Numerical studies of chiral ordering in three-dimensional XY spin glasses

Hikaru Kawamura
Faculty of Engineering and Design, Kyoto Institute of Technology, Sakyo-ku, Kyoto 606, Japan
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Chiral-glass and spin-glass orderings of the nearest-neighbor XY spin glasses in three dimensions are studied numerically by means of a Monte Carlo simulation, together with a $T=0$ domain-wall renormalization-group method. The results strongly suggest the occurrence of a finite-temperature chiral-glass ordering accompanied with broken reflection symmetry with orientational symmetry preserved. The estimated chiral-glass exponents, $\nu_{CG}=1.5\pm0.3$ and $\eta_{CG}=-0.4\pm0.2$, are close to the exponents of the three-dimensional Ising spin glass, suggesting that the chiral-glass transition in an XY spin glass belongs to the universality class of the standard Ising spin glass. By contrast, the conventional orientational spin-glass ordering seems to occur only at zero temperature, consistent with the previous results. Effects of uniform magnetic fields and random magnetic anisotropies are discussed.

I. INTRODUCTION

Owing to extensive experimental studies, it now seems well established that the spin-glass magnets exhibit an equilibrium phase transition at a finite temperature.[1] From theoretical side, there now seems to be a consensus that the lower critical dimension (LCD) of an Ising ($n=1$) spin glass with short-range interactions is between $d=2$ and 3, while the LCD of vector spin glasses ($n\geq 2$) with short-range interactions is greater than $d=3$.[1] In other words, at $d=3$, only an anisotropic Ising spin glass exhibits an equilibrium spin-glass transition at a finite temperature, whereas isotropic vector spin glasses like $XY(n=2)$ and Heisenberg ($n=3$) spin glasses exhibit only a zero-temperature transition.

It has been known that the magnetic interactions in many of real spin-glass materials are nearly isotropic, being well described by an isotropic Heisenberg model. However, they also have a weak magnetic anisotropy originated from, e.g., the Dzyaloshinski-Moriya interaction or the dipolar (pseudodipolar) interaction.[1] This means that the magnetic anisotropy inherent to real spin glasses, albeit weak in magnitude, is crucially important in inducing the experimentally observed spin-glass transition at a finite temperature.

One should bear in mind, however, that concerning the true nature of the experimentally observed spin-glass transitions there still remains a puzzle not completely understood: Namely, although most of real spin-glass materials are well approximated by an isotropic Heisenberg model, experimentally observed spin-glass transitions appear to be well described by an anisotropic *Ising* model.[1] Furthermore, no detectable sign of Heisenberg-to-Ising crossover has been observed in experiments which is usually expected to occur if the observed Ising-like critical behavior is caused by the weak magnetic anisotropy.

Meanwhile, since the pioneering work by Villain, it has been known that vector spin glasses such as XY and Heisenberg spin glasses possess a twofold Ising-like degeneracy, called "chirality," in addition to a continuous degeneracy associated with the original spin-rotation symmetries.[2] The appearance of such twofold [Z_2] chiral degeneracy is a consequence of the noncollinear or noncoplanar spin structures induced by spin frustration. Chirality physically represents the sense or the handedness of these noncollinear (or noncopolanar) spin structures. Note that, in its ordered (symmetry-broken) state, such noncollinear (noncoplanar) spin orderings break the full symmetry of the Hamiltonian, $O(n)=Z_2\times SO(n)$.

Recently, it has been suggested that such chiral degree of freedom hidden in vector spin glasses may be a key ingredient in solving the above-mentioned puzzle concerning the nature of the experimentally observed spin-glass transitions.[3] It should be stressed here that the standard criterion of a spin-glass transition concerns the appearance of the spin-glass order parameter or the divergence of the spin-glass susceptibility. Since the chirality is a multispin variable of higher-order in the original spin variables as described below, the above-mentioned property that the LCD of vector spin glasses is greater than three does not necessarily exclude the possibility of a finite-temperature *chiral-glass* transition in $d=3$. In fact, this interesting possibility has not fully been addressed in the literature until recently.

Numerical study of the chiral ordering[4] in vector spin glasses was initiated about ten years ago for the case of the two-dimensional XY spin glass, the simplest spin-glass model which can sustain a nontrivial chiral degree of freedom.[5] Thus, via a Monte Carlo simulation of the two-dimensional $\pm J$ XY model, Kawamura and Tanemura reported that the ordering tendency of the chirality appeared to be much enhanced as compared with that of the XY spin. They observed that, although both the spin and chirality order only at zero temperature, the chiral-glass susceptibility of the model behaves essentially like the spin-glass susceptibility of a pure Ising spin glass, with the associated chiral-glass susceptibility exponent, $\gamma_{CG}\gtrsim 4.5$. Analogy to the Ising spin glass was also observed in the dynamics of the chirality by Batrouni and Dagotto,[6] who found by numerical simulation of the two-dimensional XY spin glass that the dynamics of the chiral variable is of the thermal-activation type, in sharp

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contrast to the dynamics of the XY spin. Kawamura and Tanemura further claimed, on the basis of a numerical $T=0$ domain-wall renormalization-group (DWRG) calculation for the same model,[7] that there appeared to exist two distinct diverging length scales in this zero-temperature transition, one associated with the spin and the other associated with the chirality. The respective spin-glass and chiral-glass correlation-length exponents were estimated to be $\nu_{SG}=1.2\pm0.15$ and $\nu_{CG}=2.6\pm0.3$. This observation indicates that the chirality is decoupled from the spin on a long length scale, and that the chiral-glass correlation length is much longer than the spin-glass correlation length. This rather unusual property was also observed by the Monte Carlo calculation by Ray and Moore, who found $\nu_{SG}\simeq1.0$ and $\nu_{CG}\simeq2.0$.[8] Similar behavior was also reported in an $n=3$-component Heisenberg spin glass in two dimensions.[3]

Much more interesting in connection with real spin-glass materials is a possible chiral-glass ordering in the *three-dimensional* systems. For a three-dimensional XY spin glass, on the basis of their DWRG calculation, Kawamura and Tanemura reported the evidence of a finite-temperature chiral-glass ordering without a conventional spin-glass order parameter.[7] The low-temperature phase is then an unusual chiral-glass phase where reflection symmetry is broken with rotation symmetry being preserved. Similar behavior has also been reported in the DWRG calculation for the Heisenberg spin glass in three dimensions.[3]

With these DWRG results for three-dimensional vector spin glasses, it is now clearly desirable to perform Monte Carlo simulations for the same system[9] to study the possible chiral-glass ordering. In the present paper, I report on the results of an intensive Monte Carlo simulation performed for the nearest-neighbor $\pm J$ XY (plane rotator) model on a simple cubic lattice, aimed at obtaining further information about its chiral-glass ordering. Such calculation is expected to supplement the previous $T=0$ DWRG calculation[7] in several points: First, Monte Carlo simulations give direct information about finite-temperature properties of the model, whereas the previous DWRG calculation was limited to zero temperature. Second, in the previous DWRG calculation, chiral ordering was investigated somewhat indirectly by examining the response of the system against the change of the boundary conditions,[7] without an explicit calculation of the chiral variable. In a Monte Carlo simulation, by contrast, one can explicitly calculate the chiral variable and study its ordering process more directly. Third, Monte Carlo simulations enable one to estimate the critical exponents associated with a finite-temperature chiral-glass transition, if any. Then, one can get information about the universality class of the chiral-glass transition.

In addition to these Monte Carlo results, I also wish to present in this paper some results of the DWRG calculation, which supplement the previous calculation by Kawamura and Tanemura.[7] The data are improved over those in Ref. 7 in the following two points: First, in addition to the $\pm J$ model (binary distribution of the nearest-neighbor bonds) previously studied, I have made a similar calculation also for the Gaussian bond distribution in order to check whether the results are sensitive to the particular type of bond distributions or not. Second, I have improved statistics in a configurational average by increasing the number of samples by an order of magnitude.

