Magnetotransport of a two-dimensional electron gas in a spatially random magnetic field

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We study magnetotransport of a two-dimensional electron gas (2DEG) subject to a random magnetic field produced by a demagnetized NdFeB magnet. A large, positive, V-shaped magnetoresistance is observed in a perpendicular applied external field. This effect is interpreted classically in terms of a nonuniform local Hall resistivity. Depleting the 2DEG with a gate beneath the random magnet yields a crossover from positive to negative magnetoresistance at a sheet resistance $\sim h/e^2$.

A considerable theoretical debate has arisen concerning the problem of a two-dimensional electron gas (2DEG) in a spatially random magnetic field.[1,2] Kalmeyer and Zhang[2] and Halperin, Lee, and Read[2] have related this problem to the quantum Hall system at filling factor $\nu=\frac{1}{2}$ through the use of the Chern-Simons gauge-field theory. A number of experimental groups have reported metallic behavior in magnetotransport near $\nu=\frac{1}{2}$ in high-mobility GaAs heterostructures and other materials.[3,4] A common observation in these experiments is a distinctive V-shaped positive magnetoresistance around $\nu=\frac{1}{2}$ with weak temperature dependence compared to odd-denominator fractional quantum Hall states.[3] Theoretical arguments[2] and numerical calculations by Kalmeyer *et al.*[2] have also revealed a positive magnetoresistance both for the $\nu=\frac{1}{2}$ problem and for the gauge-transformed problem of fermions in a random magnetic field.

Previous experiments addressing the random-field problem directly have produced random magnetic fields using a superconducting film on top of a Hall bar to create an inhomogeneous distribution of magnetic flux tubes penetrating the film when an external magnetic field is applied.[5–7] The observations of these experiments include a negative magnetoresistance associated with weak localization in an inhomogeneous magnetic field,[5,8] and a positive magnetoresistance associated with small-angle scattering of ballistic electrons by flux tubes.[7] Another recent experiment, using the flux cancellation and trapping effect of superconducting lead grains on the surface of a Hall bar to generate a random magnetic field,[9] revealed a quadratic positive magnetoresistance, also associated with classical scattering, during the initial field sweep. The changes in resistivity for the superconductor-on-2DEG experiments are at the level of 10% or less, and the effect is quenched at magnetic field above ~ 0.1 T due to the destruction of superconductivity[9] or the smearing of field inhomogeneity from overlapping vortices.[7]

In this paper, we report a technique to investigate electron transport in a random magnetic field using a demagnetized neodymium-iron-boron (NdFeB) ferromagnet attached to the surface of a high-mobility 2DEG heterostructure. We study magnetotransport in this system by applying a uniform external field B_{ext} in addition to the random field produced by the NdFeB magnet. The characteristic length of the random field is ~ 20 μm, which is greater than the transport mean free path of the 2DEG. We find a positive, linear (V-shaped) magnetoresistance about zero applied field at both 4.2 and 77 K, suggestive of similar observations around $\nu=\frac{1}{2}$. This effect is very different from what is observed in the superconductor-on-2DEG experiments in the following

