PHYSICAL REVIEW B VOLUME 51, NUMBER 18 1 MAY 1995-II

Kondo effect in systems of reduced dimensionality

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(Received 15 August 1994; revised manuscript received 11 November 1994)

This paper describes a series of experiments concerning the Kondo effect in metal films and wires which have revealed some surprising aspects of the Kondo behavior in quasi-two- and quasi-one-dimensional systems. We have studied two different Kondo alloys, Au(Fe) and Cu(Fe), and found that in both cases the Kondo contribution to the resistivity, $\Delta\rho_K$ is sensitive to the size of the sample, and is suppressed at length scales below about 1500 Å. This critical length scale is observed in both quasi-two and quasi-one-dimensional systems. Studies of bilayer samples composed of a Kondo film in contact with a layer of the pure host material [e.g., Cu(Fe) overcoated with Cu] reveal the existence of a Kondo proximity effect, which is also length scale dependent. Results for the spin scattering rate τ_s^{-1} for conduction electrons, obtained via weak localization measurements, indicate that τ_s^{-1} is suppressed in quasi-two-dimensional systems in a manner similar to that found for $\Delta\rho_K$. Finally, we find that disorder plays a central role; increasing the amount of disorder (i.e., elastic scattering) suppresses $\Delta\rho_K$. Surprisingly, the length scale which controls the Kondo behavior seems to be essentially independent of the Kondo temperature T_K. This implies that physics beyond that contained in the simplest Kondo models is responsible.

I. INTRODUCTION AND BACKGROUND

The Kondo effect is an old and much studied problem in condensed matter physics, involving the interaction between a local magnetic moment and a sea of conduction electrons. This interaction leads to several anomalous properties, perhaps the best known of which is a logarithmic variation of the resistivity with temperature at high temperatures. This was first explained theoretically by Kondo who obtained it via perturbation theory in the interaction strength.[1] It was, of course, immediately recognized that ρ cannot vary logarithmically to arbitrarily low temperatures, since this would imply a divergent contribution to the resistivity. There has been an enormous amount of theoretical and experimental effort aimed at understanding the nature of the Kondo ground state and the associated low-temperature behavior.[2–5] It is now understood that this ground state is a singlet in which the local moment is "compensated" by the conduction electron spins. The development of a theory of the ground state turned out to be quite difficult, and was achieved only over the course of many years, culminating with the work of Wilson and others. Exact results are now available for many thermodynamic quantities.

An appealing qualitative picture of the Kondo ground state is that the local moment is enveloped by a screening cloud of conduction electron spins arising from an antiferromagnetic exchange interaction between the local moment and the electrons. This screening becomes stronger as the temperature is lowered, and below the Kondo temperature T_K, the moment is fully screened resulting in a singlet ground state. As far as we know, there are no exact results for the relevant spin correlation functions, but a number of approximate calculations have considered the spatial size of this screening cloud.[6–9] While the properties of the cloud (and even its existence) are still a matter of debate, several calculations predict that its extent should be of order

$$R_K \approx \frac{\hbar v_F}{k_B T_K} , \tag{1}$$

where v_F is the Fermi velocity. This length scale is quite large compared to other microscopic lengths; for T_K = 1 K, and assuming a typical value for v_F, $R_K \approx 2$ μm. There have been a number of experimental attempts to observe either R_K or the effects of the screening cloud.[10] Such experiments are not easy, since, while the cloud might be quite large, it is also extremely dilute, as the conduction electrons in the cloud have a total polarization of order only μ_B (corresponding to one local moment). In addition Friedel oscillations can also mask the behavior associated with the cloud at relatively short distances. Nevertheless, several experiments suggest that either the cloud does not have the size predicted by (1) or may not even exist.[10]

Whether or not the picture of a Kondo screening cloud described by the length scale R_K is correct, one would certainly expect that the Kondo effect should be characterized by *some* length scale. Surprisingly, despite the large amount of work on the Kondo problem, it appears that the question of length scales is far from settled. This state of affairs has motivated us to examine the problem from the point of view of "mesoscopics." That is, assuming that there is a length scale associated with the Kondo effect, one might expect the Kondo behavior to be different in a small system (as compared to a "bulk" sample), provided that one or more dimensions are small compared to the relevant length scale. By studying samples of different sizes one may thus be able to learn about the Kondo length scale. To this end we have studied the Kondo behavior of a variety of quasi-one- and quasi-two-dimensional systems. Our experiments have revealed that there is indeed a length scale associated with the Kondo effect, and that it is in the neighborhood of 1500 Å in two different Kondo systems, Au(Fe) and Cu(Fe). In conducting these experiments we have followed a rather tortuous path both in our understanding of the Kondo effect, and in our interpretation of the results. A coherent picture of the experiments and what they imply can only be obtained from a careful consideration of essentially all of the results together. It therefore seems worthwhile to discuss all of our results to date in one place, and that is the purpose of this paper. In this paper we (1) have attempted to show how the different experiments are logically connected, (2) present a number of new results, especially with regards to the behavior of the spin-scattering rate and the role of disorder, and (3) consider at some length the criticisms which have been raised against our experiments. We also discuss the current status of theories which have attempted to account for our results.

II. THEORETICAL BACKGROUND

A. Kondo effect

It seems quite fair to say that theoretical work on the Kondo problem has led to important new insights into many-body physics. Here we have in mind the work of Wilson on renormalization group approaches,[3] as well as other developments.[2,4,5] While interest in the single-impurity Kondo problem, i.e., the behavior when interactions between the local moments can be neglected, has declined in recent years, much attention remains focused on concentrated systems, such as the Kondo lattice, in which these interactions are important.[5] Such models are thought to be relevant to a variety of strongly correlated electronic systems including the heavy fermion metals and perhaps the high-T_c materials. In these cases a variety of length scales comes into play. Any new insights concerning length scales in the single-impurity Kondo problem should be extremely relevant to concentrated systems and thus of broad interest.

We believe that our experiments are all in the dilute limit; i.e., the interactions between local moments can be neglected. In that case the simplest model of the Kondo effect need consider a single local magnetic moment interacting with a sea of conduction electrons via an exchange interaction

$$\mathcal{H}_{\text{ex}} = J\vec{S}_i \cdot \vec{S}_e \ , \tag{2}$$

where $\vec{S}_i$ is the spin of the local moment, i.e., the impurity, $\vec{S}_e$ the spin of a conduction electron, and J is a constant which is presumed positive so that the interaction is antiferromagnetic (as required for there to be a Kondo effect). At high temperatures the interaction (2) yields a contribution to ρ which varies logarithmically with T,

$$\Delta\rho_K = -B \log T \ , \tag{3}$$

as was first calculated by Kondo.[1] Here B is a positive constant which is a function of J, the density of states, and other properties of the electron gas. The logarithmic variation in (3) means that as T is reduced the effects of the interaction (2) become larger, essentially because the electrons with antiparallel spin spend more time near the local moment; i.e., they scatter from it more strongly, thus increasing the resistivity. $\Delta\rho_K$ cannot increase without a bound, however, since there is a limit to how much a single local moment can scatter. The divergence in (3) is cut off— i.e., $\Delta\rho_K$ becomes a constant— at temperatures of order the Kondo temperature $T_K \approx T_F \exp\left(-\frac{1}{JN}\right)$, where T_F is the Fermi temperature and N is the density of states at the Fermi level.

The results we have described thus far do not tell us anything about length scales. While there are exact results for the thermodynamic behavior produced by the interaction (2),[4,5] so far as we know there are only approximate solutions for the correlation functions. Some of the earliest work on these correlations led to the screening cloud picture mentioned in the Introduction, namely, a cloud of electrons in the neighborhood of the impurity which screens its moment and leads at low temperatures to a singlet ground state. Several calculations[6–9] have found that the length scale which characterizes this cloud is just R_K in (1). A number of more recent theoretical treatments yield the same length scale in other related contexts.[11]

If we accept that R_K is the length scale that controls the Kondo behavior, then we would certainly expect to find the Kondo properties to be altered in systems in which one or more dimensions are small compared to R_K. It was just this possibility that motivated, at least in part, our initial experiments. However, our experiments do *not* rely in any way on the assumption that R_K is a relevant length scale. Indeed, this was an issue our work was designed to resolve in the first place. While the experiments certainly do show that there *is* a length scale in the neighborhood of 1500 Å which controls the Kondo behavior, the results also suggest that this length scale is *not* R_K. Nevertheless, it is still useful to discuss R_K a bit further, as this will bring out several points that will be important later.

