiabatic evolution is calculated from the exact solution (14) and given by $\Phi_{\rm dyn}^{(\pm)} \approx \pm\pi\sqrt{\omega_1^2+\omega_3^2}$. Since $\tan\beta_+ = \omega_1/(\omega_3+1) \approx \omega_1/\omega_3 = \tan\chi$, the Berry phase is the adiabatic approximation of the AA phase (15)

$$\Phi_{\rm AA}^{(\pm)} \approx \Phi_{\rm Berry}^{(\pm)} = -\pi(1\mp\cos\chi). \tag{22}$$

In Fig. 2, we summarize the AC, AA, dynamical, and Berry phases as a function of tilt angle at various values of $\frac{\mu E r}{\hbar c} = 0.5, 1.5, 2.5$, and 3.5. The values of $\frac{\mu E r}{\hbar c}$ are chosen in accordance with the value taken in Ref. 23. For small values of $\frac{\mu E r}{\hbar c}$ the AC, AA, and dynamical phases cross each other as tilt angle χ increases. In this region, there is a great difference between the values of the AA phase and the Berry phase. On the other hand, for large values of $\frac{\mu E r}{\hbar c}$, the dynamical phase becomes flat and its magnitude is approximately that of the adiabatic dynamical phase, $\Phi_{\rm dyn}^{(\pm)} \approx \pm\pi\frac{\mu E r}{\hbar c}$. The shapes of the AA and AC phases resemble each other. The AA phase is nearly equal to the Berry phase. Thus the adiabatic approximation in the limit of $\frac{\mu E r}{\hbar c} \gg 1$ is justified. Note that the value of the AA phase is $-2\pi \le \Phi_{\rm AA}^{(\pm)} \le 0$.

III. PERSISTENT SPIN CURRENTS BY THE *AC* PHASE

Under the gauge transformation (9), the Hamiltonian (5) becomes that of a free rotator, and at the same time the AC phase modifies the boundary condition of the wave function of the free rotator. Then the exact energy spectrum of the system simply becomes

$$E_{n,\alpha} = \frac{\hbar^2}{2mr^2}\left(n - \frac{\Phi_{\rm AC}^{(\alpha)}}{2\pi}\right)^2, \tag{23}$$

where $n = 0, \pm1, \pm2, \ldots$ denotes the orbital quantum number and $\alpha = \pm$ is a spin quantum number. Meir, Gefen, and Entin-Wohlman[18] obtained the general form of Eq. (23) by using the transfer-matrix method. The dependence of the AC phase on the tilt angle implies that the exact energy spectrum (23) is also a function of the tilt angle. The eigenstates are given by

$$\Psi_{n,\pm} = \frac{1}{\sqrt{2\pi}}e^{in\phi}\begin{pmatrix}\cos\frac{\beta_\pm}{2}\\ \pm e^{i\phi}\sin\frac{\beta_\pm}{2}\end{pmatrix}, \tag{24}$$