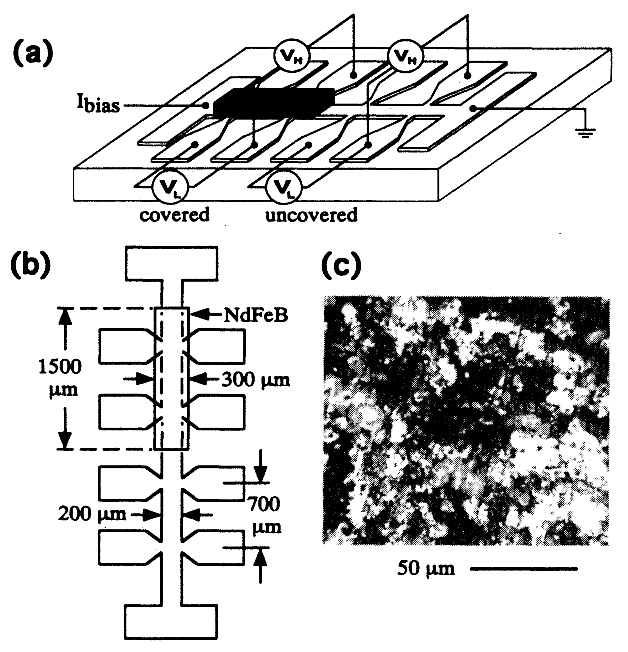

FIG. 1. (a) Schematic of our technique for realizing a random magnetic field by placing a demagnetized NdFeB magnet directly on top of the Hall bar. (b) Top view gives sample dimensions. (c) Optical micrograph of the face of the demagnetized NdFeB normal to sinter easy axis shows surface roughness, with 5–10-μm peaks on a $\sim$20-μm lateral length scale. (This face is placed on the Hall bar.)

ways. First, we observe strongly positive magnetoresistance, accounting for changes in longitudinal resistance by up to a factor of 8. Second, the magnetoresistance occurs on a much larger scale of external applied magnetic field of order 1 T. We then present a model of the magnetoresistance effect in terms of a local, nonuniform Hall resistivity. Finally, by depleting the 2DEG with a gate beneath the NdFeB, we induce a crossover from positive to negative magnetoresistance in the regime of strong localization (sheet resistance $\gtrsim h/e^2$).

Our samples consist of a $GaAs/Al_xGa_{1-x}As$ heterostructure with δ-doping layer ($N_D = 5\times10^{12}$ cm^{-2}) set back 250 Å (300 Å) from the 2DEG and a total distance of 800 Å (1000 Å) from the surface to the 2DEG. No dependence on growth scheme is observed. A 200-μm-wide Hall bar with 700 μm between voltage probes was defined by wet etching [Figs. 1(a) and 1(b)]. The typical mobility (without random magnet) at 4.2 K was 80 $m^2/(V\,s)$ and the carrier density was $\sim 3.1\times10^{15}$ m^{-2}, giving a transport mean free path of $l\sim 7.4$ μm. Securing the NdFeB slab with photoresist to one half of the Hall bar, as indicated in Fig. 1(b), allows simultaneous measurement of longitudinal and Hall voltages V_L and V_H with and without the NdFeB above the 2DEG. Samples were measured at 4.2 and 77 K using standard ac lock-in techniques at 317 Hz with a current bias of 0.5 μA.

The random magnets were thermally demagnetized sintered NdFeB,[10] cut into slabs 300 μm wide, 200 μm tall, and 1500 μm long. The easy axis of magnetization of the NdFeB powder is aligned during sintering, with this direction oriented perpendicular to the plane of the 2DEG.[11] In the demagnetized state, each sinter grain is multidomain, with the domains aligned along the easy axis with roughly equal fractions of domains pointing up and down. A relatively weak external field ($B_{ext}\sim 0.1$ T) along the easy axis results in partial magnetization along the applied field direction.[12,13] The surface of the NdFeB near the 2DEG is quite rough, as can be seen by optical microscopy [Fig. 1(c)], with 5–10-μm peaks and valleys (measured by atomic force microscopy) on a $\sim$20-μm lateral length scale. We expect the magnetic field created by the NdFeB as felt by the 2DEG to be largest at the peaks of the rough magnet surface which rest on the GaAs surface, resulting in a spatially random magnetic field at the 2DEG on this 20-μm roughness length scale. We find that the magnetization of the NdFeB (and therefore the rms strength of the random field) increases with the external field up to $B_{ext}\sim 1$ T, at which point its magnetization is nearly complete. Some unaligned domains persist at this value of B_{ext}, allowing the magnet to reversibly demagnetize as the field is lowered and reversed.[13] This reversibility lets us sweep between $B_{ext}=\pm1$ T with only minor hysteresis. Below 135 K,[11,14] the magnetization tilts $\sim30°$ away from its room-temperature easy axis; however, a strong random field perpendicular to the 2DEG is still present. While a disordered magnetic field is also present in the plane of the 2DEG, we expect the confined 2DEG to be sensitive only to the perpendicular component at these low fields.