We first note that the spin correlation function should not be expected to have a purely exponential form with a characteristic length of R_K. The effects of Friedel oscillations are also present. While these oscillations generally die out before one reaches R_K (assuming that T_K is sufficiently small), they are dominant near the local moment, since the net polarization contained in the screening cloud is small. While this is usually not a problem when interpreting analytic calculations, it can greatly complicate the analysis of numerical work. Second, R_K should only characterize the screening at low temperatures, well below T_K. One would not expect to find R_K as the dominant length scale for $T > T_K$, since in that range the ground state has not yet "condensed." Third, the form of the result for R_K in (1) is expected from dimensional analysis, since it is the simplest way to obtain a length scale from the energy scale T_K. Fourth, (1) can be derived in a qualitative sense using arguments familiar from work on transport in disordered systems. If one considers two electrons near the Fermi level which differ in energy by an amount $k_B T_K$, then R_K is the distance they can travel (ballistically) before their phases differ by an amount of order π.

We will see below that the experiments are *not* consistent with a dominant length scale given by R_K. All of the quasi-one- and quasi-two-dimensional Kondo samples that have been studied experimentally have much higher levels of disorder (i.e., much shorter elastic mean free paths) than the bulk samples generally used to study the Kondo effect. It is thus natural to consider what effects this disorder might have on R_K. Perhaps the simplest possibility can be seen by reconsidering the qualitative derivation of (1) given above. If the level of disorder is large enough that R_K is longer than the elastic mean free path ℓ_e, then the motion of the two electrons will be diffusive, and the distance they can travel before going out of phase will be

$$R_K^* = \sqrt{R_K \ell_e} \ . \tag{4}$$

While we will see below that R_K^* also does not appear to be the length scale observed in our experiments, it does raise the possibility that disorder may be important.

The role of disorder is, of course, now well appreciated, in the context of weak localization and electron-electron interaction effects. This background has prompted several workers to consider the effect of disorder on the Kondo effect. Ohkawa *et al.*[12–14] and Suga *et al.*[15,16] have considered the interplay between weak localization and the Kondo effect. They find that the presence of disorder leads to additional singular contributions to the conductivity at low temperatures, which in two dimensions are proportional to T^{-1}. Vladár and Zimányi[17] considered the behavior of T_K, and found that it could vary nonmonotonically as a function of disorder, while Tešanović[18] discussed the behavior of systems which are dominated by diffuse surface scattering, and showed that the Kondo effect is modified somewhat, becoming slightly more singular when the film thickness is reduced. Fukuyama[19] found that while the presence of localized spins enhances the electron-electron interaction contribution to the conductance, these interactions do not affect the Kondo contribution. It will turn out that none of these predictions appear to describe the behavior we have observed.

Very recently there has been renewed theoretical interest in size-dependent effects in Kondo systems. So far as we know, the only work along these lines that has been published is due to Bergmann and co-workers. In the first,[20] the effect of system size on the Friedel resonance, i.e., oscillations, was considered, and it was found that the minimum system size which could support this resonance was much smaller than R_K. It was then argued that the Kondo resonance would display similar behavior and thus that the length scale R_K was *not* relevant to the Kondo effect. It is not clear to us how to reconcile this result with the earlier theoretical work on the screening cloud mentioned above. In any case, as we have noted already, our experiments imply that R_K is *not* the length scale observed in the experiments.

In further work, Bergmann and co-workers considered the RKKY interaction in a small system.[21] They argued that the interaction can be viewed as the result of a conduction electron "polarization wave" which originates at one local moment and impinges on the other. In a large system there is only one such wave for each pair of moments (the one that travels directly between the two moments), but in a small system it is possible for additional waves, which reflect off the sample surfaces, to also contribute, thereby enhancing the RKKY interaction relative to what would be found in a bulk system. Bergmann and co-workers did not actually calculate how much this enhanced interaction would affect the experimentally measured properties, but argued nevertheless that it is responsible for the behavior we have observed. However, we do not believe that this explanation is consistent with the experiments, and will give our reasons after the relevant results are described below.

B. Other contributions to the low-temperature resistivity

Most of our analysis will involve the Kondo contribution to the resistivity in quasi-one- and quasi-two-dimensional systems, and it is therefore important to consider other possible contributions to the resistivity (or resistance) in such systems.

The usual phonon contribution ρ_{ph} varies as a power of T at low temperatures, and thus becomes increasingly important as T is increased. One could conceivably try to estimate ρ_{ph}, from measurements on similar systems, and then correct for it (i.e., subtract it out). However, this would involve large uncertainties since at high temperatures $\rho_{\text{ph}}(T)$ varies rapidly as compared with the Kondo contribution. We have therefore chosen to restrict our analyses to temperatures at which ρ_{ph} was negligible compared to $\Delta\rho_K$. This temperature range depends on the material and type of sample. For Cu(Fe) films temperatures as high as 15 K could be used, but with Au(Fe) wires the upper limit was only 3 K.

Two other phenomena make important contributions to the low-temperature electrical properties in these systems: weak localization (WL) and electron-electron interactions (EEI).[22] We can generally neglect the effect of WL in our Au(Fe) and Cu(Fe) samples, since the large amount of spin scattering makes the WL contribution temperature independent. Moreover, in all of our work on quasi-two-dimensional systems, i.e., thin films, the sheet resistance $R_\square$ was fairly low, as compared to the types of samples usually used to study WL and EEI. The contributions of WL and EEI in two dimensions are proportional to $R_\square$, and for this reason these were always negligible compared to the Kondo behavior. Only with our quasi-one-dimensional Au(Fe) samples did we have to worry about such an interfering contribution. Here the EEI effects in our smallest samples were comparable to the Kondo contribution, since the EEI effect scales as A^{-1}, where A is the cross-sectional area, while the Kondo part varied approximately as w, where w is the sample width. Rather than try to subtract off the EEI effect, which would have led to large uncertainties, we simply chose not include those data in our analysis.

The vast majority of our measurements involved the behavior of the Kondo contribution to the resistivity. It would certainly be of great interest to investigate the behavior of thermodynamic properties such as the specific heat or susceptibility in small Kondo systems. Unfortunately such measurements are very difficult because of the limited volume and small number of local moments which are involved. However, one property other than the resistivity which is accessible is the spin scattering time τ_s, the rate at which the spin of a conduction electron is flipped by scattering from the local moments. This can be measured through weak localization magnetoresistance.[22] The measurements are nowadays routine, and the methods we used have been described in detail elsewhere.[23,24]

III. EXPERIMENTAL METHOD

We have studied the Kondo alloys Au(Fe) and Cu(Fe), along with the corresponding host materials Au and Cu. All of the samples were fabricated by using lithographic methods to pattern thin films. We first discuss the deposition of the Au and Au(Fe) films. It turns out that at typical evaporation pressures ($\sim 3 \times 10^{-7}$ Torr in our case) Au and Fe have essentially identical evaporation temperatures, making it possible to produce homogeneous Au(Fe) by coevaporation to completion from a single evaporation source.[25] Since we desired Fe concentrations below 100 ppm (0.01 at. %), it was not convenient to simply weigh or measure out the appropriate amount of Fe. In our first experiments we solved this problem by evaporating a thin layer of Fe onto Au wire. We then evaporated measured amounts of this coated wire to produce the desired Au(Fe) films. Later we learned of the availability of Au-Fe wire with a concentration of 0.07% Fe,[26] and from that time on we used measured amounts of this Au-Fe wire together with the proper amount of pure Au for our depositions. The results did not depend on which evaporation materials were used. In both cases the Au/Fe was thermally evaporated rapidly from a W boat, at typical rates of 20 Å/s. Pure Au films were deposited in a similar manner.