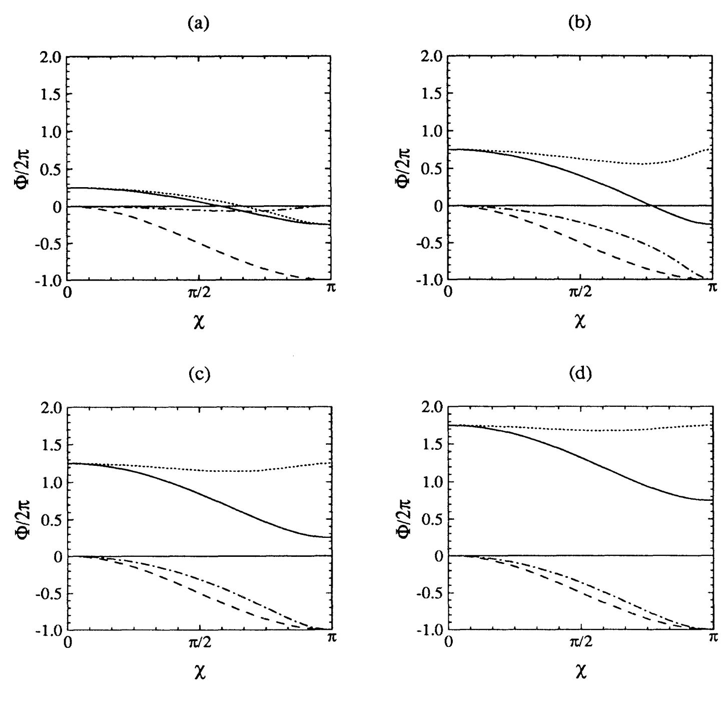

FIG. 2. AC (solid line), AA (dashed-dotted line), dynamical (dotted line), and Berry (dashed line) phases (divided by 2π) as a function of tilt angle χ for (a) $\frac{\mu E r}{\hbar c} = 0.5$, (b) 1.5, (c) 2.5, and (d) 3.5. (i) Note that $\Phi_{\rm AC}^{(\pm)} = \Phi_{\rm dyn}^{(\pm)} + \Phi_{\rm AA}^{(\pm)}$. (ii) Note that the Berry phase is approximately equal to the AA phase for large values of $\frac{\mu E r}{\hbar c}$. (iii) At $\chi = 0$, $\Phi_{\rm AC}^{(\pm)}$ equals the dynamical phase; the geometric phase vanishes.

which are single-valued functions.

The thermal equilibrium expectation value $\langle O \rangle$ of the observable O is easily calculated as

$$\langle O \rangle = \frac{1}{Z}\mathrm{Tr}[e^{-\beta H} O], \tag{25}$$

where Z is the canonical partition function and $\beta = 1/k_B T$. The interesting physical observables are the magnetization $\hbar\boldsymbol{\sigma}/2$, and (dimensionless) spin current $J^a = (P_\phi - \frac{\mu r}{c} a_\phi)\sigma^a/\hbar$ with $a = 1,2,3$ for spin indices. The thermal expectation values of the spin and the spin current are given by

$$\langle \sigma^a \rangle = \frac{1}{Z}\mathrm{Tr}[e^{-\beta H}\sigma^a], \tag{26}$$

$$\langle J^a \rangle = \frac{1}{Z}\mathrm{Tr}\left[e^{-\beta H}\,\frac{1}{\hbar}\,\left(P_\phi - \frac{\mu r}{c} a_\phi\right)\sigma^a\right]. \tag{27}$$

The canonical partition function of the one electron is

$$Z = \mathrm{Tr}(e^{-\beta H}) = \sum_{n=-\infty}^{\infty}\sum_{\alpha=\pm}\exp(-\beta E_{n,\alpha}). \tag{28}$$

Because of cylindrical symmetry of the system, the thermal expectation values $\langle \sigma^1 \rangle$ and $\langle \sigma^2 \rangle$ vanish. From $\langle \Psi_{n,\alpha}|\sigma^3|\Psi_{n,\alpha}\rangle = \cos\beta_\alpha$, $\langle \sigma^3 \rangle$ can be written as

$$\langle \sigma^3 \rangle = \frac{1}{Z}\sum_{n,\alpha}[e^{-\beta E_{n,\alpha}}\cos\beta_\alpha]. \tag{29}$$

But we have $\langle \sigma^3 \rangle = 0$ because $E_{n,+} = E_{-(n+1),-}$ and $\cos\beta_+ = -\cos\beta_-$. For the same reason that $\langle \sigma^1 \rangle$ and $\langle \sigma^2 \rangle$ vanish, we find $\langle J^1 \rangle = \langle J^2 \rangle = 0$. The thermal expectation value of the persistent spin current $\langle J^3 \rangle$ is given by

$$\langle J^3 \rangle = \frac{1}{Z}\mathrm{Tr}\left[e^{-\beta H}\,\frac{1}{\hbar}\left(P_\phi - \frac{\mu r}{c}a_\phi\right)\sigma^3\right] \tag{30a}$$

$$= \frac{1}{Z}\sum_{n,\alpha}\left[\cos\beta_\alpha\left(n - \frac{\Phi_{\mathrm{AC}}^{(\alpha)}}{2\pi}\right)e^{-\beta E_{n,\alpha}}\right]. \tag{30b}$$

Because $\Phi_{\mathrm{AC}}^{(+)} + \Phi_{\mathrm{AC}}^{(-)} = -2\pi$, $E_{n,+} = E_{-(n+1),-}$, and $\cos\beta_+ = -\cos\beta_-$, the persistent spin current $\langle J^3 \rangle$ can be finally reduced to

$$\langle J^3 \rangle = \cos\beta_+\,\frac{\sum_{n=-\infty}^{\infty}\left(n - \frac{\Phi_{\mathrm{AC}}^{(+)}}{2\pi}\right)e^{-\beta E_{n,+}}}{\sum_{n=-\infty}^{\infty}e^{-\beta E_{n,+}}}. \tag{31}$$