Figure 2(a) shows a typical longitudinal resistance $R_L = V_L/I_{bias}$ and Hall resistance $R_H = V_H/I_{bias}$ versus

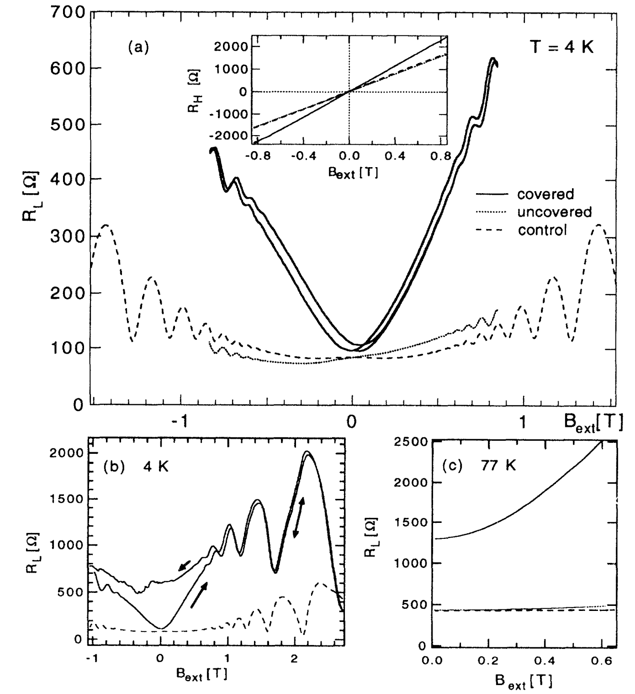

FIG. 2. (a) Longitudinal resistance R_L at 4.2 K vs external applied field B_{ext}, measured under the NdFeB magnet (covered, solid curve), on the other end of the Hall bar (uncovered, dotted curve), and after the removal of the NdFeB (control, dashed curve). Strong positive magnetoresistance exists only for the covered case. The inset shows the Hall resistance R_H for the same run, with enhanced slope in the covered region. (b) R_L for a high-field sweep up to $B_{ext}=9$ T for the covered and control cases. (c) R_L at 77 K vs B_{ext}, under the magnet (solid), at the other end of the Hall bar (dotted), and after removal of the NdFeB (dashed).

B_{ext} at 4.2 K for three cases: (i) for the half of the Hall bar covered by the random magnet (covered), (ii) for the uncovered half (uncovered); and (iii) after removing the random magnet, as measured on a subsequent cool down (control). A large positive magnetoresistance is found for the covered case, while the uncovered and control R_L show little dependence on B_{ext} over the same field range. The magnetoresistance under the random magnet is roughly linear in $|B_{ext}|$ and continues to increase up to $B_{ext}\sim 0.85$ T, at which point Shubnikov–de Haas oscillations begin. The maximum relative magnetoresistance $[R_L(B_{ext})-R_L(0)]/R_L(0)$ is $\gtrsim 500\%$ in this run, larger by at least a factor of 50 compared to the superconductor-on-2DEG experiments.[7,9] The three other lithographically identical samples we have measured show similar effects. (In one sample, R_L for the uncovered half also had a weak positive magnetoresistance, though much smaller than for the covered half). We attribute the small asymmetry in the uncovered R_L to nonuniform fringing fields from the remote NdFeB. A similar linear positive magnetoresistance is also seen at 77 K [Fig. 2(c)], accounting for a factor of 2 change in resistance above the zero-field value by $B_{ext}\sim 0.6$ T. The relative magnetoresistance at 4.2 K exceeds typical values reported in giant magnetoresistance (GMR) measurements of metallic multilayers at 4.2 K; for instance, relative resistance changes up to 80% are reported in a Co/Cu multilayer within a saturation field of ~ 0.4 T.[15] We believe the magnetoresistance of magnetic multilayers is of an entirely different origin than in the present system.

