Gaplessness and properties of layered superconductors: Application to high-T_c cuprates

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The presence of different low-dimensional structural units along with a short coherence length leads to the formation of a two-gap spectrum, that is to a double-peak structure in the superconducting density of states (e.g., Y-Ba-Cu-O). The presence of magnetic impurities leads to a gapless state. As opposed to the usual one-gap isotropic model, the region of gaplessness is extended, so that one can observe the effect of gaplessness without a noticeable shift in T_c. Strong anisotropy of the pairing interaction within the planes of a layered superconductor also leads to a similar extension of the region of gapless behavior in the presence of magnetic impurities. This gaplessness has a great impact on the spectroscopy of the cuprates, particularly for nonstoichiometric compounds. Experimental data are discussed in detail.

I. INTRODUCTION

This paper is concerned with the energy spectrum of the high-T_c oxides, namely, with the appearance of a gapless state in these materials. We discussed some aspects of this problem for the $YBa_2Cu_3O_x$ (YBCO) compound in our previous papers[1] (see also the review in Ref. 2). This paper contains a more general treatment and a detailed analysis of some recent experiments.

The phenomenon of gapless superconductivity is not new in the physics of superconductivity. It has been introduced in Ref. 3 and analyzed in Refs. 3–5 (see also the reviews in Refs. 6 and 7). However, a short coherence length and the presence of the layered structure in the cuprates lead to an unusual manifestation of this phenomenon. For example, contrary to the case of conventional superconductors, it is perfectly realistic to expect gapless behavior in the cuprates without a noticeable depression in T_c. In other words, there is a large extension of the region of gaplessness, and this aspect of high-T_c superconductivity is very important if we analyze the spectroscopy of these materials. Gapless superconductivity, which has been a remarkable but rather exotic phenomenon for conventional materials, is playing a very important role in the field of high-T_c superconductivity.

The structure of this paper is as follows. Section II contains a general theoretical treatment. Evaluation of various parameters and an analysis of experimental data for the cuprates are described in Sec. III.

II. THEORY

The phenomenon of a gapless superconductivity is caused by the pair-breaking effect of magnetic impurities. Of course, there are other reasons for gaplessness (see for example the review in Ref. 6), but we focus here on the case of magnetic impurities. The local magnetic moment of the impurity tries to align the spins of the carriers forming the pairs, that is, destroy the pairing. The analysis in Refs. 3–7 has been carried out for an isotropic one-gap model which provides a good description of conventional superconductors. At first sight, the applicability of such a model even to conventional metals looks strange, because many conventional metals have a highly anisotropic Fermi surface as well as a complicated multiband structure. Some manifestations of the energy-gap anisotropy in conventional superconductors can be observed (see for example the reviews in Refs. 8 and 9), but, nevertheless, these effects are not large in the conventional superconductors; this is due to a large coherence length in these materials. Indeed, the inequality $l \ll \xi_0$ which holds for these materials (l is a mean free path) and the scattering between different parts of the Fermi surface as well as the interband scattering in the multigap case, leads to an averaging and isotropization. Note that the multigap structure has been observed before only in the exotic superconductor, namely in Nb-doped $SrTiO_3$.[10]

The picture in the cuprates is entirely different. The small value of the coherence length allows one to observe a multigap structure as well as an energy-gap anisotropy. We described briefly[1] the disappearance of the gap in YBCO caused by oxygen depletion. Here we consider the problem of gaplessness in cuprates in more detail, and, in addition, we will describe a number of different cases.

In general, anisotropy means a deviation from the one-gap isotropic picture. A most interesting example corresponds to the presence of two different groups of carriers. For a proximity system there is a spatial separation of such groups. Another example of two energy bands corresponds to separation in momentum space.

Consider first the S_α-S_β system, where S_α and S_β are two spatially separated superconducting subsystems. Assume for concreteness that $T_c^\alpha > T_c^\beta$. An important example is an S_α-N_β sandwich (then $T_c^\beta = 0$). Note that the YBCO compound is an example of such a system.[1] Indeed, it has been established that the chains not only

provide doping for the plane but also form an additional conducting subsystem. Planes and chains are bound by the charge transfer.