Overall, the obtained results are consistent with each other, and give fairly strong support to the occurrence of a finite-temperature chiral-glass ordering in a three-dimensional XY spin glass. Furthermore, the estimated critical exponents associated with this chiral-glass transition have turned out to be close to the spin-glass exponents of the standard Ising spin glass, suggesting that the chiral variable in a three-dimensional XY spin glass behaves essentially like an Ising variable, as has been observed in the corresponding two-dimensional systems.[5-8] A preliminary account of the Monte Carlo simulation has already been reported.[10]

This paper is organized as follows. In Sec. II, the model is defined and the chirality is introduced. Details of a Monte Carlo simulation are explained in Sec. III. Monte Carlo results for the three-dimensional $\pm J$ XY model are presented and analyzed in Sec. IV. In particular, the chiral-glass transition temperature and the associated chiral-glass exponents are determined from the finite-size scaling analysis. The results of the $T=0$ DWRG calculation are presented in Sec. V both for the binary ($\pm J$) and Gaussian bond distributions. The results in Sec. V are complementary to those in Ref. 7. On the basis of the obtained numerical results, effects of uniform magnetic fields and random magnetic anisotropies are analyzed in Sec. VI on the basis of a symmetry consideration. Section VII is devoted to summary and discussion. In particular, a possible close connection to d-wave ceramic superconductors is noticed.

II. CHIRALITY

The model considered is the nearest-neighbor random-bond XY (plane-rotator) model on a $d=3$-dimensional simple cubic lattice with two-component, fixed-length spins with orientations θ_i. The bond distribution is either binary ($\pm J$) or Gaussian. The Hamiltonian is

$$\mathcal{H}=-\sum_{\langle ij\rangle}J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j=-\sum_{\langle ij\rangle}J_{ij}\cos(\theta_i-\theta_j)\ , \tag{2.1}$$

where the sum runs over all nearest-neighbor pairs $\langle ij\rangle$. In the case of the binary bond distribution, the J_{ij} are independent random variables taking the values $+J$ and $-J$ with equal probability, while in the case of the Gaussian bond distribution, they obey the Gaussian distribution with zero mean and the variance J.

The local chirality may be defined for *two* neighboring spins at the sites i and j, or on the $\langle ij\rangle$ bond, by the scalar, $\kappa_{ij}=[\mathbf{S}_i\times\mathbf{S}_j]_z=\sin(\theta_i-\theta_j)$. The scalar chirality for the XY spins is *quadratic* in the spin variables, and is distinct from the scalar chirality for the three-component Heisenberg spins: The latter may be defined for *three* neighboring spins by the scalar $\chi_{ijk}=\mathbf{S}_i\times\mathbf{S}_j\cdot\mathbf{S}_k$, being *cubic* in the spin variables.[4,5(b),11]

Often, it is more convenient to define the local chirality

at each plaquette α by,[5]

$$\kappa_\alpha = 2^{-3/2} \sum_{\alpha} \mathrm{sgn}(J_{ij}) \sin(\theta_i - \theta_j) , \tag{2.2}$$

where the sum runs over a directed contour, say of clockwise orientation, along the sides of the plaquette. Note that the chirality defined in this way is local gauge invariant of the group Z_2: Namely, κ_α remains invariant under the local transformations, $\mathbf{S}_i \rightarrow -\mathbf{S}_i$ and $J_{i,i+\delta} \rightarrow -J_{i,i+\delta}$, where δ denote all nearest neighbors of the ith site. Chirality defined by Eq. (2.2) may be regarded as a "continuous" Ising variable taking values around ± 1 for frustrated plaquettes and values around zero for unfrustrated plaquettes. Note that the present definition of chirality differs from the one used in Ref. 8, in which κ_α was defined as a "discrete" Ising variable taking values strictly ± 1 even for nearly collinear spin configurations on unfrustrated plaquettes.

In any case, the chirality is a *pseudoscalar* in the sense that it is invariant under global spin rotation whereas it changes sign under any global spin reflection $\theta_i \rightarrow \theta_0 - \theta_i$ where θ_0 specifies the axis of reflection. Evidently, chiral order can be regarded as a manifestation of the breaking of reflection symmetry.

The chiral-glass order parameter $q_{\mathrm{CG}}^{(2)}$, or the chiral-glass susceptibility χ_{CG} which is expected to diverge at the chiral-glass transition temperature $T = T_{\mathrm{CG}}$, may be defined by[5]

$$q_{\mathrm{CG}}^{(2)} = (N_p)^{-2} \sum_{\alpha,\beta} [\langle \kappa_\alpha \kappa_\beta \rangle_T^2]_J , \tag{2.3}$$

$$\chi_{\mathrm{CG}} = N_p q_{\mathrm{CG}}^{(2)} , \tag{2.4}$$

where $N_p = 3N$ is the total number of plaquettes on a simple cubic lattice with $N = L \times L \times L$ lattice sites, $\langle \cdots \rangle_T$ denotes a thermal average, and $[\cdots]_J$ indicates a configuration average over the bond distribution. The sum over α (or β) is taken over all plaquettes on the lattice.

The spin-glass order parameter $q_{\mathrm{SG}}^{(2)}$, or the spin-glass susceptibility χ_{SG}, may be defined by

$$q_{\mathrm{SG}}^{(2)} = N^{-2} \sum_{i,j} [\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle_T^2]_J , \tag{2.5}$$

$$\chi_{\mathrm{SG}} = N q_{\mathrm{SG}}^{(2)} . \tag{2.6}$$

One can also introduce the corresponding higher-order correlations $q_{\mathrm{CG}}^{(4)}$ or $q_{\mathrm{SG}}^{(4)}$ by

$$q_{\mathrm{CG}}^{(4)} = N_p^{-4} \sum_{\alpha,\beta,\gamma,\delta} [\langle \kappa_\alpha \kappa_\beta \kappa_\gamma \kappa_\delta \rangle_T^2]_J , \tag{2.7}$$

$$q_{\mathrm{SG}}^{(4)} = N^{-4} \sum_{i,j,k,l} [\langle \mathbf{S}_i \cdot \mathbf{S}_j \mathbf{S}_k \cdot \mathbf{S}_l \rangle_T^2]_J . \tag{2.8}$$

Note that these spin-glass and chiral-glass correlations are all invariant under global symmetries of the Hamiltonian, namely, under both rotations and reflections.

It is often useful to look at the dimensionless ratio,[8,12] called the Binder parameter,[13] defined by

$$g_{\mathrm{CG}} = [3 - q_{\mathrm{CG}}^{(4)} / (q_{\mathrm{CG}}^{(2)})^2]/2 , \tag{2.9}$$

$$g_{\mathrm{SG}} = 3 - 2 q_{\mathrm{SG}}^{(4)} / (q_{\mathrm{SG}}^{(2)})^2 , \tag{2.10}$$

where g_{CG} and g_{SG} are normalized so that, in the $L \rightarrow \infty$ limit, they tend to zero above T_c, and tend to unity below T_c provided the ground state is nondegenerate. At the chiral-glass (or spin-glass) transition point, curves of g_{CG} (or g_{SG}) against T for different L should intersect.

In contrast to the standard Ising variable, the magnitude of the local chiral variable κ_α, defined by Eq. (2.2), varies with temperature to some extent. In order to allow for this short-range order effect in χ_{CG} and to make the correspondence with Ising spins closer, I define a reduced chiral-glass susceptibility $\tilde{\chi}_{\mathrm{CG}}$ by dividing χ_{CG} by the appropriate power of the magnitude of the local chirality,[5(b)]

$$\tilde{\chi}_{\mathrm{CG}} = \chi_{\mathrm{CG}} / \langle \kappa^2 \rangle^2 , \tag{2.11}$$

$$\langle \kappa^2 \rangle \equiv N_p^{-1} \sum_{\alpha} [\langle \kappa_\alpha^2 \rangle_T]_J . \tag{2.12}$$

III. MONTE CARLO SIMULATION

Monte Carlo simulations based on a single-spin-flip Metropolis algorithm have been performed. The method is standard, except perhaps that, when a Monte Carlo updating is rejected, a spin reflection with respect to the molecular-field axis is made with certain probability. This energy-preserving reflection procedure tends to flip the chirality and considerably speeds up the chirality relaxation at low temperatures.