Deposition of the Cu(Fe) films required a different approach. Cu and Fe are not a good match for coevaporation, and so we made these films by sputtering from a composite target made by placing several Fe "dots" on top of a pure Cu (99.999%) sputtering target. The resulting Fe concentration in the films was estimated from the exposed area of Fe together with the relative sputtering rates of Fe and Cu, and from the Kondo behavior of very thick films. The two estimates were always in agreement. The Ar pressure during sputtering was typically 2 mTorr, and so the sputtered flux moved diffusively, causing different samples in a given batch to have the same Fe concentration. This was confirmed by studying the Kondo behavior of test batches which contained a number of samples which differed only in their location on the sample holder in the vacuum chamber. Nevertheless, midway through our work on Cu(Fe) films we modified the holder so that it could be rotated about an axis perpendicular to the direction of the sputtered flux during the deposition, to further ensure homogeneity. Pure Cu films were prepared by either sputtering or thermal evaporation.

In all cases the substrates were glass cover slips. The quasi-two-dimensional samples were patterned photolithographically from the films into strips of width $\sim$ 150 μm and length $\sim$ 60 cm. The quasi-one-dimensional samples, i.e., wires, were produced from the films using substrate step techniques.[27] Further details of sample fabrication will be given below in connection with the particular measurements. The resistance was measured as a function of T using standard techniques.[24]

IV. RESULTS

A. Au(Fe) films

Our initial experiments involved Au(Fe) films with the goal being to determine if varying only the thickness t would have any effect on the Kondo contribution to the resistivity, $\Delta\rho_K$. The samples were produced by thermal evaporation as described in Sec. III. To isolate just the effect of sample size on $\Delta\rho_K$ it is clearly desirable to maintain a fixed Fe concentration. However, we found that with our thermally evaporated Au(Fe) films we could not keep this concentration constant from evaporation to evaporation. That is, we were unable to prevent changes (typically 20%, but sometimes more) in the concentration from batch to batch. For this reason we decided to always try to make comparisons between samples produced from the *same* deposition, since these would all have the *same* concentration. Thus for the study of Au(Fe) films we needed to make a series of samples with different thicknesses t in a single deposition. This was accomplished by placing substrates at different distances from the evaporation source, and also by varying the angle between the substrate and the evaporation direction.[24] The sample thickness was estimated in two ways: from the measured source to substrate distance d, combined with the angular orientation θ of the substrate, assuming that the thickness varied as $(\cos\theta)/d^2$, i.e., that the evaporation source was pointlike and the evaporant motion ballistic; and from Matheissen's rule using the measured resistivities at 300 K and at low temperatures, along with the handbook value for the phonon contribution to the resistivity of Au at 300 K. The two methods agreed reasonably well (typically 10–20 %). We also compared the behavior of samples with the same t, but which were located at different distances and angular orientations from the evaporation source. Such samples always exhibited similar behavior.

Results for the resistivity as a function of temperature for a series of Au(Fe) films were first reported in

Refs. 28, 29, and later confirmed independently by one of us.[24] Those results showed that the Kondo contribution to the resistivity exhibits a *pronounced* dependence on film thickness. The Fe concentration in most of the experiments was $\approx$ 30 ppm; work on bulk Au(Fe) has shown that the interactions have no effect in our temperature range when the concentration is below 100 ppm.[30] We emphasize again that the strong dependence of $\Delta\rho_K$ on film thickness was found by comparing samples prepared in the *same* deposition, so that they had the same Fe concentration. In addition, the contributions of weak localization and electron-electron interaction effects were, as noted above, negligible as these samples had relatively low sheet resistances. In all cases the variation of ρ was consistent with a logarithmic form, as expected from (3) for $T \gg T_K$ [T_K for Au(Fe) is $\sim$ 0.3 K]. While the temperature range was limited, there was no evidence for any size-dependent change in the functional form of $\Delta\rho_K$. However, as the film thickness was reduced, the magnitude of this logarithmic contribution decreased dramatically. The coefficient B of the logarithmic term in (3) varied roughly linearly with film thickness up to thicknesses of approximately 3000 Å. One can define an experimental crossover length ℓ_K (note that we are careful to distinguish this length from the *theoretical* length R_K) as the thickness at which $\Delta\rho_K$ is suppressed to half its bulk value. The results in Refs. 28, 29, 24 give $\ell_K \sim 1500$ Å. This is much smaller than R_K, which for Au(Fe) is predicted to be 6 μm. The diffusive length scale R_K^* in this case is 3000 Å (the elastic mean free path was $\approx$ 300 Å) which is not far from the observed value of ℓ_K. However, we will see below that this coincidence is almost certainly accidental.

It is important at this point to consider a potential experimental difficulty, namely, the possibility that this behavior was due to oxidation of the Fe. The concerns about oxidation may be stated as follows. Oxidation of the Fe could render it nonmagnetic, in which case it would not contribute to the Kondo effect, thereby reducing the effective concentration. Such oxidation might be more of a problem in the thinner samples. For example, there could be a "penetration depth" ξ for oxygen diffusion such that all or most of the Fe within this distance of the surface is oxidized. If so, then the thinnest samples would exhibit the smallest Kondo effect, which is what was observed. We have long been aware of this potential problem (see, for example, the discussion in Ref. 29), and do not believe that such effects were significant for the following reasons.

First, it is well known from weak localization studies of spin scattering in similarly prepared Au(Fe) films[23] that samples as thin as 100 Å can exhibit large amounts of spin scattering. This implies that ξ cannot be much larger than 100 Å, and it is easy to see that such a value would not lead to behavior like that described above. For example, with $\xi = 100$ Å and $t = 500$ Å, the suppression of the Kondo effect due to such oxidation would be of order $\xi/t = 20\%$, which is much smaller than observed in the experiments.[31] Second, we have tried to minimize any oxidation by coating the films with a layer of SiO evaporated immediately after the Au(Fe) deposition, and before breaking vacuum in the chamber. Such samples exhibited behavior identical to those which were left uncoated. Third, we have coated the films with a layer of photoresist immediately after removal from the vacuum chamber, and again these behaved the same as those which were left uncoated. We therefore do not believe that oxidation was a problem in our experiments. However, it is such an important issue that we will return to it again below.

B. Au(Fe) wires

The results discussed above show that as one crosses over from three dimensions to quasi-two-dimensions $\Delta\rho_K$ is reduced significantly. We have also investigated the corresponding quasi-two- to quasi-one-dimensional crossover in studies of narrow Au(Fe) wires.[32] As mentioned in Sec. III, the samples were prepared using substrate step techniques.[27] This involves depositing a Au(Fe) film onto a substrate into which a step had previously been produced by ion milling. After film deposition the sample was ion milled further so as to remove all of the Au(Fe) except that which was in the shadow of the step. The cross section of the wire was controlled by the height of the step. We used Au(Fe) films which were 150 Å thick and steps which were larger by at least a factor of 3 (and generally much more than that); hence the cross section was that of a film essentially "wrapped around" the corner of the step, as described in Refs. 32, 24. The effective sample width was estimated in two ways: from the known step height (through calibration of the ion-milling rate) and from the measured length and resistance of the finished sample.[33] The two methods agreed to within the uncertainties, typically 10%. We only compared samples prepared from the same film deposition so as to ensure that they all had the same Fe concentration.

Results for $\Delta\rho_K$ as a function of T for a series of narrow Au(Fe) wires were presented in Ref. 32. The samples were all 150 Å thick, and had the same Fe concentration; *only* the sample width was varied. It was also possible to study the behavior of a film made in same deposition, which represented the quasi-two-dimensional limit. As the strip width was reduced, we found that the Kondo contribution became smaller. The experimental crossover length was estimated to be $\ell_K \sim 1500$ Å, which is the same as that found for the three-dimensional (3D) to 2D cross over in Au(Fe). These results thus support the picture inferred from the data for films, namely, that reducing the sample size suppresses the Kondo contribution to the resistivity, and that the length scale which controls this suppression in Au(Fe) is $\sim$ 1500 Å. We note that, while our temperature range was limited, the temperature dependence was again consistent with the logarithmic variation (2).