Thus the persistent spin current for the spin-up and spin-down is expressed as the one for the spin-up (or spin-down) only. This implies that the persistent spin current would be independent of spin polarization. It should be noticed that the magnetic field necessary for polarization of the electron is not required to observe the persistent spin current. If $\cos\beta_+ = 1$, i.e., $\chi = 0$, then the persistent spin current is a periodic function of the AC flux $\Phi_{\mathrm{AC}}^{(+)}/2\pi$ and $\langle J^3 \rangle$ of Eq. (31) is identical to the persistent charge current by the AB effect. Except for $\chi = 0$ or π, the persistent spin current is an oscillating function of the AC flux with the modulation $\cos\beta_+$. From the AC phase $\Phi_{\mathrm{AC}}^{(+)} = -\pi\left(1 - \sqrt{\omega_1^2 + (\omega_3+1)^2}\right)$, the modulation can be written in terms of the AC flux as

$$\cos\beta_+ = \frac{\sin^2\chi + \cos\chi\sqrt{\cos^2\chi + 4(y+y^2)}}{1+2y}, \tag{32}$$

where $y \equiv \Phi_{\mathrm{AC}}^{(+)}/2\pi$.

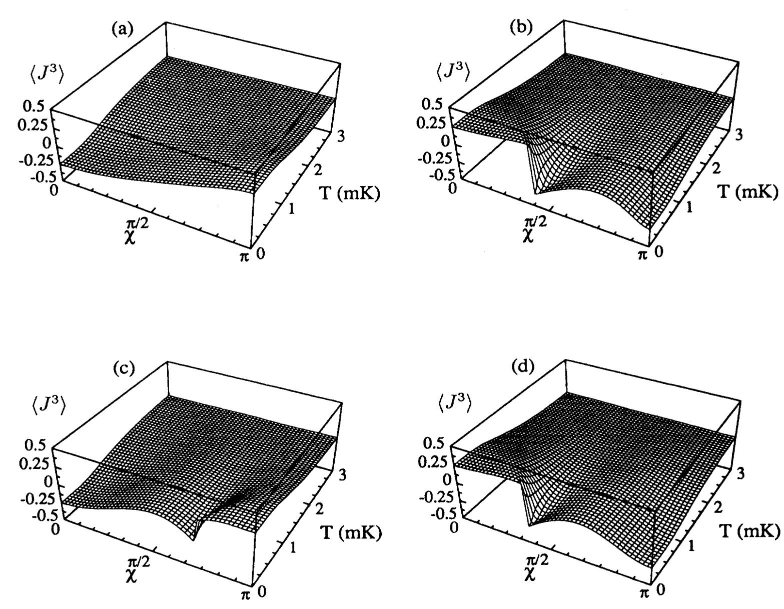

FIG. 3. The persistent spin current $\langle J^3 \rangle$ as a function of temperature T and tilt angle χ for (a) $\frac{\mu E r}{\hbar c} = 0.5$, (b) 1.5, (c) 2.5, and (d) 3.5. Temperature T is drawn in units of mK.

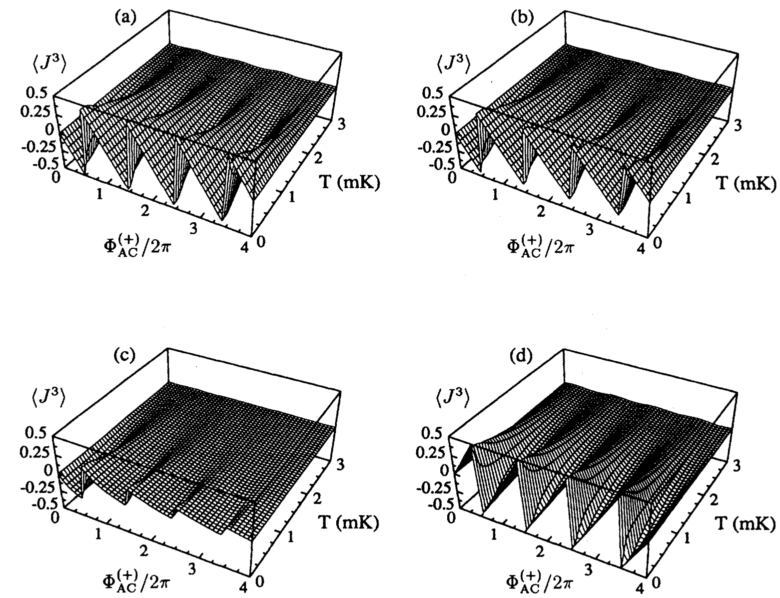

FIG. 4. The persistent spin current $\langle J^3 \rangle$ as a function of temperature T and the AC flux $\Phi^{(+)}_{AC}/2\pi$ for (a) χ =0, (b) $\pi/4$, (c) $\pi/2$, and (d) π. Notice that the persistent spin current is periodic as a function of $\Phi^{(+)}_{AC}/2\pi$ in (a) and (d). It is modulated by $\cos\beta_+$ [cf. Eq. (32) in the text] in (b) and (c).