In order to test whether the obtained Monte Carlo data are equilibrium ones, I follow Bhatt and Young and computed the correlations like $q_{\mathrm{CG}}^{(2)}$, $q_{\mathrm{SG}}^{(2)}$, $q_{\mathrm{CG}}^{(4)}$, and $q_{\mathrm{SG}}^{(4)}$ in two ways, each of which is known to give an upper or lower bound of a true equilibrium value in finite observation time.[12] For that purpose, it is convenient to rewrite the spin-glass and chiral-glass correlations (2.3), (2.5), (2.7), and (2.8) in terms of an overlap between two independent "replicas" denoted by 1 and 2. Then, the spin-glass correlations may be written in terms of *a tensor variable* $q_{\mu\nu}$ with $2^2 = 4$ independent components,

$$q_{\mathrm{SG}}^{(2)} = \sum_{\mu,\nu} [\langle q_{\mu\nu}^2 \rangle_T]_J , \tag{3.1}$$

$$q_{\mathrm{SG}}^{(4)} = \sum_{\mu,\nu,\delta,\rho} [\langle q_{\mu\nu}^2 q_{\delta\rho}^2 \rangle_T]_J , \tag{3.2}$$

$$q_{\mu\nu} \equiv (1/N) \sum_i S_{i,\mu}^{\{1\}} S_{i,\nu}^{\{2\}} \quad (\mu,\nu = x,y) . \tag{3.3}$$

Note that these spin-glass correlations, Eqs. (3.1) and (3.2), differ from the ones used in Ref. 8 by Ray and Moore. The present definitions may have some advantage in that they are invariant under both global spin rotations and reflections made independently for the two replicas, 1 and 2, while those used in Ref. 8 are not so.

Likewise, for the chiral-glass correlations, one has

$$q_{\mathrm{CG}}^{(2)} = [\langle q_\kappa^2 \rangle_T]_J , \tag{3.4}$$

$$q_{\mathrm{CG}}^{(4)} = [\langle q_\kappa^4 \rangle_T]_J , \tag{3.5}$$

$$q_\kappa \equiv (1/N_p) \sum_{\alpha} \kappa_\alpha^{\{1\}} \kappa_\alpha^{\{2\}} . \tag{3.6}$$

Again, the chiral-glass correlations, Eqs. (3.4) and (3.5),

are invariant both under global rotations and reflections made independently for the two replicas.

Then, in one way of numerically estimating the correlations, the two configurations corresponding to 1 and 2 are taken from a single Monte Carlo run but at two distant Monte Carlo (MC) times, say, at $t=t_0$ and at $t=t_f=2t_0$, where t_0 is a presumed thermalization time. As one increases t_0, these correlations are expected to approach true equilibrium values from above. In actual computation, I take an average of the instantaneous overlaps over an interval of $t_f=[2t_0,2t_0+\Delta t]$ with $\Delta t=2\times10^4$ MCS (Monte Carlo steps per spin).

In the second way of numerically estimating the correlations, the two replica configurations corresponding to 1 and 2 are taken from two independent Monte Carlo runs for the same bond realization at the same MC time. These two runs are evolved independently from different spin initial conditions and with different random-number sequences. Then, the instantaneous overlaps between these two configurations are averaged over a MC time interval $t=[t_0,2t_0]$. With increasing t_0, these overlaps are expected to approach true equilibrium values from below.

The data are accepted only when the results computed in these two ways agree within the errors. If not, the obtained data are rejected, and longer runs with larger t_0 are tried until this criterion is satisfied.

The lattices studied are $L=4,6,8,12$ with periodic boundary conditions. Sample averages are taken over 1000 ($L=4$), 500~900 ($L=6$), 400~800 ($L=8$), 200 ($L=12$) independent bond realizations. The relaxation time associated with the chirality becomes extremely long at low temperatures even for rather small lattices studied in this work. By contrast, it is generally easier to equilibrate spin degrees of freedom. In the present calculation, I could equilibrate both the spin and chiral degrees of freedom down to $T/J=0.2$ ($L=4$), 0.28 ($L=6$), 0.30 ($L=8$), and 0.42 ($L=12$). For the largest lattice, $L=12$, two data points at $T/J=0.42$ and 0.43 are newly added to the data presented in Ref. 10. The longest runs were made for $L=8$ at $T/J=0.30$, where t_0 is taken to be 10^6 MCS. The whole simulation took about 300 h of CPU time on the supercomputer VP2600 at Kyoto University.

IV. MONTE CARLO RESULTS

In this section, the results of Monte Carlo simulations are presented. Figures 1(a) and 1(b) display the size and temperature dependence of the Binder parameters for the chirality and for the spin, g_{CG} and g_{SG}, respectively. One can see from Fig. 1(a) that the data of g_{CG} for $L=4,6,8$ all come together at $T/J\simeq0.32$ suggesting the occurrence of a chiral-glass transition at $T_{CG}/J=0.32\pm0.03$. The value of g_{CG} at $T=T_{CG}$ is estimated to be $g^*_{CG}=0.72\pm0.05$. The data below T_{CG} stick to a common curve and does not splay out, which is reminiscent of the behavior found in the Binder parameter g of the three-dimensional Ising spin glass below T_{SG}.[12] One may interpret such an apparent absence of the splay out below T_{CG} as indicating that the system remains critical all the way below T_{CG}. However, it

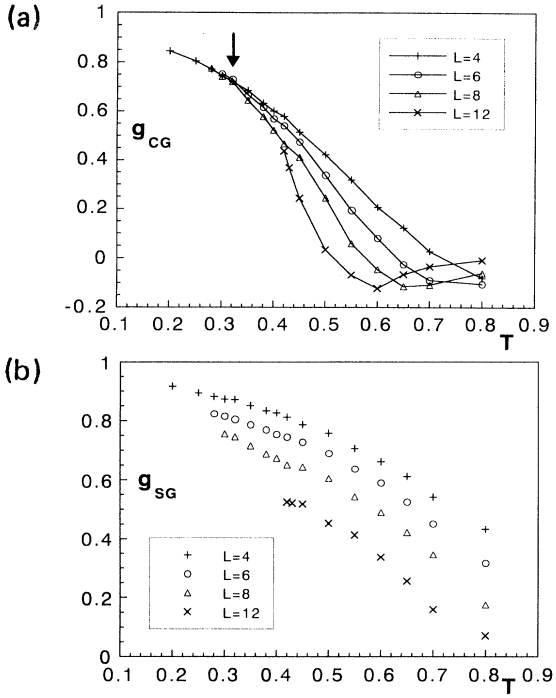

FIG. 1. The temperature and size dependence of the Binder parameters for the spin and for the chirality, g_{CG} (a) and g_{SG} (b), of the three-dimensional $\pm J$ XY spin glass. The arrow in (a) indicates a chiral-glass transition point.

seems also likely to the author that this is related to a very slow power-law decay of the chiral-glass correlations below T_{CG} to its asymptotic long-distance value $q^{(2)}_{CG}>0$.[12(b)]

Somewhat above T_{CG}, g_{CG} takes negative values unlike the g of the three-dimensional Ising spin glass, although it tends to zero at high enough temperatures. This negativity of g_{CG} probably reflects the character of the local chirality: Namely, even in the ordered configuration, a significant portion of the local chirality takes values around $\kappa_\alpha=0$ on *unfrustrated* plaquettes, in addition to the values around $\kappa_\alpha=\pm1$ on frustrated plaquettes. Due to the existence of such $\kappa_\alpha\simeq0$ component, the probability distribution of the chiral variables may well differ from that of the standard discrete Ising variables, which may account for the observed negativity of g_{CG}. I note that a negative Binder parameter g has also been observed in the three-dimensional $\pm J$ Ising spin glass with asymmetric bond distributions by Shirakura, Matsubara, and Inawashiro.[14]

In sharp contrast to g_{CG}, the Binder parameter for the spin, g_{SG}, decreases monotonically for increasing L at all temperatures studied, as can be seen from Fig. 1(b). One may safely conclude that a finite-temperature spin-glass transition, if any, must occur at a temperature significantly lower than $0.2J$. The data certainly favor the occurrence of a conventional spin-glass order only at zero temperature, consistent with the previous results.[7,9,15,16] To the author's knowledge, this is the first

calculation of the Binder parameter for three-dimensional *vector* spin glasses. As an indicator of the transition, the dimensionless Binder parameter has an advantage over some other methods such as examining the linearity of the log-log plot of the susceptibility versus temperature. Thus, the present data have given further support to common belief concerning the absence of a finite-temperature spin-glass ordering in three-dimensional vector spin glasses.[3,7,9,11,15–20] Anyway, qualitative difference observed between g_{CG} and g_{SG}, both estimated from exactly the same Monte Carlo runs, is quite striking.