The equations for the order parameters Δ^α and Δ^β in the thermodynamic Green's-function formalism can be written in the form

$$\Delta^\alpha(\omega_n)Z^\alpha(\omega_n)=\pi T\sum_{n'}(\lambda_\alpha D_{nn'}-\tilde{\mu}_\alpha)\Delta^\alpha(\omega_{n'})\varphi^\alpha_{n'} +\pi T\sum_{n'}\lambda_{\alpha\beta}D_{nn'}\Delta^\beta(\omega_{n'})\varphi^\beta_{n'} +\Gamma^{\alpha\beta}\Delta^\beta(\omega_n)\varphi^\beta_n\ , \tag{1}$$

$$\Delta^\beta(\omega_n)Z^\beta(\omega_n)=\pi T\sum_{n'}(\lambda_\beta D_{nn'}-\tilde{\mu}_\beta)\Delta^\beta(\omega_{n'})\varphi^\beta_{n'} +\pi T\sum_{n'}\lambda_{\beta\alpha}D_{nn'}\Delta^\alpha(\omega_{n'})\varphi^\alpha_{n'} +\Gamma^{\beta\alpha}\Delta_\alpha(\omega_n)\varphi^\alpha_n\ . \tag{2}$$

Z_α and Z_β are the renormalization functions:

$$Z^\alpha(\omega_n)=1+\pi T\sum_{n'}\lambda_\alpha D_{nn'}\omega_{n'}\varphi^\alpha_{n'} +\pi T\sum_{n'}\lambda_{\alpha\beta}D_{nn'}\omega_{n'}\varphi^\beta_{n'}+\Gamma^{\alpha\beta}\varphi^\beta_n\ , \tag{3}$$

$$Z^\beta(\omega_n)=1+\pi T\sum_{n'}\lambda_\beta D_{nn'}\omega_{n'}\varphi^\beta_{n'} +\pi T\sum_{n'}\lambda_{\beta\alpha}D_{nn'}\omega_{n'}\varphi^\alpha_{n'}+\Gamma^{\beta\alpha}\varphi^\alpha_n\ . \tag{4}$$

In Eqs. (1)–(4) $\omega_n=(2n+1)\pi T$, λ_α and λ_β are the coupling constants, describing the intrinsic pairing in the α and β subsystems (we assume that the pairing is mediated by the phonon exchange, and $D_{nn'}=\tilde{\Omega}^2[\tilde{\Omega}^2+(\omega_n-\omega_{n'})^2]^{-1}$ is a phonon Green's function; $\tilde{\Omega}$ is the characteristic phonon frequency; however, the approach can be easily generalized for different mechanisms), $\varphi^i=[\omega_n^2+(\Delta^i(\omega_n))^2]^{-1}$, $\Gamma^{ik}=|T^{ik}|\nu_k$, T^{ik} is the tunneling matrix element, ν_i is the density of states, $i,k\equiv\{\alpha,\beta\}$. The terms containing $\Gamma^{\alpha\beta}$ and $\Gamma^{\beta\alpha}$ describe the proximity effect (see Ref. 11). The thermodynamic Green's-function formalism in the theory of the proximity effect was used by one of the authors in Ref. 12. The proximity channel of the charge transfer in cuprates, e.g., the charge transfer between the planes and the chains in YBCO, is called the intrinsic proximity effect, and this stresses the fact that the phenomenon occurs on the scale of the unit cell.

The terms containing the coupling constants $\lambda_{\alpha\beta}$ and $\lambda_{\beta\alpha}$ describe an inelastic charge-transfer channel. More specifically, the carrier from the α subsystem radiates a phonon and makes a transition to β. Another carrier absorbs the phonon and also makes the transition to β; as a result of this phonon exchange, these two carriers form a pair in the β subsystem. The coupling constants contain the matrix element which describes the inelastic tunneling process. It is worth noting that we are talking about the exchange by virtual, not real, phonons; as a result, the energy is conserved in the whole process. Critical temperature can be obtained from Eqs. (1)–(4); one should set $T=T_c$ and $\Delta_\alpha=\Delta_\beta=0$ in the denominators of the right-hand sides of these equations.

Consider the case when both the subsystems contain magnetic impurities. Scattering by these impurities can be described by additional terms in the right sides of Eqs. (1), (2), and in the normalization functions (see, for example, Ref. 13). One should add an additional term $\Gamma_2^\alpha\Delta^\alpha\varphi_\alpha$ to the right side of Eq. (1) and the term $\Gamma_1^\alpha\varphi_\alpha$ to the expression for $Z^\alpha(\Gamma_i^\alpha=(\tau_i^\alpha)^-1$, $i=\{1,2\}$, τ_i are the relaxation times introduced in Ref. 3). Similar terms should be added to the right sides of Eqs. (2) and (4). Then we obtain