The Kondo contribution $\Delta\rho_K$ in the narrow wires was much reduced from the bulk value, and this made the contribution from phonon scattering (at high temperatures) and electron interaction effects (in the narrowest samples) relatively more important than in the thin film experiments. The phonon contribution was comparable to $\Delta\rho_K$ above about 3 K. At higher temperatures it would have been very difficult to subtract it out with sufficient accuracy. We therefore limited our analysis to temperatures below 3 K, so as to avoid this complication. A similar problem arose (not unexpectedly) when we investigated samples with widths below about 700 Å. As noted above, the contributions from one-dimensional electron-electron interaction (EEI) effects become larger as the sample is made narrower, and these are the samples in which the Kondo effect becomes quite small. The EEI contribution can be estimated from the theory to be much smaller than the Kondo effect in our samples for widths above about 1000 Å. However, we found[32] that for a sample with a width of 360 Å the EEI contribution was larger than the Kondo contribution, which was again expected from the theory of EEI's. One could try to isolate the Kondo contribution for these smaller samples by subtracting off the EEI effect, but given the uncertainties in the latter together with the fact that the Kondo contribution was very small in this case, we did not feel that such results would be trustworthy. For this reason we did not attempt to analyze in any detail the results for samples smaller than about 1000 Å. This approach to the analysis was not followed in other recent work on Au(Fe) wires;[34] we will discuss this point further in Sec. V.

It is useful to now revisit the oxidation issue which was discussed in the previous section, as the results for the narrow wires allow us to rule out oxidation problems in a very unambiguous way. All of the wire samples had the *same* thickness, and this thickness was much less than the sample width. Hence, oxidation of the Fe would occur to essentially the *same* degree in *all* of the wire samples. An explanation of the width dependence we have observed for $\Delta\rho_K$ in the wires in terms of oxidation is thus not tenable.

In addition, these results allow us to rule out another potential problem. While Au and Fe evaporate at essentially the same temperature, one can imagine that the films were still somehow inhomogeneous. This might occur, for example, if the Fe were to agglomerate. Such metallurgical problems could conceivably render some of the Fe nonmagnetic and hence affect the Kondo behavior. However, since our wires all had the *same* thickness and this thickness was much less than the width, such behavior should again be essentially the *same* in all of the samples. It is thus very hard to see how to account for our results for narrow wires, except in terms of a size-dependent Kondo effect.

C. Cu(Fe) films

The experiments with Au(Fe) described above show that the length scale which controls the Kondo behavior, ℓ_K, is ~ 1500 Å for both the quasi-2D and quasi-1D cases. To determine if ℓ_K is associated with either of the theoretical length scales R_K or R_K^* discussed in Sec. II, it is useful to consider how ℓ_K depends on T_K. For this reason we have investigated the Kondo behavior of Cu(Fe) films. For this material $T_K \sim 20$ K which is two orders of magnitude larger than for Au(Fe).[35] Another motivation for studying Cu(Fe) is that its higher T_K makes it much more convenient to study the behavior near and below T_K, as compared with Au(Fe) [all of the results described above for Au(Fe) were limited to temperatures well above T_K]. In addition, the Cu(Fe) films were deposited by sputtering as contrasted with the thermal evaporation used to produce the Au(Fe), thus allowing us to test if some structural or metallurgical aspect of the films played an unexpected role.

The Cu(Fe) films were deposited onto substrates which had different angular orientations with respect to the sputtering source, so that a single deposition produced a series of samples with different thicknesses, but with the *same* Fe concentration. The thickness was estimated as in the Au(Fe) experiments, and checked on occasion with interferometry. Typical concentrations were ~ 300 ppm; measurements with bulk Cu(Fe) alloys have shown that this is sufficiently small that interactions between impurities make a negligible contribution to the resistivity in our temperature range.[30] Other batches showed similar behavior for Fe concentrations as small as 100 ppm, the smallest value we studied.

The results for Cu(Fe) films have been presented elsewhere,[36] and thus will only be described here. The Kondo contribution to the resistivity was found to depend strongly on film thickness. While, as described above, the variation of $\Delta\rho_K$ was logarithmic for Au(Fe) films, the Cu(Fe) results exhibited significant curvature, as $\Delta\rho_K$ flattened off considerably at low T. This difference was due to the different Kondo temperatures for the two materials, and we will return to this issue in the next section. Here we want to emphasize that the thickness dependence of $\Delta\rho_K$ for Cu(Fe) was very similar to that found for Au(Fe), with the Kondo contribution to the resistivity being suppressed as the thickness was reduced. The length scale for this suppression was $\ell_K \sim 1600$ Å, which is quite close to that found for Au(Fe). This implies that ℓ_K does *not* depend strongly on T_K, which we find to be surprising. This very weak dependence of ℓ_K on T_K also suggests that the approximate agreement between ℓ_K and R_K^* for Au(Fe) was accidental, since R_K^* varies as $T_K^{-1/2}$. It thus appears that the physics which determines ℓ_K is not contained in either R_K or R_K^*.

In our discussion of the theory we noted several calculations which have considered how disorder, in the spirit of weak localization and electron-electron interactions, affects the Kondo behavior. The results depended on the specific models, but the predictions were either that there would be no change or that the Kondo contribution to the resistivity would become more singular when disorder is introduced. Neither of these predictions explains our results.

D. Behavior of T_K

The results for both Au(Fe) and Cu(Fe) indicate that the Kondo contribution to the resistivity is suppressed when the size of the system is made smaller than $\ell_K \sim 1500$ Å. It is natural to consider what, if anything, happens to T_K in such systems. In particular, there are (at least) two possibilities that immediately come to mind: (1) T_K might be independent of system size, so that as the size is made smaller, $\Delta\rho_K$ is simply reduced by the same factor at all temperatures, or (2) T_K could be suppressed along with $\Delta\rho_K$ as the size is reduced.

This question has prompted us to examine the behavior in the vicinity of T_K in both Au(Fe) and Cu(Fe) films. Results for two Au(Fe) films are shown in Fig. 1. These were prepared as described in Sec. III, and were made from the same deposition. In Fig. 1(a) we show the raw data for the two samples, and the suppression of $\Delta\rho_K$ is clearly seen. The rollover of $\Delta\rho_K$ at temperatures comparable to $T_K \sim 0.3$ K is also evident. To estimate T_K from these results we could fit them to the theory, but unfortunately there are no exact predictions for this quantity, even for the simplest Kondo models. While we could try to use one of the approximate theoretical forms, we have chosen to take a different approach. Our goal is to determine only if the two samples have the same or different values of T_K, which can be accomplished by comparing their temperature dependences to each other. To this end Fig. 1(b) shows the data from Fig. 1(a) with the results for $\Delta\rho_K$ for the thinner sample scaled up by a constant (temperature independent) factor. The two curves in Fig. 1(b) are seen to be in semiquantitative agreement, implying that they have the *same* temperature dependence and hence the *same* values of T_K. It is not easy to estimate the uncertainties in this kind of analysis since it does not involve any specific functional form. In addition, the experiments are difficult since at these temperatures it is hard to know if the electron temperature is the same as that of the lattice; this is a frequent problem in studies of weak localization, etc., in similar systems. In any case, to within some hard-to-specify uncertainties, we conclude that the results for Au(Fe) imply that T_K is independent of film thickness.

We next return to the results for Cu(Fe). Since T_K in this case is much higher than for Au(Fe), it was much easier to examine the behavior for $T < T_K$ without complications from electron heating effects. To check for any variation of T_K we again scaled the results for each thickness by a constant, temperature-independent factor. The results of such a "scaling analysis" are given in Ref. 36, where it was shown that all of the data collapse nicely onto a common curve, demonstrating again that T_K is independent of thickness.

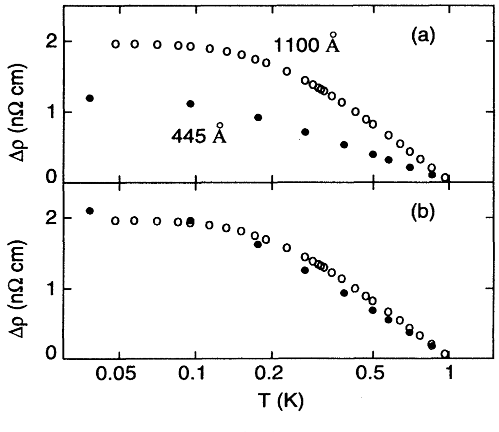

FIG. 1. Results for two Au(Fe) films at low temperatures. (a) Resistivity as a function of T. The thicknesses are indicated, and the Fe concentration was ≈ 50 ppm. (b) Same data except that the results for the 445 Å sample have been scaled up by a constant, temperature-independent factor.