Next, let us proceed to the determination of chiral-glass critical exponents, assuming the occurrence of a chiral-glass transition at $T_{CG}/J=0.32$. With use of the standard finite-size scaling relation for g_{CG} of the form

$$g_{CG}=\bar{g}_{CG}(L^{1/\nu_{CG}}|T-T_{CG}|)\ , \tag{4.1}$$

and setting $T_{CG}/J=0.32$, the chiral-glass correlation-length exponent is estimated from a one-parameter scaling fit as $\nu_{CG}=1.5\pm0.3$. The corresponding finite-size scaling plot is displayed in Fig. 2.

The temperature and size dependence of the reduced chiral-glass susceptibility $\tilde{\chi}_{CG}$, defined by Eq. (2.11), is shown in Fig. 3. Standard finite-size scaling analysis is also applied based on the relation

$$\tilde{\chi}_{CG}=L^{2-\eta_{CG}}\bar{\tilde{\chi}}_{CG}(L^{1/\nu_{CG}}|T-T_{CG}|)\ . \tag{4.2}$$

With setting $T_{CG}/J=0.32$ and $\nu_{CG}=1.5$, the chiral-glass critical-point decay exponent is estimated from a one-parameter scaling fit as $\eta_{CG}=-0.4\pm0.2$. The corresponding finite-size scaling plot is displayed in Fig. 4.

If one uses here standard scaling relations for exponents, the chiral-glass susceptibility and the chiral-glass order-parameter exponents are estimated to be $\gamma_{CG}\simeq3.6$ and $\beta_{CG}\simeq0.45$.

In contrast to the spin-glass susceptibility χ_{SG} which is found to be a nondecreasing function of L at all temperatures studied, χ_{CG} behaves in this way only at $T/J\lesssim0.5$, but tends to rather decrease for large L at $T/J\gtrsim0.55$.

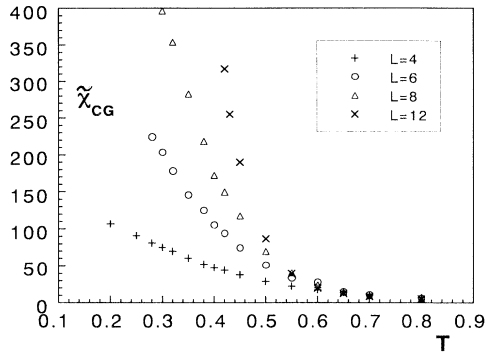

FIG. 3. The temperature and size dependence of the reduced chiral-glass susceptibility $\tilde{\chi}_{CG}$, defined by Eq. (2.11) in the text, of the three-dimensional $\pm J$ XY spin glass.

Since the susceptibility in the critical regime should be an increasing function of L, this observation suggests that the critical region associated with a chiral-glass transition might be rather narrow. Similar narrowness of the critical region associated with a chiral-glass transition was also reported in two dimensions by Ray and Moore,[8] although the chiral-glass transition in two dimensions takes place at zero temperature.[5–8]

Remarkably, the obtained chiral-glass exponents are close to the spin-glass exponents of a three-dimensional Ising spin glass: Indeed, for the $\pm J$ Ising spin glass, Monte Carlo simulation gave $\nu=1.3\pm0.3$ and $\eta=-0.3\pm0.24$,[12] or $\nu=1.3\pm0.1$ and $\eta=-0.22\pm0.05$,[21(b)] high-temperature series expansion gave $\nu=1.3\pm0.2$ and $\eta=-0.25\pm0.17$,[22(b)] while for the Gaussian distribution, Monte Carlo simulation gave $\nu=1.6\pm0.4$ and $\eta=-0.4\pm0.2$.[12(b)] Thus, our present result is entirely consistent with the claim that the chiral-glass transition of an XY spin glass lies in the same universality class as that of an Ising spin glass. This coincidence is further supported by the observation that the value of the Binder parameter at the transition point

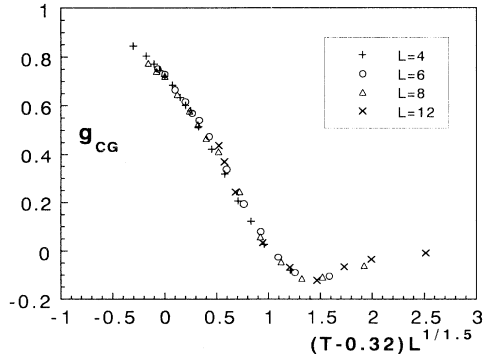

FIG. 2. Finite-size scaling plot of the Binder parameter for the chirality, g_{CG}, of the three-dimensional $\pm J$ XY spin glass.

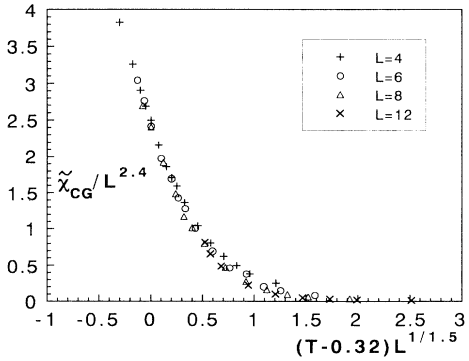

FIG. 4. Finite-size scaling plot of the reduced chiral-glass susceptibility $\tilde{\chi}_{CG}$, defined by Eq. (2.11) in the text, of the three-dimensional $\pm J$ XY spin glass.

$g^{*}_{CG} \simeq 0.72$, which is expected to be a universal quantity, is nearly the same as the corresponding value for the three-dimensional Ising spin glass.[12,23] Of course, in view of the uncertainties associated with the numerical estimates of exponents, one cannot draw a truly definite conclusion from numerical simulations.

Such correspondence is not totally unexpected, since the chirality is essentially an Ising-like variable, at least from a symmetry viewpoint. However, one should also remember that the chirality is a multispin variable, not independent of the XY spins at the microscopic level. So, generally speaking, symmetry alone is not sufficient to conclude that the chirality behaves like an Ising variable independent of the XY spins. Indeed, in regularly frustrated XY spin systems in three dimensions, chirality behaves as a composite operator parasitic to the XY spins, not as an independent Ising variable.[24,25] A remarkable point in the present result is that, on longer length scales, or upon enough renormalization, chirality appears to be decoupled from the XY spins, or more precisely, its proper-rotation part. In other words, the two orthogonal parts of the order-parameter space, $Z_2 \times SO(2)$, appear to be decoupled on longer length scales into the Z_2 part, corresponding to reflection, and the $SO(2)$ part, corresponding to proper rotation. In fact, it has been shown that such decoupling really occurs in frustrated XY spin systems *in one dimension* both in regular[26] and in spin-glass[27] systems, although both the spin-glass and the chiral-glass orderings occur only at $T=0$ in one dimension. Since the randomness often causes an effective reduction of the space dimensionality of the system, the occurrence of such decoupling of spin and chirality is not so unlikely even in three dimensions *in random systems.*