$$\Delta^\alpha(\omega_n)=[\Delta^{\rm ph}_\alpha(\omega_n)+\Gamma^{\alpha\beta}\Delta^\beta(\omega_n)\varphi^\beta(\omega_n)]Z^{-1}_{\alpha M}(\omega_n)\ , \tag{5}$$

$$\Delta^\beta(\omega_n)=[\Delta^{\rm ph}_\beta(\omega_n)+\Gamma^{\beta\alpha}\Delta^\alpha(\omega_n)\varphi^\alpha(\omega_n)]Z^{-1}_{\beta M}(\omega_n)\ , \tag{6}$$

where

$$Z_{\alpha M}=1+\lambda_\alpha+\Gamma^{\alpha\beta}\varphi^\beta(\omega_n)+\Gamma^\alpha_M\varphi^\alpha(\omega_n)\ , \tag{7}$$

$$Z_{\beta M}=1+\Gamma^{\beta\alpha}\varphi^\alpha(\omega_n)+\Gamma^\beta_M\varphi^\beta(\omega_n)\ . \tag{8}$$

Here $\Gamma^i_M=(\tau^{-1}_{s,i})^i\propto n^i_m\nu^i$ is related to the magnetic relaxation time (see Ref. 3), n^i_m and ν^i are the concentrations of the impurities and densities of states, correspondingly; $\Delta^{\rm ph}_\alpha$ represents the first and second terms in Eq. (1); $\Delta^{\rm ph}_\beta$ has a similar meaning. We assume also that $\lambda_\beta\ll1,\lambda_{ik}\ll1$.

Let us consider the energy spectrum of the superconductor. Then one should perform an analytical continuation of Eqs. (1)–(8) at $T=0$ K, or to write directly the equations for the time-dependent Green's function:

$$\Delta^\alpha(\omega)=Z^{-1}_{\alpha M}(\omega)\Delta^{\rm ph}_\alpha(\omega)+\Gamma^{\alpha\beta}Z^{-1}_{\alpha M}(\omega)F^+_\beta(\omega)\ , \tag{9}$$

$$\Delta^\beta(\omega)=Z^{-1}_{\beta M}(\omega)\Delta^{\rm ph}_\beta(\omega)+\Gamma^{\beta\alpha}Z^{-1}_{\beta M}(\omega)F^+_\alpha(\omega)\ , \tag{10}$$

and corresponding equations for Z^α, Z^β, $F_i(\omega)=\Delta_i(\omega)[\Delta_i^2(\omega)-\omega^2]^{-1}$; $i=\{\alpha,\beta\}$. The energy gap corresponds to the region where the superconducting density of states

$$\nu^i_s=\nu^i_n\,{\rm Re}\frac{\omega}{[\omega^2-(\Delta^i(\omega))^2]^{1/2}} \tag{11}$$

is equal to zero. An increase in the concentration of the magnetic impurities n_m leads to an increase in the value Γ^i_M. At some value Γ^c_M one can observe the gapless state, that is the appearance of a nonzero value of the density of states $\omega\geq0$. This feature is a consequence of the appearance of the imaginary parts of the order parameters $\Delta^i(\omega)$ in the low-frequency region; the critical value Γ^c_M corresponds to the order parameters going to zero at $\omega\to0$ (cf. Ref. 13). The region $\omega>0$ is characterized by the presence of an imaginary part of $\Delta^i(\omega)$.

With the use of Eqs. (9) and (10) at $\omega=0$, after some manipulations we obtain

$$\Delta^\alpha=\Gamma^{\beta\alpha}\Delta^\beta(\Delta^{\rm ph}_\beta+\Gamma^{\beta\alpha}-\Delta^\beta-\Gamma^\beta_M)^{-1}\ , \tag{12}$$

$$\Delta^\beta=R[(1+\lambda_\alpha)\Gamma^{\beta\alpha}+\Delta^{\rm ph}_\alpha-\Gamma^{\alpha\beta}-\Gamma^\alpha_M]^{-1}\ , \tag{13}$$

where

$$R=(\Delta_\alpha^{\rm ph}+\Gamma^{\alpha\beta}-\Gamma_M^\alpha)(\Delta_\beta^{\rm ph}+\Gamma^{\beta\alpha}-\Gamma_M^\beta)-\Gamma^{\alpha\beta}\Gamma^{\beta\alpha}\ . \quad (14)$$

Here

$$\Delta^i\equiv\Delta^i(0);\Delta_i^{\rm ph}\equiv\Delta_i^{\rm ph}(0);i=\{\alpha,\beta\}\ .$$