We numerically calculate the spin current $\langle J^3 \rangle$ by varying $\frac{\mu E r}{\hbar c}$, tilt angle χ, and temperature T. For an InAs ring of radius $r \approx 1$ μm, $ma\kappa$ is taken as 1.8 in Ref. 23. This value is equivalent to $\frac{\mu E r}{\hbar c} = 3.6$, since $ma\kappa$ corresponds to $-\frac{\mu E r}{2\hbar c}$. Thus we take $\frac{\mu E r}{\hbar c} = 0.5$, 1.5, 2.5, and 3.5. The temperature ranges from 0 to 3 mK. Since temperature $T \approx 1$ mK and $\frac{\mu E r}{\hbar c} \approx 1$, the contribution of terms with $n \geq 10$ in the summation of (30b) is negligible.

Figure 3 shows the persistent spin current $\langle J^3 \rangle$ as a function of tilt angle χ and temperature T for $\frac{\mu E r}{\hbar c} = 0.5$, 1.5, 2.5, and 3.5. As the tilt angle χ varies from 0 to π, the change of $\langle J^3 \rangle$ is noticeable. There is no jump of the persistent spin current in Fig. 3(a), because in this case $-0.5 \leq \Phi^{(+)}_{AC}/2\pi \leq 0.5$ as shown in Fig. 2(a). Only at the values of $\Phi^{(+)}_{AC}/2\pi = n \pm 0.5$ with n an integer, the persistent spin current changes direction suddenly, keeping the magnitude (see Fig. 5). The abrupt changes in Figs. 3(b), (c), and (d) occur at the specific tilt angles at which $\Phi^{(+)}_{AC}/2\pi$ becomes 0.5, 0.5, and 1.5, respectively. In Fig. 4, the persistent spin current $\langle J^3 \rangle$ is shown to disappear when the temperature is beyond 3 mK. This indicates that the persistent current is a quantum me-

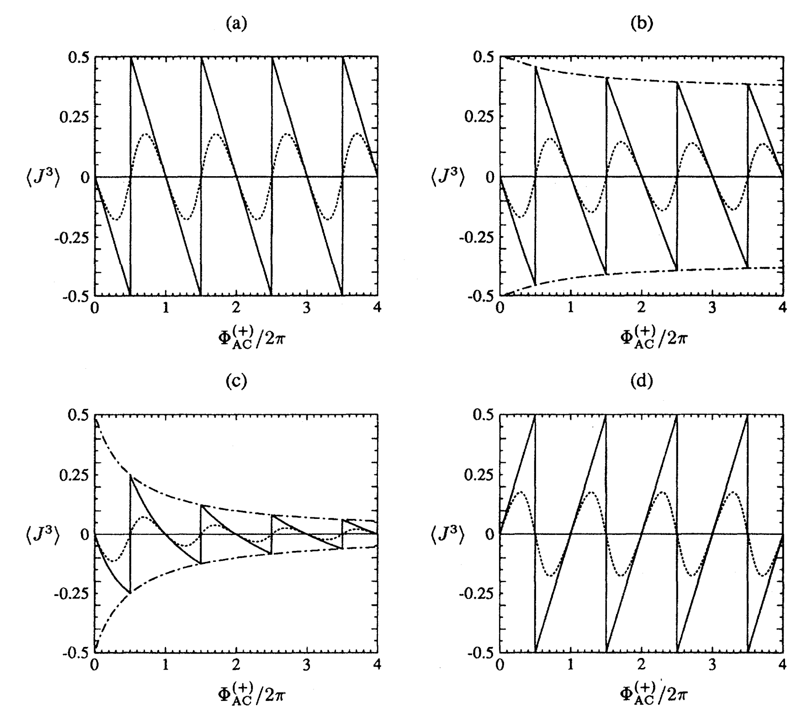

FIG. 5. The persistent spin current vs the AC flux $\Phi^{(+)}_{AC}/2\pi$ for (a) χ =0, (b) $\pi/4$, (c) $\pi/2$, and (d) π at temperature $T = 0$ (solid line) and $T = 1$ mK (dotted line). The dashed-dotted line represents the modulation $(1/2)\cos\beta_+$. Fixed temperature slices through Fig. 4 (at $T = 0$ K and 1 mK) are plotted in Fig. 5. These slices show more clearly the periodicity and modulation effects noted in Fig. 4.

chanical effect. The persistent current vanishes when the thermal energy k_BT is larger than the level spacing $\Delta E \approx \hbar^2/2mr^2$. For $T = 3$ mK, and $r \approx 1$ μm, $k_BT \approx 4.14 \times 10^{-26}$ J and $\Delta E \approx 6.10 \times 10^{-27}$ J. The rest electron mass m has been used in our calculation.