The temperature and size dependence of the spin-glass susceptibility χ_{SG} is shown in Fig. 5. One can also apply the standard finite-size scaling analysis to χ_{SG} with assuming $T_{SG}=0$. A reasonable fit has been obtained by choosing $\nu_{SG} \sim 2.0$ and $\eta_{SG} \sim -0.8$, as shown in Fig. 6. These exponents are close to the values determined by Jain and Young from their Monte Carlo data taken for $L=16$ lattices but at much higher temperatures

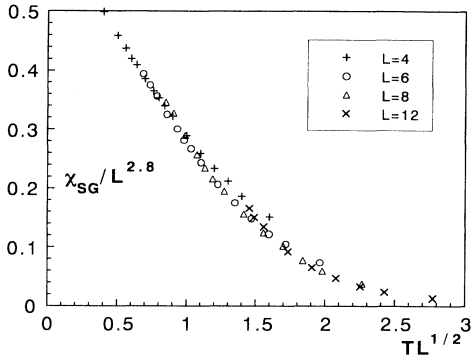

FIG. 6. Finite-size scaling plot of the spin-glass susceptibility χ_{SG} of the three-dimensional $\pm J$ XY spin glass.

$T/J \geq 0.65$.[9] However, a one-parameter scaling plot for g_{SG}, with $T_{SG}=0$ and $\nu_{SG}=2.0$, yields a rather poor fit as shown in Fig. 7. In fact, the data for g_{SG} do not scale well for any values of the assumed T_{SG} and ν_{SG}, in sharp contrast to the case of its chiral counterpart g_{CG}. The reason for this is probably the following: If the chiral-glass order really takes place at $T_{CG}/J \simeq 0.32$, it will have some effects on the XY spin degree of freedom, although the spin-glass susceptibility itself does not diverge at $T=T_{CG}$. (Here, recall that the XY spins themselves are transformed nontrivially under reflections.) Then, it is likely that the asymptotic critical behavior associated with the $T=0$ spin-glass transition is realized only at $T \ll T_{CG}$. Around T_{CG}, in addition to the spin-glass correlation length ξ_{SG}, there exists another hidden diverging length scale, the chiral-glass correlation length ξ_{CG}. The existence of such second length scale may well deteriorate the quality of the one-parameter scaling form written solely in terms of ξ_{SG}. In order to examine the asymptotic critical behavior associated with the $T=0$ spin-glass transition, one has to go down to much lower temperature $T \ll T_{CG}$ where equilibrium data could not

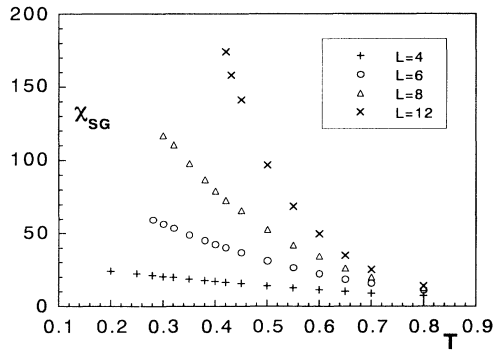

FIG. 5. The temperature and size dependence of the spin-glass susceptibility χ_{SG} of the three-dimensional $\pm J$ XY spin glass.

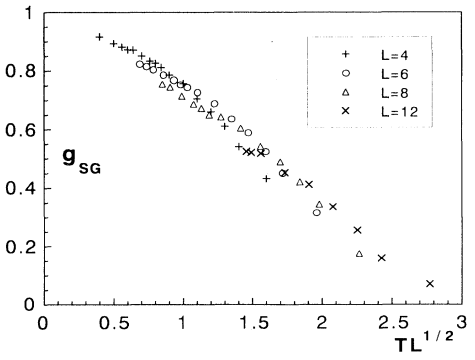

FIG. 7. Finite-size scaling plot of the Binder parameter for the spin, g_{SG}, of the three-dimensional $\pm J$ XY spin glass.

be obtained in the present simulation. Therefore, a good fit obtained for χ_{SG} with the exponents consistent with those of Ref. 9 may in fact be fortuitous. Remember, one could often manage to get an apparently good fit if one is allowed to use two or more free fitting parameters. Although it seems quite likely from the data of g_{SG} in Fig. 1(b) that the conventional spin-glass ordering occurs only at zero temperature, further study utilizing the data at $T < T_{CG}$ is required in order to reliably estimate the exponents associated with this zero-temperature spin-glass transition.

Before concluding this section, I wish to give a comment on the relation to a possible phase transition of a gauge glass of random type-II superconductors in magnetic fields. For the gauge glass, recent Monte Carlo simulations[28,29] as well as numerical DWRG calculations[30,31] suggest the existence of a finite-temperature transition in three dimensions, while there is no finite-temperature ordering in two dimensions.[32] Indeed, for the gauge-glass model in three dimensions, Huse and Seung pointed out its close resemblance to the Ising spin glass,[28] while Reger *et al.* gave an estimate $\nu = 1.4 \pm 0.4$,[29] which is close both to the Ising spin-glass exponent and to the chiral-glass exponent determined in this work.

However, one should realize that there is a fundamental difference between the chiral-glass and the gauge-glass problems. In the case of the chiral-glass ordering in an XY spin glass, the original Hamiltonian possesses both rotation and reflection symmetries. Chiral-glass ordering concerns the spontaneous breaking of reflection symmetry with preserving rotation symmetry. By contrast, in the case of the gauge glass, the original Hamiltonian possesses rotation symmetry only. Gauge-glass ordering concerns the spontaneous breaking of rotation symmetry, while reflection symmetry has already been broken energetically at the Hamiltonian level. In fact, by deriving the Landau-Ginzburg-Wilson Hamiltonians and examining the number of critical modes, Gingras argued that the Ising spin glass and the gauge glass should belong to mutually different universality classes.[33]

V. DOMAIN-WALL RENORMALIZATION-GROUP CALCULATION

In this section, I report on the results of the numerical DWRG calculation at zero temperature on the chiral ordering of the three-dimensional XY spin glass both with the binary ($\pm J$) and Gaussian bond distributions. This section should be regarded as an addendum to Ref. 7. New results presented here are (i) new data taken for the Gaussian bond distribution, and (ii) improved statistics achieved by taking a sample average over an order of magnitude more bond realizations.

The DWRG method has successfully been used by various authors in attacking the vector spin-glass problems.[15,17,18,34] Here, we follow Ref. 7 and adapt it to the form appropriate for the study of the chiral ordering. In the standard DWRG method, the domain-wall energy for a given sample of linear dimension L is defined as a difference between the two ground-state energies for periodic and antiperiodic boundary conditions. (Note that by the "antiperiodic boundary condition" quoted here and below we mean antiperiodic boundary condition in one direction and periodic boundary conditions in the remaining two directions, while by the "periodic boundary condition" we mean periodic boundary conditions in all three directions.) Usually, the variance of the distribution of this energy difference over samples is taken as a measure of the averaged domain-wall energy $W_s(L)$:

$$W_s(L) = [\epsilon_s^2]_J^{1/2} \tag{5.1}$$

with

$$\epsilon_s \equiv E_P - E_{AP} , \tag{5.2}$$

where E_P and E_{AP} are the total ground-state energies for periodic and antiperiodic boundary conditions, respectively. When $W_s(L)$ behaves as $W_s(L) \propto L^{-y_s}$ for $L \gg 1$, either positive or negative y_s is associated with a zero-temperature or a finite-temperature *spin-glass* transition, respectively.