One can see from Eq. (13) that the gapless superconducting state arises at $\Gamma_M^\beta=\Gamma_M^c$ (then $R=0$), where

$$\Gamma_M^c=\frac{(\Delta_\alpha^{\rm ph}-\Gamma_M^\alpha)(\Delta_\beta^{\rm ph}+\Gamma^{\beta\alpha})+\Gamma^{\alpha\beta}\Delta_\beta^{\rm ph}}{\Delta_\alpha^{\rm ph}+\Gamma^{\alpha\beta}-\Gamma_M^\alpha}\ . \quad (15)$$

It is important that, according to Eqs. (12) and (13), the gapless state arises in both subsystems. Indeed $\Delta_\alpha\propto\Delta_\beta\propto R$. Note also that in the case of an S-N system (then $\lambda_B=0$, and we are dealing with an induced superconducting state in the β subsystem) we obtain

$$\Gamma_M^{\rm cr}\cong\varepsilon_\beta\cong\frac{\lambda_{\beta\alpha}}{\lambda_\alpha}\varepsilon_\alpha+\Gamma_{\beta\alpha}\ . \quad (16)$$

One can estimate also the value of the critical concentration. We obtain

$$l_M^{\rm cr}\cong v_F^\beta\tau_M^{\rm cr}\cong hv_F^\beta/\Gamma_M^{\rm cr}\cong\xi_\beta\ , \quad (17)$$

where ξ_β is the coherence length for the β system. We will evaluate the parameters describing the gapless state in the cuprates in the next section.

Therefore, the relatively small amount of magnetic impurities leads to an appearance of the gapless state. However, and this is particularly true for S-N or S_α-S_β ($T_c^\alpha\gg T_c^\beta$) systems, the gaplessness does not lead to a drastic change in the value of T_c. Indeed, the value of T_c can be determined from Eqs. (1)–(4). An explicit expression for T_c depends on the strength of the coupling.[1] For example, in the weak-coupling case ($\lambda_\alpha\ll1$), one can obtain

$$T_c=T_c^0\exp(f)\ ,$$

$$f=\lambda_{\alpha\beta}\lambda_{\beta\alpha}\lambda_\alpha^{-3}+\frac{\pi}{8}(\lambda_{\alpha\beta}/\lambda_\alpha)\gamma_{\beta\alpha}^0-\frac{\pi}{16}\gamma_{\alpha\beta}^0\ . \quad (18)$$

Here $\gamma_{ij}^0=\Gamma^{ik}/\pi T_c^0$, T_c^0 is the critical temperature for an isolated α group. The opposite case (strong coupling $\lambda_\alpha\gg1$) is described by the relation

$$T_c\cong(2\pi)^{-1}\Lambda_{\rm eff}^{1/2}\tilde\Omega\ ,$$

$$\Lambda_{\rm eff}=(\lambda_\alpha-2\lambda_{\alpha\beta}\gamma_{\beta\alpha})[1+\gamma_{\alpha\beta}(1-\gamma_{\beta\alpha})^{-1}+2\tilde\mu]^{-1}\ . \quad (18')$$

For a specific example, consider the case when the magnetic impurities are in the β subsystem. Then one should make the replacement in the renormalization function Z^β:$\Gamma^{\beta\alpha}\to\tilde\Gamma^{\beta\alpha}=\Gamma^{\beta\alpha}+\Gamma_M^\beta$. With use of Eqs. (1)–(4), we obtain

$$\begin{aligned}\Delta^\alpha(\omega_n)&\left[1+(\lambda_\alpha+\lambda_{\alpha\beta})\pi T\sum_{n'}D_{nn'}\omega_{n'}/|\omega_{n'}|+\Gamma^{\alpha\beta}/|\omega_n|\right]\\
&=\pi T\sum_{n'}(\lambda_\alpha D_{nn'}-\tilde\mu)\Delta^\alpha(\omega_{n'})/\omega_{n'}|+\lambda_{\alpha\beta}\lambda_{\beta\alpha}(\pi T)^2\sum_{n',n''}D_{nn'}D_{n'n''}\Delta^\alpha(\omega_{n''})|\omega_{n'}|^{-1}|\omega_{n''}|^{-1}(1+\tilde\Gamma^{\beta\alpha}/|\omega_{n'}|)^{-1}\\
&+\lambda_{\alpha\beta}\Gamma^{\beta\alpha}\pi T\sum_{n'}D_{nn'}\Delta^\alpha(\omega_{n'})|\omega_{n'}|(|\omega_n|+\tilde\Gamma^{\beta\alpha})^{-1}+\lambda_{\beta\alpha}\Gamma^{\alpha\beta}\pi T\sum_{n'}D_{nn'}\Delta^\alpha(\omega_{n'})|\omega_{n'}|(|\omega_n|+\tilde\Gamma^{\beta\alpha})^{-1}\\
&+\frac{\Gamma^{\alpha\beta}\Gamma^{\beta\alpha}}{|\omega_n|+\tilde\Gamma^{\beta\alpha}}\frac{\Delta^\alpha(\omega_n)}{|\omega_n|}\ ,\end{aligned} \quad (19)$$