As discussed above, small external fields ($|B_{\rm ext}| \lesssim 1$ T) introduce only minor hysteresis in $R_L(B_{\rm ext})$ under the magnet. However, extending the sweep to several T [Fig. 2(b)] yields a strong hysteresis in R_L. Following a sweep to 9 T, the positive magnetoresistance around $B_{\rm ext}=0$ nearly vanishes as compared to that observed in sweeps confined to low field. This can be understood by noting that, following a sweep to 9 T, the NdFeB will be fully magnetized with a large remnant magnetization when $B_{\rm ext}=0$. Thus the return sweep through $B_{\rm ext}=0$ from high field enables us to explore magnetotransport in a different class of random magnetic field than the low-field sweep: For the low-field sweep, the strength of the randomness varies with $B_{\rm ext}$; for the high-field sweep, the strength of the random magnetic field (due to the rough surface) upon return to $B_{\rm ext} < 1$ T is relatively insensitive to $B_{\rm ext}$.

The low-field Hall resistance R_H in the covered region is linear in $B_{\rm ext}$, but with a steeper slope than in the uncovered and control cases [Fig. 2(a), inset]. Apparently, the magnetization of the NdFeB results in an average effective field at the 2DEG of $\sim 1.5 B_{\rm ext}$ in the covered region in addition to a spatially varying part. This interpretation assumes that the sheet density is not altered by the random magnet, which we believe to be the case based on data at high fields when the NdFeB is saturated.

At the temperatures studied, the magnetotransport effects observed are expected to be predominantly of a classical nature, which we model in terms of a local resistivity tensor that is nonuniform due to the random magnetic field since the transport mean free path is shorter than the length scale of the inhomogeneous magnetic field. Macroscopic Hall and sheet resistances, R_{xy} and R_{xx}, are then found by averaging over a spatially varying local resistivity.[16] A simple version of the problem involves a random division of the two-dimensional plane into two regions with differing local magnetic fields. Since differing fields will predominantly affect the local Hall resistivity, we assign resistivities ρ_{xx} and ρ_{xy}^a to one region and ρ_{xx} and ρ_{xy}^b to the other. Assuming that these two regions are statistically equivalent, this problem can be solved by an exact duality transformation,[17] giving macroscopic resistances $R_{xy}=(\rho_{xy}^a+\rho_{xy}^b)/2$ and $R_{xx}=(\rho_{xx}^2+(\rho_{xy}^a-\rho_{xy}^b)^2/4)^{1/2}$. For the low-field sweeps, we model the local Hall resistivities with a random component proportional to $B_{\rm ext}$: $\rho_{xy}^a=\alpha B_{\rm ext}(1+\delta_a)$ and $\rho_{xy}^b=\alpha B_{\rm ext}(1+\delta_b)$, where δ_a and δ_b are positive constants and $\alpha \equiv 1/ne$ is the Hall slope (measured in the control case). The average value $(\delta_a+\delta_b)/2$ appears as a correction to the Hall resistance, $R_{xy}=\alpha B_{\rm ext}[1+(\delta_a+\delta_b)/2]$, which can be measured directly by comparing Hall resistances in the covered and uncovered regions [Fig. 2(a), inset]. This form of the ρ_{xy}'s gives an enhancement to the longitudinal sheet resistance which depends on $B_{\rm ext}$ due to the random field,

$$R_{xx}=\left[\rho_{xx}^2+\left[\frac{\alpha B_{\rm ext}}{2}(\delta_a-\delta_b)\right]^2\right]^{1/2} \sim \frac{\alpha|B_{\rm ext}|}{2}|\delta_a-\delta_b| \;. \qquad (1)$$

The resulting V-shaped magnetoresistance agrees with the experiment. From the Hall resistances and $R_{xx}(B_{\rm ext})$ in the covered region [Fig. 2(a)], we can determine the constants $\delta_a \sim 0.66$ and $\delta_b \sim 0.48$ within the model. Note that $R_{xx}(0) \sim \rho_{xx}$ in the low-field sweep.