A crucial observation made in Ref. 7 is that, in case of an XY spin glass, the application of antiperiodic boundary conditions does *not* cause a flipping of chirality. This means that the antiperiodic boundary condition introduces only a spin domain wall into the sample which accompanies a proper rotation of XY spins, but not a chiral domain wall which accompanies the flipping of chirality. Note that, by the "spin domain wall" above, we mean all possible low-energy excitations which do not accompany the flipping of chirality. In particular, in addition to spin-wave-type locally small deformations, it may contain vortex-type, or possibly, droplet-type excitations which involve not necessarily small spin deformations. In order to detect the chiral domain wall, a new "reflection boundary condition" was introduced in Ref. 7, in which boundary spins were *reflected* with respect to a fixed axis in the spin space. Evidently, reflection boundary conditions accompany the flipping of chirality and introduce the chiral domain wall into the sample. Then the difference between the ground-state energies for the reflection boundary condition E_R and that for the usual periodic boundary condition $\epsilon' = E_P - E_R$, should contain the contribution of the chiral domain wall in addition to that of the usual spin domain wall. In actual computations, in order to suppress the contribution of the spin domain wall, a slightly different definition has been employed as a measure of the chiral-domain-wall energy $W_c(L)$:

$$W_c(L) = [(\epsilon_c - [\epsilon_c]_J)^2]_J^{1/2} \tag{5.3}$$

with

$$\epsilon_c \equiv \min(E_P, E_{AP}) - E_R . \tag{5.4}$$

The reason for this choice has been explained in Ref. 7. When $W_c(L)$ behaves as $W_c(L) \propto L^{-y_c}$ for $L \gg 1$, either positive or negative y_c is associated with a zero-temperature or a finite-temperature *chiral-glass* transition, respectively.[35]

The lattice sizes studied are $L = 3, 4, 5, 6$ for both cases of the binary and Gaussian bond distributions. Sample

averages are taken over 50 000 ($L=3$) 10 000 ($L=4$), 2000 ($L=5$), and 1000 ($L=6$) independent bond realizations. Note that the number of samples for $L=6$, the largest size studied, is increased by an order of magnitude as compared with that in Ref. 7, while that for $L=5$ is doubled. The ground-state energy is estimated by repeating a spin-quench algorithm many times,[3,7,15,17,18,34] where randomly chosen spin initial conditions are used in each trial. In fact, I have made 5 ($L=3$), 50 ($L=4$), 500 ($L=5$), and 5000 ($L=6$) trials for each sample. Sufficiency of the number of trials has been checked by performing longer runs for a subset of samples in which at least a few times more trials are made.

The L dependence of the calculated W_s and W_c is displayed in Fig. 8 for both cases of the binary ($\pm J$) and Gaussian bond distributions. The attached error bars represent one standard deviation associated with the sample average. From the figure, one finds that $W_s(L)$ iterates toward weak coupling for both types of bond distributions, which indicates that the conventional spin-glass transition occurs only at zero temperature.

By contrast, we have found a markedly different behavior for $W_c(L)$. Indeed, $W_c(L)$ iterates towards *strong* coupling at $L \geq 4$ for both distributions, although it initially iterates towards weak coupling for very small lattices in case of the $\pm J$ distribution. A small even-odd effect discernible in the $\pm J$ distribution may be due to the fact that the concentration of antiferromagnetic bonds for odd-L samples is not strictly equal to 50%, but rather to 48.1% ($L=3$) and 49.6% ($L=5$), unlike the cases of even-L samples. Initial decrease of $W_c(L)$ observed for the $\pm J$ distribution may be due to the residual contribution of the spin domain wall, which should be increasingly negligible for larger lattices. Hence, the data for $W_c(L)$ strongly suggest the occurrence of a finite-temperature chiral-glass ordering. It corroborates the Monte Carlo results in the previous section, and gives fairly convincing evidence for the occurrence of a finite-temperature chiral-glass ordering in a three-dimensional XY spin glass.

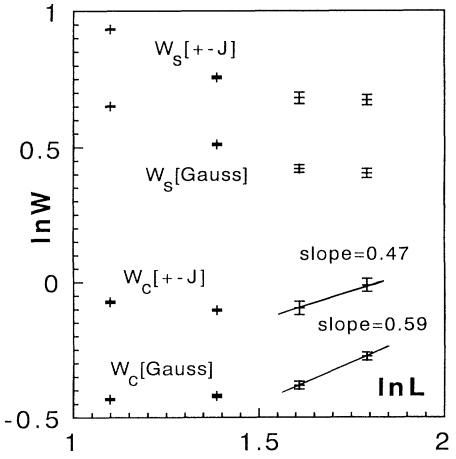

FIG. 8. The L dependence of the spin-domain-wall energy $W_s(L)$, defined by Eqs. (5.1) and (5.2) in the text, and of the chiral-domain-wall energy $W_c(L)$, defined by Eqs. (5.3) and (5.4) in the text, on a log-log plot for the three-dimensional XY spin glass on a $L\times L\times L$ simple cubic lattice for both cases of the binary ($\pm J$) and Gaussian bond distributions. The error bar on each point represents one standard deviation associated with a sample average.

Before concluding this section, I wish to add a comment about the behavior of the spin-domain-wall energy, $W_s(L)$. Often, one extracts a stiffness exponent from the slope of the straight-line fit to the data as given in Fig. 8. Then, a plausible scaling argument is used to identify the inverse of the stiffness exponent with the spin-glass correlation-length exponent ν_{SG} at the $T=0$ transition.[36] The present data of $W_s(L)$, however, have a systematic tendency to level off as one goes to larger L, which does not enable one to estimate ν_{SG} unambiguously. Taken literally, a larger value of ν_{SG} results if the data for larger L are used in the fit.

In this connection, it should be mentioned that the conventional practice of identifying the inverse of the stiffness exponent with ν_{SG} was recently challenged by Kawashima, Hatano, and Suzuki.[37] Studying the two-dimensional Ising spin glass with the Gaussian bond distribution, they found a large discrepancy between the ν_{SG} value estimated from the stiffness exponent via the standard DWRG calculations and the values estimated from other more direct methods, the former being much larger than the latter. Kawashima, Hatano, and Suzuki then claimed that the conventional practice of relating the stiffness exponent to the correlation-length exponent might be wrong. Here, I will not enter into this problem any further, but only mention that the spin-domain-wall energy calculated in this section certainly shows a tendency to level off, but it does not turn over and goes to weak coupling in the range of the sizes studied.

VI. EFFECTS OF UNIFORM MAGNETIC FIELDS AND RANDOM MAGNETIC ANISOTROPIES

In view of the numerical results in the previous sections suggesting the occurrence of a finite-temperature chiral-glass ordering, I wish to discuss in the present section the effects of uniform magnetic fields and of random magnetic anisotropies. These perturbations work to reduce the full $O(2)$ symmetry of the system, and it is interesting to study their effects on the phase diagram and on the nature of the chiral- and spin-glass orderings. The results in this section are all derived based on naive symmetry arguments without performing further numerical calculations, on the assumption that a fully isotropic XY spin glass exhibits a finite-temperature chiral-glass transition with broken reflection symmetry but with orientational symmetry preserved.

As mentioned, magnetic anisotropies are believed to play very important roles in the spin-glass ordering of real materials.[1] Here we shall consider two representative types of random exchange anisotropies: One is the dipolar (or pseudopolar) anisotropy, and the other is the Dzyaloshinski-Moriya anisotropy.[1,38] The random *ex-*

change anisotropy may be written in the generalized form as

$$\sum_{i,j}\sum_{\mu,\nu} D_{ij}^{\mu\nu} S_{i,\mu} S_{j,\nu} , \tag{6.1}$$

where the anisotropy tensor $D_{ij}^{\mu\nu}$ is assumed to be a random variable with zero mean and with the variance D. In the case of the dipolar-type anisotropy, $D_{ij}^{\mu\nu}$ is *symmetric,*

$$D_{ij}^{\mu\nu} = D_{ij}^{\nu\mu} , \tag{6.2}$$

whereas in the case of the Dzyaloshinski-Moriya-type anisotropy, $D_{ij}^{\mu\nu}$ is *antisymmetric*

$$D_{ij}^{\mu\nu} = -D_{ij}^{\nu\mu} . \tag{6.3}$$

In case of the two-component XY spins, these two types of anisotropies have very different symmetry properties as described below.[33]

In what follows, I shall discuss the cases of (a) uniform magnetic field, (b) random dipolar-type anisotropy, and (c) random Dzyaloshinski-Moriya-type anisotropy, separately.