Figure 4 exhibits the persistent spin current as a function of the AC flux and temperature. If tilt angle $\chi = 0$ or π, then $\langle J^3 \rangle$ is a periodic function of the AC flux $\Phi_{\rm AC}^{(+)}/2\pi$. But if $\chi \neq 0$ or π, then the persistent spin currents are oscillating functions of the AC flux $\Phi_{\rm AC}^{(+)}/2\pi$ with the modulation $\cos\beta_+$. Figure 5 shows this phenomenon more clearly. In the case of $\chi \neq 0$ or π, the persistent spin current $\langle J^3 \rangle$ decreases as the AC flux $\Phi_{\rm AC}^{(+)}/2\pi$ increases. Since $\Phi_{\rm AC}^{(+)}$ depends on the strength of the electric field E, for a strong electric field in the z direction, the persistent spin current $\langle J^3 \rangle$ diminishes. For $\chi = 0$ or π, $\langle J^3 \rangle$ is identical to the persistent charge current of the AB effect.

IV. CONCLUSION

We have studied the geometric phases and the persistent spin current associated with the AC effect in a mesoscopic ring embedded in a cylindrically symmetric electric field. Using the $SU(2)_{\rm spin}$ gauge theory, we have obtained the AC phase as a function of the strength and tilt angle of the applied electric field. The AC phase is shown to be a sum of the AA phase and the dynamical phase of the spin evolution. The $SU(2)_{\rm spin}$ gauge transformation gives rise to the energy spectrum rather easily. The thermal equilibrium value of the magnetization vanishes. The persistent spin current $\langle J^3 \rangle$ has been numerically analyzed by varying the tilt angle, $\frac{\mu E r}{\hbar c}$, and temperature. Especially, we have noted that the persistent spin current, which is a periodic function of the AC flux, is modulated by $\cos\beta_+$.

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APPENDIX

Since the Hamiltonian (5) possesses cylindrical symmetry, the z component of the total angular momentum, $J_z = P_\phi + (\hbar/2)\sigma^3$ is conserved. One can easily obtain the exact energy eigenvalues of the Hamiltonian (5) by diagonalizing the Hamiltonian.[14] The simultaneous eigenstates of J_z and σ^3 are

$$J_z|l;\alpha\rangle = \hbar l|l;\alpha\rangle\,, \tag{A1a}$$

$$\sigma^3|l;\alpha\rangle = \alpha|l;\alpha\rangle\,, \tag{A1b}$$

where $l = \pm 1/2, \pm 3/2, \ldots$ and $\alpha = \pm$. The matrix elements of the Hamiltonian (5) in the basis of $\{|l;\alpha\rangle\}$ are given by

$$\langle l';\alpha'|H|l;\alpha\rangle = \frac{\hbar^2}{2mr^2}\delta_{l',l} \times \begin{pmatrix} l - \frac{1}{2}(\omega_3+1) & -\frac{\omega_1}{2} \\ -\frac{\omega_1}{2} & l + \frac{1}{2}(\omega_3+1) \end{pmatrix}^2 . \tag{A2}$$

Then the exact energy eigenvalues are readily obtained as

$$E_{l,\pm} = \frac{\hbar^2}{2mr^2}\left[l^2 + \frac{1}{4}\left((\omega_3+1)^2 + \omega_1^2\right) \pm l\sqrt{\omega_1^2 + (\omega_3+1)^2}\right] \tag{A3a}$$

$$= \frac{\hbar^2}{2mr^2}\left[n - \frac{1}{2}(1-\lambda_\pm)\right]^2 \tag{A3b}$$

$$= \frac{\hbar^2}{2mr^2}\left[n - \frac{\Phi_{\rm AC}^{(\pm)}}{2\pi}\right]^2 , \tag{A3c}$$

where $n = 0, \pm 1, \pm 2, \ldots$. Thus the energy spectrum obtained by diagonalization is equal to that calculated via the gauge transformation. Apparently matrix diagonalization is an easier way to obtain the eigenvalue than the $SU(2)_{\rm spin}$ gauge transformation that we have taken. However, the latter approach reveals the internal structure of the AC effect more clearly and may be used to identify the additional physical effects such as the spin-dependent motive forces and the non-Abelian Faraday law.[26]

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