This result for T_K was not totally unexpected. It is consistent with previous studies of weak localization and spin scattering in Au(Fe) and Cu(Cr) films which found that T_K in thin films was similar to that of bulk systems.[37,38]

A thickness-independent T_K together with the suppression of the magnitude of $\Delta\rho_K$ implies that the so-called "unitarity limit"[5] is not reached, in contrast to the usual case in Kondo systems. When a magnetic impurity is placed into a host material, the maximum scattering cross section can be calculated from its scattering phase shifts and the appropriate sum rule. While this does not specify how the effective cross section, and hence the contribution to the resistivity, will vary with T, it does place an upper limit on how much a single impurity can contribute to $\Delta\rho$. In most cases this limit is exhausted at $T = 0$, but the results imply that such is not the case in our films. We know of no reason why this limit must always be attained. We should also note that the theoretical work in Ref. 17 predicts that T_K can change when the degree of disorder is varied, which is not what we observe. However, it is not clear if our samples are sufficiently disordered that this theory should apply.

E. Spin-scattering rate in Au(Fe) films

We have used the weak localization magnetoresistance, Fig. 2, to study the dephasing rate of conduction electrons due to spin scattering in Au(Fe) films as a function of their thickness. Since the spin-orbit scattering in Au is strong, the theoretical predictions involved only two parameters,[22] the sheet resistance $R_\square$ and the phase breaking length L_ϕ. Since $R_\square$ could be measured independently, there was just one free parameter in each fit. The only complication was that the values of $R_\square$

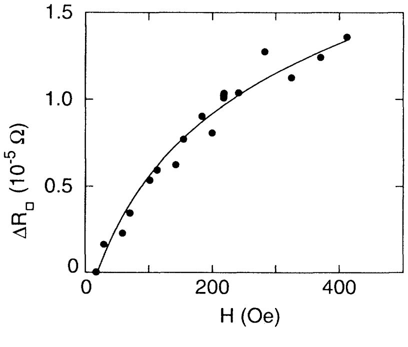

FIG. 2. Magnetoresistance of a Au(Fe) film. Here $R_\square$ is the sheet resistance. For this sample $R_\square = 5$ Ω, and the Fe concentration was 50 ppm. The solid curve is a fit to weak localization theory in the strong spin-orbit scattering limit Note that the zero of the vertical axis was arbitrary.

were rather low, sometimes as small as 0.1 Ω, which is much smaller than usually encountered in weak localization studies. This made the relative magnetoresistance $\Delta R_\square / R_\square$ fairly small, and for this reason slight drifts in temperature, etc., added greatly to the uncertainties (recall that ρ had a significant temperature dependence due to the Kondo effect).

Results for L_ϕ for several values of thickness are shown in Fig. 3, which also shows the corresponding behavior of $\Delta\rho_K$ for these samples. Each point in Fig. 3(b) was obtained from a separate magnetoresistance measurement (as in Fig. 2); the scatter is the result of the slight drifts mentioned above. For all but the thinnest sample, L_ϕ was the same, to within the errors, at 1.4 and 4.2 K. Since other sources of phase breaking are temperature dependent, this implies that spin scattering dominates in these cases, i.e., $L_s \approx L_\phi$. [This is also consistent with previous work on Au(Fe) which has shown that L_s is temperature independent in this range.[37]] For the thinnest sample, L_ϕ was slightly longer at 1.4 K, implying that at 4.2 K both spin scattering and another scattering mechanism, which was temperature dependent, were important. This is in accordance with previous studies of weak localization in similar Au and Au(Fe) films,[23] which found that the electron-phonon phase breaking rate is ~ 8000 Å at 4.2 K and ~ 15000 Å at 1.4 K. These values imply that the phase breaking at 1.4 K in this sample was due completely to spin scattering, and so we can obtain the value of L_s from the result for L_ϕ at 1.4 K.

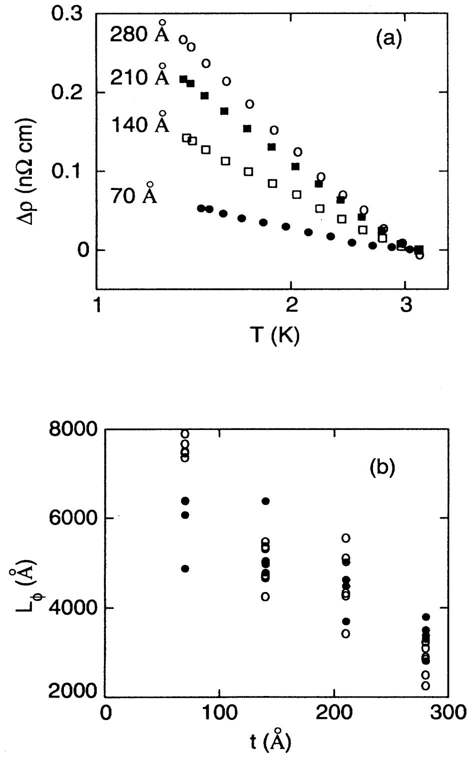

FIG. 3. (a) Results for the resistivity as a function of T for a series of Au(Fe) films. The Fe concentration was 50 ppm, and the thicknesses are given in the figure. (b) Phase breaking length L_ϕ for the samples considered at the top. The values of L_ϕ were obtained from fits to the magnetoresistance like that shown in Fig. 2. The solid symbols were obtained at 4.2 K, while the open symbols correspond to 1.4 K.

The results thus derived for L_s are shown in Fig. 4 which shows L_s and also the spin scattering time τ_s as functions of t. Here we have obtained τ_s from the relation $L_s = \sqrt{D\tau_s}$ where D is the electron diffusion constant, which we estimate to be 30 cm^2/s from previous work.[23] Comparing with Fig. 3, we see that the spin-scattering is suppressed (i.e., τ_s becomes longer) in the thinner samples in a manner very similar to the behavior of $\Delta\rho_K$. The size dependence of the Kondo behavior is thus not limited to the resistivity, but is found also for the spin-scattering rate.

As mentioned above, there have been several previous studies of spin scattering in Kondo systems using weak localization measurements.[37,38] However, those works were concerned with the temperature dependence, and unfortunately did not study the behavior as a function of sample thickness.

F. Bilayer samples: Kondo proximity effect

In addition to experiments involving single films or wires, we have also studied the bilayer sample geometry shown in Fig. 5. We have already seen that for single films the magnitude of $\Delta\rho_K$ was suppressed as the thickness was reduced. With the bilayers we started with films

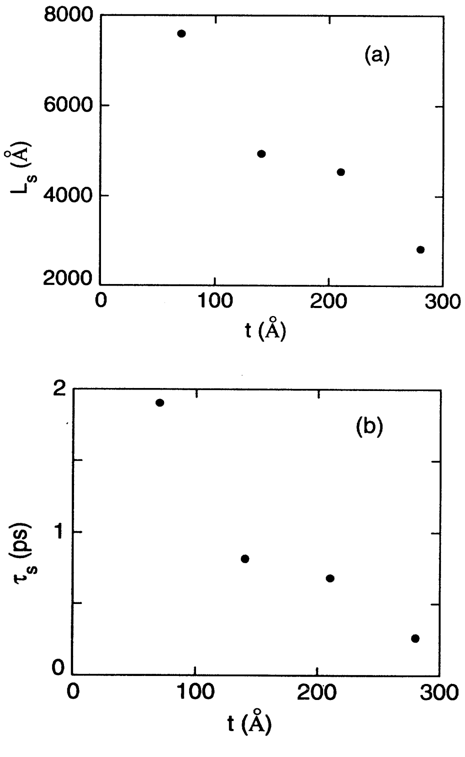

FIG. 4. Results for (a) the spin-scattering length L_s and (b) the spin-scattering time τ_s, obtained from the results in Fig. 3 as explained in the text.