A. Uniform magnetic field

Let us begin with the case of uniform magnetic fields. Applied uniform fields reduce the symmetry of the Hamiltonian from the original $O(2)=Z_2\times SO(2)$ to Z_2 associated with global spin reflection with respect to the magnetic-field axis. Note that the remaining Z_2 symmetry gives rise to the chiral degeneracy as long as the applied field is not so strong and the spin ordering remains noncollinear. Thus, even under magnetic fields, the system still remains *chiral* and can exhibit a chiral-glass transition. In particular, one can still take the chirality, defined by Eq. (2.2), as an order parameter of this chiral-glass transition.

Note that the symmetry change induced by applied fields occurs only in the spin-rotation part, $SO(2)\rightarrow 1$. By contrast, the Z_2 chiral degeneracy is kept intact by applied fields. [The only modification is that, due to the loss of $SO(2)$ rotation symmetry, the invariant axis of reflection is now limited to the direction of the applied field while it can take an arbitrary direction in zero field.] Since the $SO(2)$ spin-rotation part has already been decoupled from the chirality part on a longer length scale, the chiral-glass transition in applied fields should be governed by the same fixed point as in the zero-field case.

As such, effects of applied fields on the chiral-glass ordering should be relatively minor: (i) For weak fields, spin orderings remain noncollinear, but spins now tend to point more to the direction of the magnetic field, which should reduce the magnitude of the local chirality (2.2). (ii) In applied fields, the longitudinal component of the XY spin takes a finite value at any finite temperature. Under such circumstances, the transverse component of the XY spin behaves essentially in the same way as the chirality, and can also be regarded as an order parameter of the chiral-glass transition. In other words, the chiral-glass order in applied fields accompanies a conventional spin-glass order.

From the discussion above, the chiral-glass (or spin-glass) transition temperature under a magnetic field of strength H, $T_{\rm CG}(H)$, is expected to be *an analytic function of* H. Remember, there is no changeover of the fixed points for the chiral-glass transition in a field. Thus, from (i), one may expect,

$$T_{\rm CG}(H)\approx T_{\rm CG}(0)-cH^2 , \tag{6.4}$$

for weak fields. The expected phase diagram is sketched in Fig. 9.

It might be interesting to point out here that the qualitative behavior of the transition line for weak fields, Eq. (6.4), happens to be the same as that of the so-called Gabay-Toulouse (GT) line of an infinite-range vector spin-glass model (SK model).[39] Its origin, however, is entirely different. In the spin-glass transition of the SK model, the symmetries broken at a zero-field transition and at a finite-field transition differ. This means that the quadratic H dependence of the GT line should not simply be regarded as analytic. By contrast, in the present chiral-glass transition of the short-range model, broken symmetries remain the same both in zero and nonzero fields, and the quadratic H dependence should be regarded as genuinely analytic. Furthermore, in spite of its success in explaining experimental results, the GT line of the SK model cannot directly be applicable to real three-dimensional systems, since a fully isotropic vector spin glass in $d=3$ is believed to exhibit no finite-temperature transition.[3,7,9,11,15–20]

By contrast, the present chiral-glass transition line, though having an appearance of the GT line, persists as a true transition line even in three dimensions, as long as the effects of magnetic anisotropies are negligible. Thus, it is very interesting to relate the experimentally observed

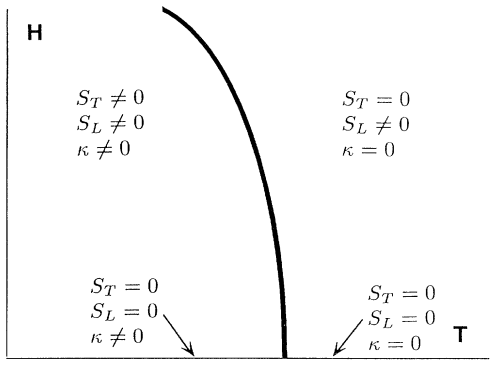

FIG. 9. Schematic phase diagram of a short-range XY spin glass in three dimensions in the temperature-magnetic field (T-H) plane. S_T, S_L, and κ represent the transverse component of the spin, the longitudinal component of the spin and the chirality, respectively. Each inequality and/or equality in the figure refers to the existence and/or nonexistence of the associated glass-type long-range correlations.

GT line to the present chiral-glass transition line, rather than to the GT line of the SK model.

B. Random dipolar-type anisotropy

Next, I wish to discuss the case of the random dipolar-type symmetric anisotropy. In the presence of the random dipolar interaction, the system no longer has an invariant reflection axis, and thus, has no chiral degeneracy. The local chirality takes a finite value induced by the anisotropy at any finite temperature, and there cannot be a chiral-glass transition in contrast to the case of uniform magnetic fields. A continuous $SO(2)$ symmetry associated with proper spin rotation is reduced to its discrete subset Z_2 associated with global spin *inversion*, $\mathbf{S}_i \rightarrow -\mathbf{S}_i$. Note that this Z_2 should not be confused with the Z_2 associated with the chiral degeneracy: In case of the XY spins, spin inversion does not accompany the chirality flipping since the chiral variable is *quadratic* in the XY spins.

The residual Z_2 degeneracy, though *achiral* in nature, is also Ising-like and is expected to be broken at some finite temperature. The associated transition should also be Ising-like with the ordered phase characterized by nonzero spin-glass order parameter $q_{\rm SG}^{(2)} > 0$. Since the residual Z_2 degeneracy is a part of the $SO(2)$ spin-rotation degeneracy, the spin-glass transition temperature under finite anisotropy $T_{\rm SG}(D)$ should merge to $T=0$ as $D \rightarrow 0$, in contrast to the case of uniform magnetic fields discussed above. Here recall that, in a fully isotropic case, the conventional spin-glass order probably occurs only at $T=0$. In this case, the symmetries broken at $D=0$ and at $D \neq 0$ are different, and there should be a changeover of the fixed points between a $T=0$ XY-like fixed point for $D=0$ and a $T>0$ Ising fixed point for $D \neq 0$. In particular, the spin-glass transition temperature under finite anisotropy should be a nonanalytic function of D, characterized by the anisotropy-crossover exponent ϕ at the $T=0$ XY-like fixed point,

$$T_{\rm SG}(D) \approx D^{1/\phi} . \tag{6.5}$$

An argument given in Ref. 15 gives $\phi = 1 + (3/2)\nu_{\rm SG}$, where $\nu_{\rm SG}$ is the spin-glass correlation-length exponent at the $T=0$ fixed point. Previous numerical calculations gave $\nu_{\rm SG} \sim 2$,[7,9,15] which means $\phi \sim 4$. The present calculations suggest, however, that this may not be a true asymptotic value. An expected phase diagram is sketched in Fig. 10.

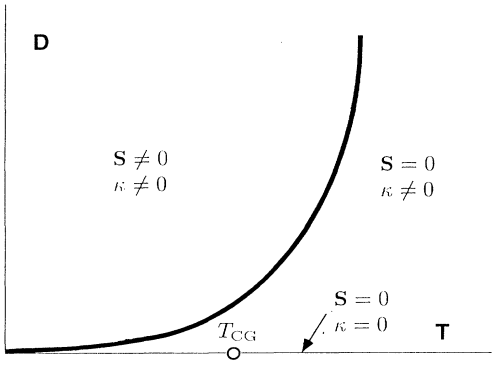

FIG. 10. Schematic phase diagram of a short-range XY spin glass in three dimensions with random dipolar-type anisotropy of magnitude D in the temperature-anisotropy (T-D) plane. $\mathbf{S}$ and κ represent the XY spin and the chirality, respectively. Each inequality and/or equality in the figure refers to the existence and/or nonexistence of the associated glass-type long-range correlations.