where $\tilde\Gamma^{\beta\alpha}=\Gamma^{\beta\alpha}+\Gamma_M^\beta$.

The appearance of the gapless state corresponds to $\gamma_M^\beta=\gamma_M^c\cong\varepsilon_\beta/\pi T_c$. One can show that the shift in T_c is relatively small. Indeed, let us evaluate the shift in T_c caused by magnetic impurities at $n_m=n_m^c$. The analysis is based on Eq. (19). Consider, at first, the weak-coupling case ($\lambda_\alpha\ll1,\lambda_{ik}\ll1$). Introducing the function

$$\delta(\omega_n)=\Delta^\alpha(\omega_n)\left[1+\frac{\Gamma^{\alpha\beta}}{|\omega_n|}\frac{|\omega_n|+\Gamma_M}{|\omega_n|+\tilde\Gamma}\right]\ ,$$

we arrive at the following equation:

$$\begin{aligned}\delta(\omega_n)=&\pi T\sum_{n'}(\lambda_\alpha D_{nn'}-\tilde\mu)\delta(\omega_{n'})|\omega_{n'}|^{-1}S(\omega_{n'})+\lambda_{\alpha\beta}\Gamma^{\beta\alpha}\pi T\sum_{n'}D_{nn'}\delta(\omega_{n'})|\omega_{n'}|^{-1}(|\omega_n|+\tilde\Gamma)^{-1}S(\omega_{n'})\\
&+\lambda_{\beta\alpha}\Gamma^{\alpha\beta}\pi T\sum_{n'}D_{nn'}\delta(\omega_{n'})|\omega_{n'}|^{-1}(|\omega_n|+\tilde\Gamma)^{-1}S(\omega_{n'})\ ,\end{aligned} \quad (20)$$

where

$$S(\omega_n)=\left[1+\frac{\Gamma^{\alpha\beta}}{|\omega_n|}\frac{|\omega_n|+\Gamma_M}{|\omega_n|+\tilde\Gamma}\right]^{-1};\quad\tilde\Gamma=\Gamma^{\beta\alpha}+\Gamma_M\ .$$

One can seek the solution in the form: $\delta(\omega_n)=c+\delta'(\omega_n)$; $c=\text{const};\delta'\ll c$. Assume also the small tunneling proba-

bility ($\gamma_{ik} \ll 1$). As a result, we obtain the following expression for the shifting T_c which corresponds to the appearance of the gapless state:

$$\frac{\Delta T_c}{T_c} = f'; f' = -\gamma_M^c \gamma_{\alpha\beta}\gamma_{\beta\alpha} \sum_{n'} D_{nn'}(2n'+1)^{-2}(|2n'+1|+\Gamma)^{-2} \approx -\gamma_M^c \gamma_{\alpha\beta}\gamma_{\beta\alpha} \ll 1 \ . \tag{21}$$

In the strong-coupling case ($\lambda \gg 1$), one can use the matrix method, developed in Ref. 16 (see Refs. 1, 2, and 17). The equation which determines T_c in this case has the form

$$\varphi_n = \sum_m^n R_{nm}\varphi_m \ ,$$
$$R_{nm} = S_m S_n \tilde{F}_{n,m;\nu} \ , \tag{22}$$

where

$$S_k = (2k+1)^{-1/2}\left[\frac{2k+1+\gamma}{2k+1+\gamma_{\beta\alpha}} + \alpha_k^M\right]^{1/2} \quad \text{and} \quad \alpha_k^M = \gamma_M \frac{\gamma_{\alpha\beta}\gamma_{\beta\alpha}}{|2k+1|(\gamma_{\beta\alpha}+|2k+1|)^2}; \quad k = \{1,2,\} \ ,$$