Following a high-field sweep to several T, the random field becomes relatively insensitive to $B_{\rm ext}$. In this case, we can model the local resistivities as $\rho_{xy}^a=\alpha(B_{\rm ext}+\Delta_a)$ and $\rho_{xy}^b=\alpha(B_{\rm ext}+\Delta_b)$, with constants Δ_a,Δ_b. The Hall resistance will now have a nonzero intercept but a slope that is unaffected by the random field, $R_{xy}=\alpha[B_{\rm ext}+(\Delta_a+\Delta_b)/2]$, while $R_{xx}=[\rho_{xx}^2+\alpha^2(\Delta_a-\Delta_b)^2/4]^{1/2}$ will be enhanced due to the fixed random field but does not depend on $B_{\rm ext}$. The absence of magnetoresistance is consistent with our observations for the high-field sweep, as is the fact that $R_{xx}(0) > \rho_{xx}$ following a return from high field [Fig. 2(b)]. From the data in Fig. 2(b), we find $\Delta_a \sim 0.28$ T and $\Delta_b \sim 0.13$ T. We emphasize that the usefulness of the model for the random magnetic-field problem is not in determining the particular values of the constants but rather in its qualitative agreement with experiment, in particular the linear dependence of the magnetoresistance on the external magnetic field.

The experimental values for δ_a,δ_b and Δ_a,Δ_b can be shown to be consistent with expected magnetic fields from a rough NdFeB surface, which we model as an assembly of magnetized spheres with alternating distances from the GaAs surface (Fig. 3). We model the NdFeB

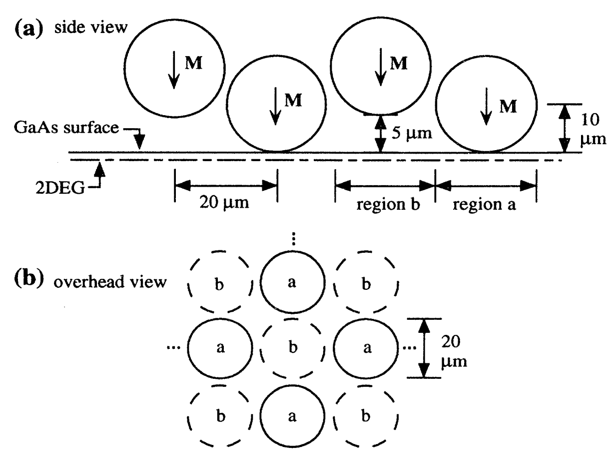

FIG. 3. (a) Side view of the rough NdFeB surface, modeled as uniformly magnetized spheres. Surface roughness is simulated by alternating distance between the spheres and the GaAs surface. (b) Overhead view of the model of the rough NdFeB surface. Solid spheres contact the GaAs surface and dashed spheres are displaced 5 μm from GaAs.

surface as composed of 20-μm-diameter spheres of magnetic material to reflect the lateral length scale for the surface roughness. Along the surface, the spheres are taken to be alternatively in contact with the GaAs and set back 5 μm, reflecting the size scale for the peaks and valleys in the surface roughness. For $B_{ext} \gtrsim 1$ T, each sphere is further assumed to be uniformly magnetized along the direction of B_{ext} and to produce a dipolar magnetic field with perpendicular component

$$B_{\perp} = (4/3\pi R^3) M (3\cos^2\theta - 1)/r^3 , \tag{2}$$

where $R = 10$ μm is the radius of the sphere and M is the magnetization [$=1.85$ T for NdFeB at saturation at 4 K (Ref. 11)]. The regions a and b defined in the binary magnetoresistance model correspond to regions of the 2DEG beneath the close spheres and the distant spheres, respectively. The magnetic field in region a can be estimated as due to the single close sphere only, while in region b the set-back sphere plus its four close neighbors all contribute significantly. At saturation ($B_{ext} \sim 1$ T) this model yields an average $B_{\perp}$ of ~ 0.44 T in region a and ~ 0.28 T in region b, giving $\delta_a = 0.44$ and $\delta_b = 0.28$. Between saturation and the return to $B_{ext} = 0$, Hall measurements indicate that the average magnetic field produced by the NdFeB decreases by a factor of ~ 0.6; that is, the remnant magnetization is 0.6 of the saturation magnetization. Using this factor gives $\Delta_a \sim 0.6 \times \delta_a \times 1$ T~ 0.27 T, and $\Delta_b \sim 0.6 \times \delta_b \times 1$ T~ 0.18 T. These model values for both δ_a, δ_b and Δ_a, Δ_b agree with the measured values from the binary magnetoresistance model within a factor of 2. Given the crudeness of our magnetized-sphere model, such agreement appears quite reasonable.

