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Tunnel junctions of unconventional superconductors

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The phenomenology of Josephson tunnel junctions between unconventional superconductors is developed further. In contrast to s-wave superconductors, for d-wave superconductors the direction dependence of the tunnel matrix elements that describe the barrier is relevant. We find the full I-V characteristics and comment on the thermodynamical properties of these junctions. They depend sensitively on the relative orientation of the superconductors. The I-V characteristics differ from the normal s-wave resistively-shunted-junction-like behavior.

The symmetry of the superconducting order parameter of the high-T_c compounds is the subject of a heated debate. Quite early there were theoretical suggestions[1–4] that these materials are unconventional superconductors that have $d_{x^2-y^2}$ symmetry of the order parameter. They have stimulated both experimental and theoretical work. The experimental situation is still unclear.[5] Several experiments indicate d-wave pairing, e.g., the temperature dependence of the penetration depth $\lambda(T)$ (Ref. 6) and the NMR relaxation rates.[7,8] The most convincing set of experiments up to now are the superconducting quantum interference device (SQUID) experiments,[9,10] which show that a SQUID loop consisting of a high-T_c crystal closed by an ordinary s-wave superconductor shows a phase shift of the order of π in the dependence of the critical current on the flux, and the experiments on flux quantization in multidomain rings.[11,12] There are other experiments, however, that cannot be reconciled with the $d_{x^2-y^2}$ model and have been interpreted to indicate (possibly anisotropic) s-wave pairing,[13,14] see, however, Ref. 15. Since a microscopic theory is still lacking, there are also different theoretical opinions on the symmetry of the order parameter. Some phenomenological calculations, assuming d-wave pairing, have been done and they yield partial understanding of some experimental data.

In this paper we will further develop the phenomenology of unconventional superconductors, assuming BCS-like behavior for the density of states. Our results are not specific for high-T_c materials, but may be relevant also for other unconventional superconductors such as heavy-fermion compounds. We will consider tunnel junctions between two pieces of bulk superconductor as shown in Fig. 1. The superconductor is assumed to have a k-dependent order parameter of the form $\Delta_k=\Delta_0[\hat{k}_x^2-\hat{k}_y^2]$. In contrast to earlier work,[16–19] we include the direction dependence of the tunnel matrix elements that describe tunneling across the insulating barrier between the superconductors. For s-wave superconductors this dependence drops out, but for d-wave superconductors it is essential.[20,21]

Along the lines of Ref. 22 we calculate the full current-voltage characteristic (I-V curves) for different relative orientations. We find that the presence of gap nodes strongly influences the I-V characteristics. For a specific relative orientation the quasiparticle current will be proportional to the square of the voltage, which leads to new behavior for the total current, which is different from the usual resistively-shunted-junction-(RSJ-) like overdamped junction. We will also comment on the thermodynamic properties[23] of d-wave tunnel junctions and the relevance of our work for thin granular high-T_c films.[24]

A convenient starting point is the tunneling Hamiltonian

$$H=H_L+H_R+\int_{r\in L}\int_{r'\in R}\sum_{\sigma}\,[T(r,r')\psi^{\dagger}_{r,\sigma}\psi_{r',\sigma}+\text{H.c.}]\ . \tag{1}$$

Here H_L and H_R denote the unperturbed Hamiltonians of the left and right superconductor. From a second-order perturbation expansion in the tunnel matrix elements one finds the Ambegaokar-Baratoff[22] formula that expresses the current across a tunnel junction in second-order perturbation theory in the tunnel matrix elements as

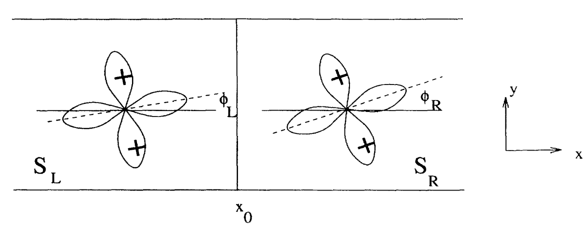

FIG. 1. The system we consider consists of two coupled d-wave superconductors. Their orientation is characterized by the angles ϕ_L and ϕ_R.

$$I=2e\ \mathrm{Im}\int\frac{d^3k}{(2\pi)^3}\int\frac{d^3k'}{(2\pi)^3}\int\frac{d\omega}{2\pi}\int\frac{d\omega'}{2\pi}[f_L(\omega)-f_R(\omega')]$$
$$\times\left\{|T_{k,k'}|^2\frac{A(k,\omega)A(k',\omega')}{\omega-\omega'+i\eta}+T_{k,k'}T_{-k,-k'}\frac{\bar{B}(k,\omega)B(k',\omega')}{\omega-\omega'+i\eta}e^{i[\varphi+2(\mu_L-\mu_R)t]}\right\}\ . \tag{2}$$