C. Random Dzyaloshinski-Moriya-type anisotropy

Next, I wish to briefly discuss the case of the random Dzyaloshinski-Moriya-type antisymmetric anisotropy. The Dzyaloshinski-Moriya-type anisotropy also breaks reflection symmetry of the Hamiltonian. In fact, this interaction may be regarded as a random chiral field acting on the $\langle ij \rangle$ bond. Again, a finite-temperature chiral-glass transition in zero anisotropy goes away under an infinitesimal anisotropy. By contrast, the Dzyaloshinski-Moriya-type anisotropy keeps the $SO(2)$ rotation symmetry intact unlike the case of the dipolar anisotropy. This situation is specific to the XY spins. [In the case of $n=3$-component Heisenberg spins, the random Dzyaloshinski-Moriya-type anisotropy reduces the original $O(3)$ symmetry only to Z_2 associated with spin inversion. From a symmetry viewpoint, there is no essential difference between the dipolar-type and the Dzyaloshinski-Moriya-type anisotropies in the Heisenberg case.] Note that an XY spin glass in the presence of the Dzyaloshinski-Moriya-type anisotropy has the same symmetry as the gauge glass.[33]

It is not clear at the present stage whether, under a finite Dzyaloshinski-Moriya-type anistropy D, the residual $SO(2)$ symmetry is broken at zero temperature or at some finite temperature. A similarity to the gauge glass in its symmetry appears to favor the latter possibility. Here recall that the recent numerical calculations for the gauge glass in three dimensions suggest a finite-temperature ordering.[28–31] If this is true, the same $SO(2)$ symmetry should be broken at a finite temperature for $D>0$, and at zero temperature for $D=0$. This may seem somewhat strange, but may be related to the fact that, in the $D=0$ case, the Hamiltonian itself has a full $O(2)=Z_2 \times SO(2)$ symmetry, the chiral Z_2 symmetry being broken *spontaneously* via a chiral-glass transition at a higher temperature $T=T_{\rm CG}$, whereas, in the $D \neq 0$ case, the Hamiltonian has only a $SO(2)$ symmetry, the chiral Z_2 symmetry being broken *energetically* already at the Hamiltonian level.

In concluding this section, I add one comment. As mentioned, many real spin-glass materials are well described by the $n=3$-component Heisenberg spin glass with weak magnetic anisotropy. In this connection, effects of magnetic fields and random magnetic anisotropies on the chiral- and spin-glass orderings in Heisenberg spin glasses are of particular interest. This problem will be discussed in detail in a separate paper.[40] Let me only emphasize here that the behavior of these Heisenberg sys-

tems under magnetic fields or anisotropies are often qualitatively different from those of the XY systems analyzed in this section. For example, in the limit of vanishing anisotropy $D \to 0$, the spin-glass transition temperature of a weakly anisotropic Heisenberg spin glass tends to *a finite value,* not to $T=0$, unlike the case of an XY spin glass.

VII. SUMMARY AND DISCUSSION

In summary, I have presented the results of Monte Carlo simulations on a three-dimensional XY spin glass, together with the results of the numerical DWRG calculation. The results strongly suggest the occurrence of a finite-temperature chiral-glass ordering without the conventional spin-glass order. The associated chiral-glass exponents estimated from a Monte Carlo simulation, $\nu_{CG}=1.5\pm0.3$ and $\eta_{CG}=-0.4\pm0.2$, are very close to those of the three-dimensional Ising spin glass. The results are consistent with the view that the chiral-glass transition in vector spin glasses belongs to the universality class of the standard Ising spin glass, as expected from a naive symmetry consideration. Then, assuming the existence of a finite-temperature chiral-glass transition, effects of uniform magnetic fields and random magnetic anisotropies are discussed. In particular, the present chiral-glass picture predicts the existence of a GT-like transition line under magnetic fields which has an origin entirely different from the conventional GT-line of an infinite-range SK model.

Finally, I wish to add a few comments. First, the behavior of the purely two-component XY (plane rotator) spin glass studied in this paper might somewhat differ from that of the three-component, XY-like, anisotropic Heisenberg spin glass, particularly when the XY-like (easy-plane-type) anisotropy is weak. Usually, the transition behaviors of the two-component XY (plane rotator) model and the three-component XY-like model are qualitatively the same. However, this is not necessarily the case in spin glasses. One can see this by noting that, in case of a weakly anisotropic XY-like Heisenberg spin glass, the *noncoplanar* spin ordering is expected to occur with a finite z component of the spin, $\langle S_{i,z} \rangle_T$. In such an occasion, the transition behavior of a weakly anisotropic XY-like spin glass might further be complicated due to the possible symmetry breaking of the Z_2 degeneracy associated with this S_z component.[41] Further discussion on the ordering of such XY-like Heisenberg spin glasses will be given in a separate paper.[40]

Second, I note that the present pure XY (plane rotator) spin-glass model might have some direct experimental relevance to *ceramic* or *granular* superconductors with a *d-wave* pairing symmetry. Recently, the possibility of a d-wave superconductivity has been discussed in high-T_c cuprates. In contrast to the standard s-wave superconductors, the Josephson coupling between the grains of d-wave superconductors can be either ferromagnetic (0 junction) or negative (π junction), depending on the spatial direction of the junction and of the crystal grain on both sides. As such, a d-wave granular superconductor may be regarded as an ideal realization of the three-dimensional XY spin glass. Note that in this case, an angle variable in the Hamiltonian θ_i represents a phase of the superconducting order parameter at the ith grain, which has nothing to do with the physical spin degree of freedom. In fact, a chiral-glass ordering is also expected to occur in these d-wave ceramic superconductors, which might have an interesting consequence on the properties of high-T_c cuprates. More details will be published elsewhere.[42]

Finally, the possibility of a finite-temperature chiral-glass ordering was suggested also for an $n=3$-component Heisenberg spin glass from the recent numerical calculation.[3] In connection to the real spin-glass materials, the Heisenberg system has much more physical significance. A Monte Carlo simulation on a possible chiral-glass ordering of Heisenberg spin glasses has also been made, and the results will be reported in a separate paper.[43]

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references cited therein. Within the replica formalism, these authors have shown that there is no spin-glass order in a $d=3XY$ spin glass *provided* that a certain correlation, such as c_1 in this reference, remains nondivergent at any finite temperature. In fact, however, this correlation is essentially the chiral-glass correlation, although Ozeki and Nishimori stated otherwise. The results in the present paper indicate that the assumption in the proof by Ozeki and Nishimori most probably does *not* hold at low temperatures. Therefore, an exact proof for the absence of the spin-glass order in a $d=3$ vector spin glass still remains open, particularly in the possible occurrence of a finite-temperature chiral-glass ordering.

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[23]By contrast, the form of the finite-size-scaling function $\bar{g}_{CG}(x)$ appears to differ considerably from the one of the standard 3D Ising spin glass observed in Ref. 12, the latter being monotomic and always positive while the former being not so. It is usually believed that the finite-size-scaling function is universal among systems in the same finite geometry and with the same boundary conditions: See, e.g., M. N. Barber, in *Phase Transitions and Critical Phenomena,* Vol. 8, edited by C. D. Domb and J. L. Lebowitz (Academic, New York, 1983), p. 145. However, since the precise correspondence between the lattice geometry and the boundary conditions of the spin variable of the 3D Ising spin glass and of the chiral variable of the 3D XY spin glass is not straightforward, it is not necessarily clear whether one should expect a common finite-size-scaling function between these two systems, even when they are to belong to the same universality class. Other possible explanation of the apparent difference in the scaling function may be that the true scaling regime has not been reached in the present simulation due to the possible narrowness of the chiral critical region.

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[35]In Ref. 27, Ney-Nifle *et al.* showed that, in a random one-dimensional XY ladder, both the spin- and chiral-domain-wall energies are characterized by a common exponent $y_s=y_c=\nu_{CG}^{-1}=(0.5263\ldots)^{-1}$. Again, as discussed above, this coincidence is originated from the inequality $\nu_{CG}<\nu'_{SG}$ specific to this one-dimensional model.

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