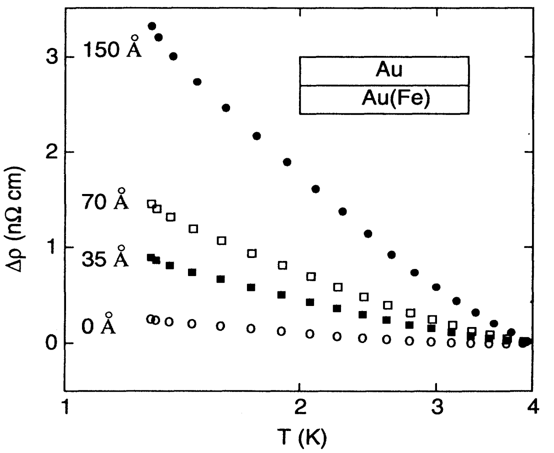

FIG. 5. Results for resistivity of *just* the Au(Fe) layer, for a series of Au(Fe)/Au bilayers. The Au thickness for each sample is given in the figure.

in which the Kondo effect was suppressed, i.e., the bottom (Kondo) layer in Fig. 5, and asked if $\Delta\rho_K$ could be *increased* by increasing the overall thickness through the addition of the *pure* layer. While the pure material should not contribute any Kondo effect of "its own" one might suspect, given our earlier results, that increasing the total thickness could restore some of the (suppressed) $\Delta\rho_K$ of the bottom layer.

We have studied bilayers consisting of Au(Fe) films coated with a layer of Au and Cu(Fe) coated with Cu.[39,36,24] In each sample batch the bottom (i.e., Kondo) layers were all deposited at the same time and were thus identical. The pure layers then were deposited separately, allowing us to study how changes in the properties of the pure layer affected the Kondo effect in the bottom layer. In this section we will be concerned with how the Kondo behavior depends on the thickness of the pure layer.

The films were deposited using the methods described in the previous sections. All of the substrates were oriented perpendicular to the evaporant beam, and all were the same distance from the source, so that all of the Kondo films had the same thickness and Fe concentration. This was confirmed by measurements of the Kondo behavior of several test batches in which no pure layers were deposited. A mask system was then used to expose one sample at a time for the evaporation of Au [for the Au(Fe)/Au samples] or sputtering of Cu [for the Cu(Fe)/Cu samples]. These depositions were all done in the space of a few minutes, without breaking the vacuum. Film thicknesses were monitored with a quartz thickness monitor which had been calibrated with interferometry. To within our uncertainties, the resistivities of the pure layers were always the same as those of the Kondo layers.[24]

Measurements with the bilayer samples yielded the behavior of the *entire* bilayer, i.e., the resistivity of the two layers, the Kondo layer, and the pure layer, in parallel. Results of this kind are shown in Ref. 39 for Au(Fe)/Au bilayers. However, what we are really after is the behavior of the Kondo, e.g., Au(Fe), layer *alone*. The thickness and resistivity of each layer were known (the Kondo contribution is only a relatively small part of the total), and since the resistivity of the Au layer was independent of temperature (in the range we are considering), we could extract the behavior of the Au(Fe) alone. The results for Au(Fe)/Au bilayers are shown in Fig. 5, where we see that there was indeed a large *enhancement* of $\Delta\rho_K$ as the thickness of the Au was increased.

It is useful to define an enhancement factor E by

$$E \equiv \frac{\Delta\rho_b(d_{\mathrm{pure}})}{\Delta\rho_b(d_{\mathrm{pure}}=0)} - 1 \,, \tag{5}$$

where d_{pure} is the thickness of the pure layer, which in Fig. 5 was Au. The case $E = 0$ thus corresponds to no enhancement, but still a nonzero Kondo effect. The behavior of E as a function of the thickness of the Au layer is shown in Fig. 6, where we give results for two batches which had Au(Fe) layers of different thicknesses. In both cases the enhancement effect was a somewhat nonlinear function of the thickness of the pure layer. The samples with the thinner Au(Fe) layers showed the largest enhancements (for a given value of d_{Au}), which is to be expected since they exhibited the largest suppression of $\Delta\rho_K$ in the first place. The enhancements are seen to be as large as a factor of 10 or more in some cases.

We have performed similar experiments with Cu(Fe)/Cu bilayers.[36] The behavior of the enhancement factor for Cu(Fe)/Cu is compared with that found for Au(Fe)/Au in Fig. 6. The results for E are seen to be similar, but not quite quantitatively the same, in the two cases. This implies that the pertinent length scales in the two systems are similar, although not identical.

In the measurements with single films we were able to reach thicknesses at which the Kondo contribution to the resistivity became independent of thickness, and we identified this as the "bulk" limit. It would have been desirable in the bilayer experiments to also reach this regime, which would correspond to a large value for the thickness of the pure layer. However, we were unable to obtain useful results in this limit for the following rea-

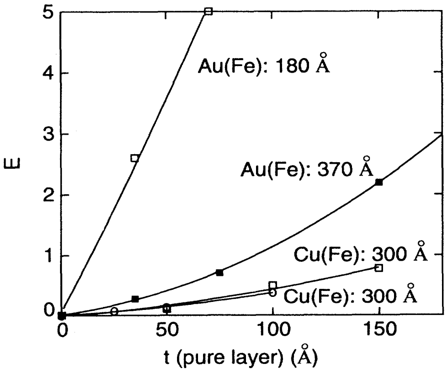

FIG. 6. Enhancement factor E as a function of the thickness of the pure layer for two sets of Au(Fe)/Au bilayers and two sets of Cu(Fe)/Cu bilayers. The thicknesses of the Kondo layers [Au(Fe) or Cu(Fe)] are given in the figure. The solid curves are simply guides to the eye.

son. As the pure layer was made thicker, a larger and larger fraction of the measuring current passed through this layer, and so it dominated the resistance of the entire bilayer more and more. For very thick pure layers this made it extremely difficult to accurately extract the behavior of the parallel Kondo layer, and it was therefore not possible to reach values of the pure layer thickness corresponding to the bulk limit.

In our analysis of the bilayer results we used simple (classical) addition of conductances to extract the behavior of the Kondo layer alone. However, implicit in this was the assumption that the resistivity of the pure layer was independent of temperature. While this was easily verified for the isolated pure films of both Au and Cu,[32,24] it is conceivable that the presence of the Kondo layer could *induce* a Kondo effect in the adjacent pure layer. We do not believe that such behavior can be ruled out on the basis of our data alone. If this was indeed the case, then the discussion given above would clearly have to be modified in ways which are obvious. In either case, however, the experiments clearly show that the Kondo effect in the bilayers is *not* just the sum of the Kondo effects from the constituent layers. There is definitely a Kondo proximity effect, although it is not certain at present whether this is best viewed as an enhancement of the Kondo effect in the Kondo layer, an induced Kondo effect in the pure layer, or perhaps some combination of the two.

The possible role of oxidation in our experiments has been discussed above where we showed that a number of results for single films and thin wires demonstrated that oxidation was not a problem. The bilayer experiments reinforce that conclusion in a particularly strong manner. Recall that in the bilayer experiments, the Kondo layers of all of the samples in a given batch were fabricated at the same time and had the same thickness. They were then covered with pure layers of different thicknesses. If there was any oxidation of the Fe before the pure layers were deposited, it would have been the *same* for all of the samples.[40] Hence, it is hard to see how this mechanism could explain the large differences seen as a function of the thickness of the pure layer. One might argue that the oxidation occurs after the pure layers are deposited, and that more oxygen is able to penetrate through the thinner pure layers, thus reducing the Kondo effect in those cases. We believe that this is not a tenable proposal in light of the evidence against post-deposition oxidation in the pure layers which was described in Secs. IV A and IV C. Nevertheless, we have investigated this possibility with special bilayer experiments in which the pure layers were deposited *first*, i.e., on the bottom, with the Kondo layer deposited onto all of the samples as the last step. We studied such inverted bilayers of both Au/Au(Fe) and Cu/Cu(Fe), and the behavior was the same as that found with the pure layers on top. We do not see how oxidation could possibly account for these results.