At 77 K, the samples continue to show a large positive magnetoresistance in the covered region—up to a relative change of 200%in 0.6 T [Fig. 2(c)]—though the local susceptibilities (δ's) of the NdFeB inferred from the above model are different from the 4.2-K values. Unlike at 4.2 K, the 77-K data also show an enhanced $R_{xx}(0)$ coexisting with a large V-shaped magnetoresistance—a combination outside either of the two limiting cases considered above. We believe the qualitative difference results from the reduced mobility (and hence transport mean free path) at 77 K, which makes the diffusive electrons more sensitive to field inhomogeneities on a smaller

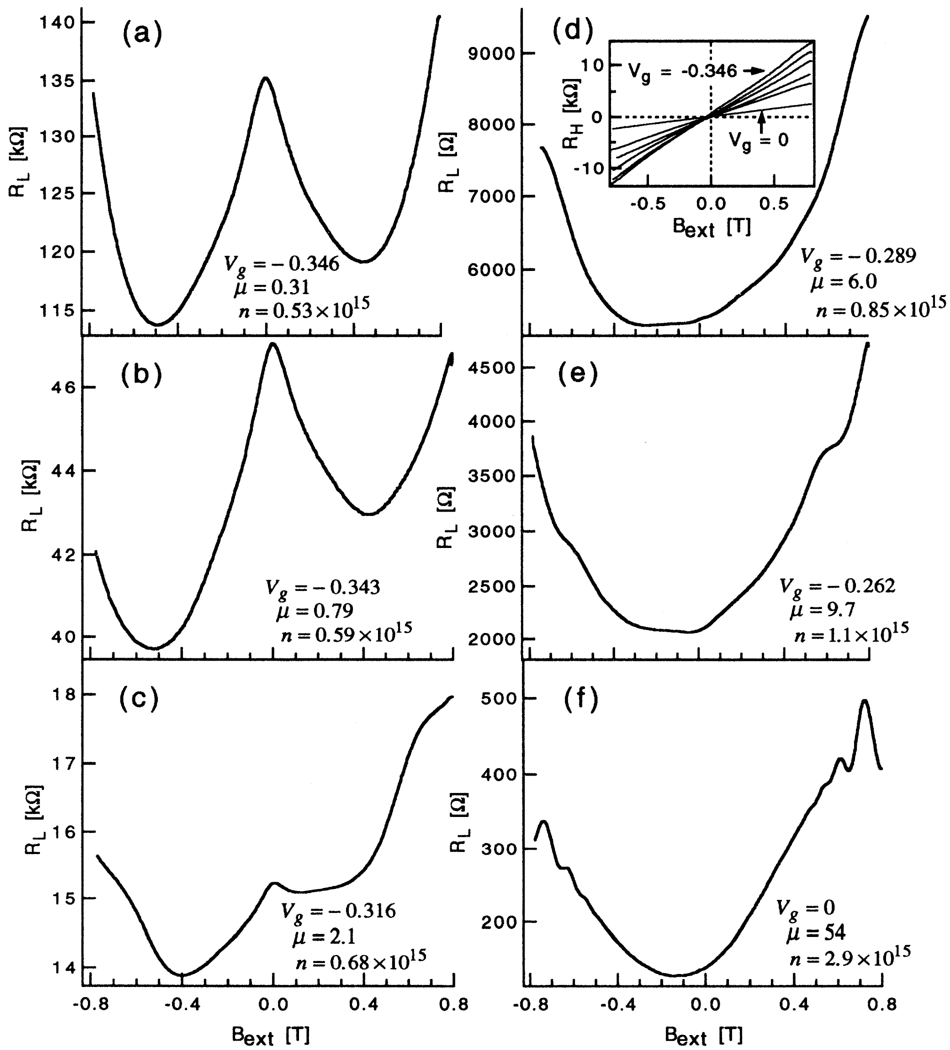

FIG. 4. Longitudinal resistance R_L under gate and random magnet for six gate voltages V_g (V), with associated mobilities μ [m^2/(V s)] and sheet densities n (m^{-2}). More negative gate voltage leads to increased sample disorder. The magnetoresistance changes from positive to negative as the sample becomes strongly localized; Hall resistance R_H remains classical.

size scale, perhaps due to <1-μm domains[11] in addition to the 20-μm roughness. This could be accounted for in a more complicated model that mixes the effects of δ and Δ.