Here A and B are the spectral densities of the normal and anomalous Green's functions, φ is the phase difference between the two superconductors, and $f_{L/R}$ the Fermi distributions at the chemical potential $\mu_{L/R}$. The left and right chemical potential differ by the applied voltage, $\mu_L-\mu_R=eV$. The $T_{k,k'}$ are the matrix elements that transfer electrons from a state k in one superconductor to a state k' in the other. The first term in (2) describes the quasiparticle current I_{QP} and the second term the supercurrent which is proportional to the critical current I_{CR}. The spectral densities have the form

$$A(k,\omega)=\pi\left[\left(1+\frac{\epsilon_k}{E(k)}\right)\delta(\omega-E(k))+\left(1-\frac{\epsilon_k}{E(k)}\right)\delta(\omega+E(k))\right],$$

$$B(k,\omega)=\pi\frac{\Delta(\hat{k})}{E(k)}[\delta(\omega+E(k))-\delta(\omega-E(k))]\ ,$$

where $E(k)=\sqrt{|\Delta(\hat{k})|^2+\epsilon_k^2}$. The strategy is to take seriously the orientation dependence.[20,21] This is necessary, since with the standard assumption that the tunnel matrix elements $T_{k,k'}$ are independent of momenta,[22,23] Eq. (2) yields a critical current $I_{\mathrm{CR}}=0$ after an angular average with $\Delta_k=\Delta_0[\hat{k}_x^2-\hat{k}_y^2]$. The results of Ref. 19 are therefore difficult to understand, as neglecting the direction dependence of the tunneling matrix elements leads to independent angle averages in both superconductors, and the angle average of $d\Delta_k=\Delta_0[\hat{k}_x^2-\hat{k}_y^2]$ is zero. Xu *et al.*[19] avoid this consequence by omitting a term proportional to $\hat{k}_x^2$ in the numerator of their Eq. (4).

Thus, in order to obtain physical results, the direction dependence of tunneling has to be taken into account. We will consider the case of a smooth barrier for which momentum parallel to the barrier is conserved during tunneling. A reasonable assumption seems to be to take the tunnel matrix elements $T(r,r')$ nonvanishing only when r and r' are both close to the barrier, i.e.,

$$T(r,r')=\tilde{T}\delta^{(3)}(r-r')\delta^{(1)}(r_x-x_0), \tag{3}$$

where x_0 is the location of the barrier, as indicated in Fig. 1. This ensures that the momentum parallel to the tunnel barrier is conserved, since the Fourier transform of the matrix element is $T_{k,-k'}\sim\delta^{(2)}(k_\parallel-k'_\parallel)$. In the following we will use the ansatz

$$T_{k,-k'}=(2\pi)^2\tilde{T}\delta^{(2)}(k_\parallel-k'_\parallel)f(|\hat{k}_x|)\Theta(k_xk'_x) \tag{4}$$

for the tunneling matrix elements. The last factor is a restriction on the direction of tunneling (Θ denotes the Heaviside function). Before tunneling the electron should move towards the barrier, after tunneling away from the barrier. The function $f(|\hat{k}_x|)$ is a weight function that makes tunneling perpendicular to the barrier more probable than tunneling parallel to the barrier. If one models the barrier by a slab of finite thickness and takes the tunneling probability to be exponentially small in the traversed distance in the barrier, one deduces $f(|\hat{k}_x|)\sim\exp(-c/|\hat{k}_x|)$, see Ref. 25. Numerical constants may depend on the exact choice for f, but the principal behavior should not depend on it. In the following we take it to be $f(|\hat{k}_x|)=|\hat{k}_x|$, which corresponds to vanishing thickness of the barrier.

With this explicit form of the tunnel matrix elements, both the quasiparticle current I_{QP} as well as the critical current I_{CR} following from Eq. (2) can be evaluated by numerical integration for different temperatures. For normalization we calculate the normal-state current for $\Delta_0=0$ with the help of (2). This allows us to identify the normal-state conductance G_N of the barrier as $G_N=8\pi^5\tilde{T}^2N(0)^2k_F^{-2}A/R_K$, where A denotes the area of the junction and $R_K=h/e^2$ the Klitzing quantum of resistance. The expression for the critical current that we find from Eq. (2) with $\mu_l=\mu_r$ is

$$I_{\mathrm{CR}}=2e\int d\omega\,d\omega'\int\frac{d\Omega}{4\pi}\frac{d\Omega}{4\pi}\frac{f(\omega)-f(\omega')}{\omega-\omega'}\times\frac{\Delta_L(\Omega)}{\omega}\frac{\Delta_R(\Omega')}{\omega'}N_L(\omega)N_R(\omega')|T(\Omega,\Omega')|^2\ , \tag{5}$$