Finally, we should mention one more bilayer experiment. We have spoken in terms of a Kondo proximity effect which is transmitted across the boundary between the Kondo and pure layers. It is interesting to consider what role this boundary plays, and in particular if a "dirty" boundary might destroy the proximity effect. To this end we studied bilayer batches of both Au(Fe)/Au and Cu(Fe)/Cu in which the samples were all exposed to a laboratory atmosphere immediately after deposition of the Kondo layer for a period of approximately 5 min. The vacuum chamber was then reevacuated and the pure layers deposited. These batches exhibited behavior similar to that found when the vacuum was maintained between depositions, demonstrating that, at least in this case, a somewhat dirty boundary does not suppress the Kondo proximity effect. It will be interesting to study what aspects of the boundary have an effect on this behavior.

G. Role of disorder

All of the results described above show that both the Kondo contribution to the resistivity and the spin-scattering rate are sensitive functions of the sample size. We also showed that our results could not be explained in terms of the diffusive length scale, R_K^*, discussed in Sec. II. Nevertheless, given what is known about the importance of disorder with regards to transport in small systems,[41,22] it is still worth considering if disorder plays any role in the Kondo behavior. We have conducted two experiments to investigate this question.

The simplest way to investigate the role of disorder would be to make a series of Kondo films with the same thickness and same concentration of local moments, and vary only the amount of disorder. We attempted to do this by making a series of Cu(Fe) films via sputtering as described in Sec. III. The films were sputtered from a Cu target onto which were attached several small Fe "dots" approximately 1 mm^2 in exposed area, and the disorder was varied by varying the pressure of the Ar sputtering gas. In this way we could vary the residual resistivity and, hence, the elastic mean-free-path by more than a factor of 2. As discussed in Sec. III, the Fe dots were found to erode somewhat over the course of several sputterings. Such erosion, if it were large enough, could cause the Fe concentration to vary significantly, and so to test for this we made each sample batch as follows. In a single pump-down of the vacuum chamber we deposited a sequence of samples; each was made with a different Ar pressure, except that the first and last samples were made with the same pressure, and hence had the same amount of disorder. The behavior of these two samples could then be compared to check if the Fe concentration changed significantly during the fabrication, and we found that it did not. Results for a typical batch are shown in Fig. 7, where we see that the Kondo contribution to the resistivity was a strong function of the amount of disorder. One measure of the degree of disorder is the elastic mean free path (the values of which are given in the caption to Fig. 7). Since all of these samples were made so as to have the *same* thickness and *same* Fe concentration, it appears that disorder alone suppresses the magnitude of $\Delta\rho_K$. However, there is a potential problem with this experiment, having to do with the sputtering process used to make the films. The magnetron sputtering gun we employed makes use of a magnetic field to confine and focus the plasma

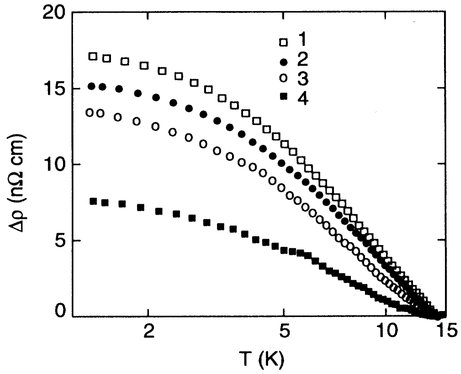

FIG. 7. Results for the resistivity as a function of T for a series of Cu(Fe) films. All were 750 Å thick. The resistivities (at 4 K) and elastic mean free paths were the following: sample (1) 3.5 $\mu\Omega$ cm and 140 Å, sample (2) 4.5 $\mu\Omega$ cm and 110 Å, sample (3) 6.8 $\mu\Omega$ cm and 75 Å, and sample (4) 8.2 $\mu\Omega$ cm and 60 Å.

which sputters material off the target. The intensity and spatial distribution of this plasma are functions of the Ar pressure, as was readily apparent from visual inspection of the plasma. This can affect film deposition in at least two ways. First, the angular distribution of material sputtered from the target will be a function of the plasma configuration, and this will affect the thickness measurements. We compensated for this by calibrating the thickness monitor, using interferometry, as a function of the Ar pressure.[24] Second, since they were magnetic, the Fe dots on the sputtering target altered the plasma distribution in their vicinity, and one might imagine that this alteration could itself change as the Ar pressure was varied, although we should emphasize that we have no evidence that this was the case. This could conceivably make the relative amount of Fe in the sputtered beam vary with Ar pressure, and thereby cause the Fe concentration in the films to vary. Unfortunately we were unable to devise a way to accurately measure the Fe concentration as a function of sputtering pressure, and therefore we cannot rule out the possibility that the results in Fig. 7 may simply be due to variations of the Fe concentration rather than changes in the amount of disorder.

While we believe that the results in Fig. 7 show that disorder is important, further evidence is certainly desirable. We have employed bilayer samples to investigate this question as follows. We first deposited onto several different substrates a layer of Cu(Fe), via sputtering. A separate layer of Cu was then deposited on top of each of these Cu(Fe) films. The Cu layers had the *same* thickness, but *different* levels of disorder. That is, the Kondo layers in the samples were all the same; only the amount of disorder in the pure layers was varied. The Cu layers were deposited by sputtering with different Ar pressures (from a second sputtering gun) and by thermal evaporation; in all cases the depositions were performed within a few minutes of the Cu(Fe) deposition, and without opening the vacuum chamber. This experimental strategy avoided the problem of maintaining the same Fe concentration while varying the disorder; it is much simpler to make a sequence of pure Cu films with different levels of disorder.

The results of this bilayer experiment showed that the enhancement of $\Delta\rho_K$ was largest in the samples with the least disorder.[43] Thus, we again conclude that disorder suppresses $\Delta\rho_K$, as was also observed in the single film experiments.

Constraints of the sample holder used to make these samples allowed us to make only four samples in any one batch. We chose two of these to be single Cu(Fe) layers and two to be Cu(Fe)/Cu bilayers; so we were only able to study two different levels of disorder in any one batch. In order to examine the behavior as a function of the disorder for more than two samples it is necessary to compare the behavior of samples from different batches, and this is done in Fig. 8. Here we plot the enhancement factor E as a function of the low-temperature resistivity of the Cu layer. We see that E is a strong function of the degree of disorder in the Cu layer. All of these samples had 350 Å thick Cu(Fe) layers and 200 Å Cu layers. Unfortunately, we were not able to keep the resistivity of the Cu(Fe) layers fixed; it varied from batch to batch, with values ranging from 2.4 to 8.9 $\mu\Omega$cm, and we believe that this is the source of most of the scatter in Fig. 8.[42] Nevertheless, the variation of E with the resistivity of the Cu layer is clear. There is no doubt that the Kondo contribution to the Cu(Fe) resistivity is a strong function of the disorder of the Cu component of the bilayer.

We have observed this dependence on disorder in a number of different sample batches. The same qualitative result has been found with Cu thicknesses of 50–400 Å, the entire range we have studied. We found similar behavior for several different values of the Fe concentration, for concentrations which varied by about a factor of 3 from batch to batch (the results for all of these samples are included in Fig. 8). Finally, we have observed the same behavior when the Kondo layer was the *top* layer, i.e., when the Cu layers were deposited first (the results for these samples are also included in Fig. 8).

The evidence is thus very strong that both size *and* disorder play a role in the Kondo behavior.

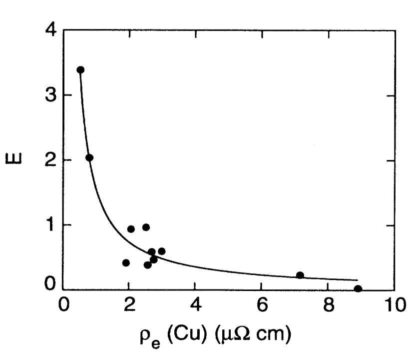

FIG. 8. Enhancement factor E as a function of ρ_e(Cu), the low temperature (residual) resistivity of the Cu layer. For all of these samples the thickness of the Cu(Fe) layer was 350 Å while the thickness of the Cu layer was 200 Å. The smooth curve is a fit to the function $E = A + B\,\rho_e^{-1}$, and is intended only as a guide to the eye.