Finally, we investigate the effect of increased potential disorder in the presence of a random magnetic field. We use a nominally identical Hall bar and NdFeB magnet, now including Ti-Au gates on both the uncovered half of the sample and on the covered half beneath the NdFeB magnet. By applying a negative voltage to the covered gate with respect to the 2DEG, the sheet density under the magnet was reduced, which in turn lowered the mobility. Lock-in measurements were made at 11.7 Hz with a current bias of 10, 20, or 50 nA depending on the sheet resistance. Over a range of gate voltage V_g from 0 to -0.346 V, the mobility decreased by more than a factor of 100, from 54 to 0.30 $m^2/(V\,s)$, while the sheet density varied from 2.9×10^{15} to 0.53×10^{15} m^{-2}. The gate on the uncovered half remained floating with respect to the 2DEG for all sweeps. Figure 4 tracks the behavior of R_L along with the corresponding gate voltage, mobility, and sheet density. For the case $V_g=0$ V in Fig. 4(f), we obtain results equivalent to those of Fig. 2(a). As the gate voltage becomes more negative, the relative magnetoresistance decreases [Figs. 4(e) and 4(d)], and finally we observe a crossover from positive to negative magnetoresistance as the sheet resistance becomes of order h/e^2 [Fig. 4(c)]. This crossover is presumably associated with a transition to the strong localization regime, similar to previous observations[18] in highly disordered 2DEG's (without random field) in the regime of variable-range hopping. Our observations are also consistent with numerical results of Kalmeyer *et al.*,[2] which show a crossover from positive to negative magnetoresistance for the random magnetic-field problem as potential disorder is increased. Throughout the crossover to strong localization, the Hall slope remains classical, showing no nonlinearity [Fig. 4(d), inset]. The only effect of the gate depletion on the Hall resistance is a change in slope reflecting the decrease in sheet density. This result is again similar to what was seen without a random magnet.[19]

While our experiment demonstrates a striking similarity and connection[2] between the magnetotransport behavior of the $\nu=\frac{1}{2}$ quantum Hall system and the 2DEG under a random magnetic field, it is important to point out that, in the present experiment and model, the V-shaped positive magnetoresistance was found to be connected to the dependence of the random field on the external field. Drawing a parallel between our results and the positive magnetoresistance around $\nu=\frac{1}{2}$ suggests that the self-consistent Chern-Simons gauge field involved in the mapping from the $\nu=\frac{1}{2}$ problem likely also has some dependence on the external field, though many details remain to be worked out.

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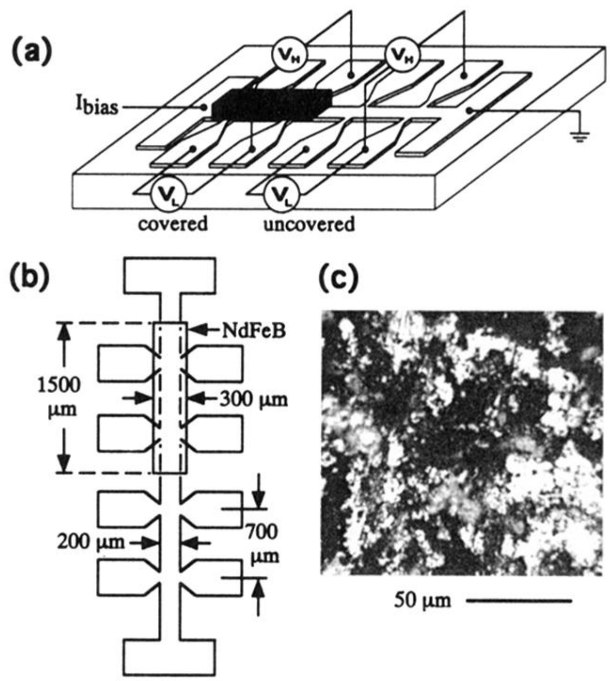

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