where $N_L(\omega)=N(0)|\omega|\Theta(|\omega|-\Delta_L(\Omega))/\sqrt{\omega^2-\Delta_L^2(\Omega)}$ denotes the (angle-resolved) density of states of the left side, $N_R(\omega')$ is defined similarly for the right side, $d\Omega=d\phi\,d\theta\sin\theta$, and $\Delta_{L/R}(\Omega)=\Delta_0\cos[2(\phi-\phi_{L/R})]$. The angles ϕ_L and ϕ_R determine the relative orientation of the two superconductors. A natural choice which applies to many junctions is $\phi_L=\phi_R=0$. The results are shown in Figs. 2 and 3(a) for I_{QP} and I_{CR} respectively. The magnitude and sign of the critical current show a strong orientation dependence, see Fig. 3(b).

The quasiparticle current behaves mostly like for an s-wave superconductor $I_{\mathrm{QP}}\sim\exp\{-\Delta_{\mathrm{eff}}/k_BT\}$, where the leading behavior is determined by tunneling from a gap node in one superconductor into the effective gap Δ_{eff} in the other. However, for those special relative orientations for which the gap nodes in the left and right superconductor are parallel, the behavior is different. The quasiparticle current $I_{\mathrm{QP}}\sim V^2$ for voltages $2\Delta_0>eV\geqslant k_BT$ and $I_{\mathrm{QP}}\sim V$ for voltages $eV\leqslant k_BT$. This can be understood as follows. The dominant contribution to I_{QP} arises from "node to node" tunneling. The available phase space around the gap nodes scales with

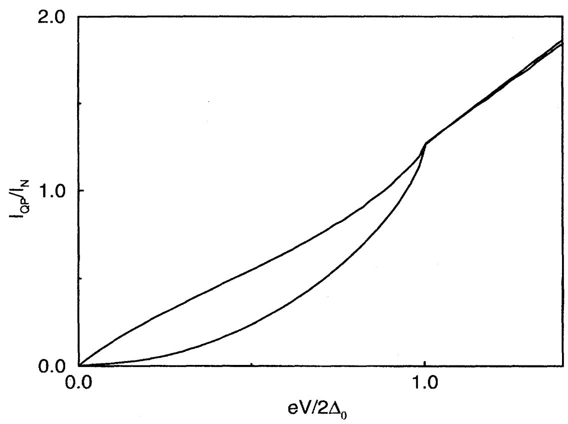

FIG. 2. The quasiparticle current $I_{\rm QP}$ as a function of the applied voltage for temperatures $T/\Delta_0=1$ (upper curve) and 0.1 (lower curve) for the case $\phi_L=\phi_R=0$. Here and in the following figures, $I_N \equiv I_N(eV=2\Delta_0)$.

the larger of eV and T. With the usual factor V from the difference in f_L-f_R this yields the quoted behavior.

The total current I may be obtained by integration over time of Kirchhoff's equation in the phase representation, $I=I_{\rm CR}\sin(\phi)+I_{\rm QP}(2eV=\hbar\dot{\phi})$. In this way the average time

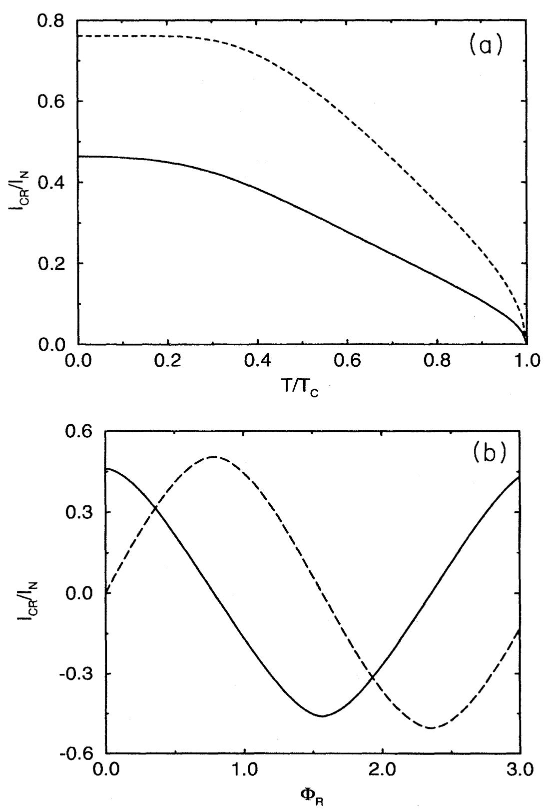

FIG. 3. (a) The critical current $I_{\rm CR}$ as a function of temperature for the case $\phi_L=\phi_R=0$ (full line). For comparison the s-wave result is also shown (dashed line). (b) The critical current $I_{\rm CR}$ as a function of the relative orientation $\phi_R[\phi_L=0$ (full line), and $\phi_L=\pi/4$ (dashed line)] at $T=0$.