V. DISCUSSION

A. Possible artifacts

A number of potential artifacts have been mentioned throughout our discussion of the experiments. There is no need to discuss again all the arguments given above, but we would like to make a few general comments.

Problems due to oxidation of the Fe at some time during or after fabrication have been mentioned repeatedly as possible explanations of our results. We have been concerned with this since our first experiments, and so have conducted numerous tests of this hypothesis. These tests have been described in detail above, and they all indicate that oxidation was *not* a problem.

Another potential problem is connected with agglomeration of the Fe. One could imagine that the Fe is, for some reason, not distributed homogeneously. This is perhaps most plausible for Au(Fe) since even though the two constituents evaporate at nearly the same temperature, such a match will never be perfect, and moreover the Fe could diffuse after deposition to form clusters, etc. However, such behavior would be surprising with Cu(Fe), since sputtering is believed to produce homogeneous alloys. Evidence that such effects were not important comes from the results for thin Au(Fe) wires and the bilayers. For the wires all of the samples were made from films of the same thickness, and so any clustering problems should have been the *same* for all. Similar arguments can be applied to the bilayer experiments, and so it seems safe to conclude that clustering or other similar metallurgical problems did not play a significant role in our experiments.

B. Conclusions and outlook

Prior to our work very little was known about the behavior of the Kondo effect in systems of reduced dimensionality. Earlier studies of the Kondo effect in thin films of which we are aware either did not look for variations of the Kondo behavior with thickness or assumed implicitly that there were no such variations. Several years ago, two groups[37,38] studied the variation of the spin-scattering rate with temperature near and below T_K, but it appears that neither examined the thickness dependence of either τ_s or $\Delta\rho_K$.

More recently, two groups have investigated the size dependence of $\Delta\rho_K$ in experiments similar to ours. DiTusa and co-workers[44] have studied the behavior of Cu(Cr) wires, and observed a size dependence similar to that observed in our narrow wires (their work actually predates that of Ref. 32). The length scale found for Cu(Cr) was ~ 1 μm which is somewhat larger than we have found for Au(Fe) and Cu(Fe). The significance of this result is not clear. For Cu(Cr), $T_K \sim 1$ K which is intermediate between the values for Au(Fe) and Cu(Fe), and so this would seem to imply a nonmonotonic variation of ℓ_K with T_K. However, the concentration of local moments in the Cu(Cr) samples was rather high, as one goal of that work was to investigate spin-glass behavior (the spin-glass freezing temperature was ~ 0.8 K), and it would be interesting to repeat those measurements with a lower Cr concentration.

Chandrashekar and co-workers[34] have recently reported experiments with Au(Fe) wires, and find no change of $\Delta\rho_K$ with sample size. This contradicts the results of Ref. 44 and the present work.[32] This discrepancy is particularly troubling with regards to our own previous work, since it and that in Ref. 34 concerned the same Kondo system, Au(Fe), and the same 1D geometry. The source of the discrepancy between these two experiments is not entirely clear to us, but some differences in the two should be noted. First, as noted above, the size dependence of the Kondo contribution is only evident for samples with widths in the range $w \sim 1000-2000$ Å. For larger widths the behavior was independent of w, while for smaller values the suppression of the Kondo contribution was obscured by electron-electron interaction effects. However, for samples with w in this range we observed a sizable suppression of the Kondo contribution, without the need for any "background" corrections; in particular, the contribution of electron-electron effects could be ignored in this range. In Ref. 34, there were, unfortunately, very few samples with widths in this range. In fact, there appear to have been only two such samples, and since they had the same value of w to within a few percent, there was essentially only one data set in the range that our work found to be the important one. Results for smaller values of w were reported in Ref. 34, but in those cases it was necessary to subtract off the electron-electron interaction contribution to the resistance. This contribution was comparable to or larger than the Kondo contribution for w below about 1000 Å, and so a small error in the subtraction procedure could conceivably have masked changes in $\Delta\rho_K$. We emphasize again that in the analysis of our 1D Au(Fe) results (Sec. IV B), such a subtraction was not necessary. It is not clear to us if this is the reason for the discrepancy between our results and those of Ref. 34. Resolution of this discrepancy will evidently require further work.

In summary, we have discussed a series of experiments concerning the Kondo effect in systems of reduced dimensionality. We have studied two different Kondo alloys Au(Fe) and Cu(Fe) in several different geometries, and a coherent picture has emerged. The Kondo contribution to the resistivity is suppressed when either the sample thickness or width is reduced below about 1500 Å. A portion of this suppressed $\Delta\rho_K$ can be restored by depositing a pure layer on top of the Kondo sample. While T_K is not a function of system size, the electron-spin-scattering rate exhibits a suppression similar to that found for $\Delta\rho_K$. Finally, the Kondo effect is also sensitive to the level of disorder; increasing the disorder suppresses $\Delta\rho_K$.

We know of no theory which can explain these results. As discussed above, there was theoretical work a number of years ago on the interplay between disorder, dimensionality, and the Kondo effect, but those calculations do not seem capable of accounting for our results. While our work was initially motivated by the simple screening cloud picture discussed in Secs. I and II, we want to emphasize yet again that our results are at odds with this picture; in particular, we do not see how to use a screening cloud model to explain the fact that ℓ_K is approximately independent of T_K. Indeed, the close agreement between the values of ℓ_K observed for Au(Fe) and Cu(Fe) is quite puzzling, since we would expect it to depend on *some* feature(s) of the material. In view of the somewhat different value of ℓ_K found for Cu(Cr),[44] it seems likely that the close correspondence of the values for Au(Fe) and Cu(Fe) is accidental.

We have, of course, attempted to construct a model to explain our results, but have been unable to produce one that is even qualitatively successful. One model which we find attractive is based on weak localization ideas. It is possible to view the Kondo effect as arising from repeated scattering of an electron from the local moment. We know from weak localization that in systems of reduced dimensionality the probability of repeated scattering from a given site (i.e., "returning to the origin") is enhanced relative to what one would find in three dimensions, and that disorder leads to even more enhancement. This phenomenon is at the heart of weak localization. It is at least conceivable that such repeated scattering of an electron from a local moment might alter (reduce?) the overall scattering rate,[45] thereby reducing both $\Delta\rho_K$ and τ_s^{-1} without affecting T_K. However, the return probability for our samples (which is proportional to the magnitude of the weak localization effects) is very small, typically $\sim 10^{-5}$, and it is hard to see how this could lead to the large reductions of $\Delta\rho_K$ which we have observed. As a final comment, while this paper was being reviewed, we were made aware of the work in Refs. 46 and 47. These authors were concerned with changes in the Kondo behavior produced by the presence of nonmagnetic impurities. It was found that the addition of Pt to the Kondo alloy Cu(Mn) significantly suppressed the Kondo contribution to the resistivity,[47] a result which was explained qualitatively as follows. The Kondo effect can be viewed as being due to multiple scattering of electrons from the magnetic impurity. Anything that alters the intermediate states which are involved could therefore suppress the effect altogether. It was proposed that the presence of the Pt caused an enhancement of the Mn spin-lattice relaxation rate, thereby altering the intermediate states. Such an explanation is, at least in spirit, very similar to our proposal based on weak localization mentioned above. However, it remains to be seen if this idea can be transformed into a quantitative theory. In any case, it is interesting that adding Pt to Cu(Mn), which increased the level of disorder to values similar to those of our samples, suppressed the Kondo behavior.

In conclusion, our work has revealed an unexpected wrinkle in an old and much studied phenomenon. The Kondo problem remains a problem.

ACKNOWLEDGEMENTS

We thank A. A. Abrikosov, S. M. Girvin, G. F. Giuliani, K. Lane, J. Parpia, A. Zawadowski, and especially P. F. Muzikar for many enlightening discussions. The help of G. Chen in the early stages of these experiments was invaluable. We are also grateful to B. C. Crooker and P. M. Shand for the use of their dilution refrigerator, and J. Appel for bringing the work in Refs. 46 and 47 to our attention. This work was supported by the National Science Foundation through Grant No. DMR-9220455.

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