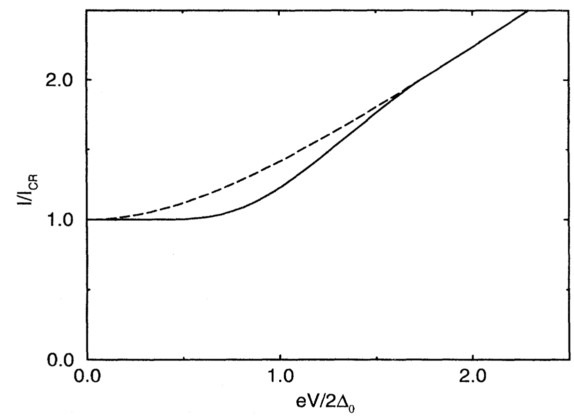

FIG. 4. The complete I-V characteristic for a d-wave tunnel junction with $\phi_L=\phi_R=0$ at $T=0$ (full line). For comparison also the RSJ result is shown (dashed line).

derivative of the phase is found as a function of the total current through the junction. The result is shown in Fig. 4 for low temperatures $k_BT \ll \Delta_0$ and $\phi_L=\phi_R=0$, i.e., the geometry in which node to node tunneling appears and $I_{\rm QP}\sim V^2$. The I-V characteristics deviate clearly from the well-known RSJ behavior. For higher temperatures T of the order of the gap Δ_0 we have $I_{\rm QP}\sim V$ (see Fig. 2) and the RSJ behavior for the I-V curve is recovered.

We now continue with the discussion of the thermodynamic properties that are described by an effective action for the phase difference across the junction. This is especially relevant for small mesoscopic tunnel junctions with a low capacitance C.[26] From a second-order perturbation expansion in the tunnel matrix elements one finds the following effective action for the phase difference φ across the junction:[23]

$$S[\varphi]=\int_0^\beta d\tau \frac{C}{8e^2}\left(\frac{\partial\varphi(\tau)}{\partial\tau}\right)^2 + \int_0^\beta d\tau d\tau' \cos\left(\frac{\varphi(\tau)-\varphi(\tau')}{2}\right)\alpha(\tau-\tau') + \int_0^\beta d\tau d\tau' \cos\left(\frac{\varphi(\tau)+\varphi(\tau')}{2}\right)\beta(\tau-\tau') \quad . \tag{6}$$

The kernels α and β describe quasiparticle tunneling and the Josephson coupling, respectively. They are given by

$$\left.\begin{matrix}\alpha(\tau)\\ \beta(\tau)\end{matrix}\right\} = \int \frac{d^3k}{(2\pi)^3}\int\frac{d^3k'}{(2\pi)^3}\left\{\begin{matrix}G(k,\tau)G(k',-\tau)\\ \bar{F}(k,\tau)F(k',-\tau)\end{matrix}\right\}|T_{k,k'}|^2 \ , \tag{7}$$

where the normal and anomalous Green's functions G and F were introduced. The usual approximation, in which the Fourier transform of the tunnel matrix element $T_{k,k'}$ is taken to be independent of momenta, leads for s-wave superconductors to the standard expressions in the literature, i.e., both α and β decay exponentially on the time scale Δ^{-1}. The same approximation for a d-wave junction will lead to unphysical results. For instance the beta term is found to be zero, as the average over orientations of Δ_k is zero.

To remedy this shortcoming, one again has to retain the direction dependence of the tunneling matrix elements (4). The gap nodes have a pronounced effect on the long-time behavior of α and β. If $\phi_L = \phi_R = 0$, the gap nodes in the two superconductors are in the same direction (the "node to node" geometry) and the asymptotic behavior of $\alpha(\tau)$ is τ^{-3}. This corresponds to a low-frequency behavior $\sim \omega^2 \ln\omega$, which is "super-ohmic." An investigation of the $\omega^2 \ln\omega$ dissipation remains a subject for further study,[27] and may be relevant for the phase diagram and superconductor-insulator transition in thin granular high-T_c films.[24] For relative orientations without node-to-node tunneling we find exponential decay, as was also found for s-wave tunnel junctions.

In conclusion we have calculated the full I-V characteristic for a tunnel junction between two unconventional superconductors by making a new ansatz for the direction dependence of the tunnel matrix elements. The temperature dependence and a strong dependence on relative orientation are found. For specific orientations the I-V characteristics differ from the usual RSJ-like behavior. All of our predictions have experimental consequences and should be verifiable using thin-film junctions on bicrystal substrates.

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