$$\tilde{F}_{n,m;\nu} = A\left[[\nu^2+(n-m)^2]^{-1}+[\nu^2+(n+m+1)^2]^{-1}-\delta_{nm}\sum_{l=0}^{2n}[\nu^2+(n-l)^2]^{-1}\right]$$
$$+B\left[\{[\nu^2+(n-m)^2]^{-1}+[\nu^2+(n+m+1)^2]^{-1}\}(|2n+1|^{-1}+|2m+1|^{-1})\right.$$
$$\left.-\delta_{nm}\sum_{n''}[\nu^2+(n-n'')^2]^{-1}(|2n+1|^{-1}+|2n''+1|^{-1})\frac{2n''+1}{|2n''+1|}\right]-2\mu \ ,$$

$$A = \lambda_\alpha \nu^2, \quad B = \lambda_{\alpha\beta}\gamma_{\beta\alpha}\nu^2, \quad \nu = \tilde{\Omega}/2\pi T_c \ .$$

The critical temperature in the strong-coupling case is determined by the equation (cf. Ref. 12)

$$T_c = \frac{\tilde{\Omega}}{2\pi}\lambda_{\text{eff}}^{1/2} \ ,$$
$$\lambda_{\text{eff}} = \frac{\lambda_\alpha + 2\lambda_{\alpha\beta}\gamma_{\beta\alpha}}{1+\gamma_{\alpha\beta}(1+\gamma_{\beta\alpha})^{-1}+2\tilde{\mu}+\alpha_M} \ , \tag{23}$$

where $\alpha_M = \gamma_M\gamma_{\alpha\beta}\gamma_{\beta\alpha}(1+\gamma_{\beta\alpha})^{-1}$. At $\gamma_M = \gamma_M^c$ the superconducting becomes gapless. One can see from Eq. (23) that the shift ΔT_c is also small [cf. Eq. (21)], so that

$$\Delta T_c \approx \tfrac{1}{2}\gamma_M^c\gamma_{\alpha\beta}\gamma_{\beta\alpha} \ll 1 \ . \tag{24}$$

As a result, we obtain

$$\frac{\Delta T_c}{T_c} \cong \gamma_M \frac{\gamma_{\alpha\beta}\gamma_{\beta\alpha}(1+\gamma_{\beta\alpha})^3}{2(1+\gamma)^4} \ll 1 \ . \tag{24'}$$

Therefore, gaplessness does not lead to a noticeable shift in T_c. If, for example, the value of the smaller gap $\varepsilon_\beta \cong 0.8T_c$ and $\gamma_{\alpha\beta} \cong 0.3, \gamma_{\beta\alpha} \cong 0.25$ (these values are realistic YBCO, see Sec. III), we obtain that $\Delta T_c/T_c \cong 10^{-2}$.

Therefore, $\Delta T_c \ll T_c$ and the appearance of the gapless state in the studied layered system does not lead to a larger depression of T_c. Let us note, for the comparison, that for conventional superconductors the shift $\Delta T_c/T_c \cong 0.9$ for $n_M = n_M^c$. Thus, indeed, the case of a layered two-gap superconductor is characterized by a clear and noticeable extension of the gapless region.

Two-band system; anisotropy

Consider now the case when there is no spatial separation of the subsystems. For example, this is the case when a three-dimensional system or an isolated layer contains two overlapping energy bands. Then the Fermi surface has two very distinguishable regions. Since there is no spatial separation of the subsystems, the magnetic impurities directly scatter the carriers from both groups.

The order parameters Δ^α and Δ^β are described by equations similar to (5)–(8). Since there is no spatial separation, the terms containing $\Gamma^{\alpha\beta}$ and $\Gamma^{\beta\alpha}$ should be omitted. The terms containing $\lambda_{\alpha\beta}$ and $\lambda_{\beta\alpha}$ describe in this case the interband transitions caused by phonon exchange. In other words, the problem is reduced to the usual two-band model, which was introduced in a weak-coupling approximation in Ref. 14 and studied in detail in Ref. 15. If $\lambda_\beta = 0$, we are dealing with induced two-band superconductivity. Consider, for example, this simpler case (the generalization for the case $\lambda_\beta \neq 0$ is straightforward). In the presence of the impurities (magnetic and nonmagnetic scatterers), the equations for the order parameters and renormalization functions can be written in the form [cf. Eqs.(5)–(8)]

$$\Delta^i(\omega_n) = [\Delta_i^{\text{ph}}(\omega_n) + \Gamma_{\text{imp}}^{ik}\Delta^k\varphi^k](Z_M^i)^{-1} \ , \tag{25}$$

$$Z_M^i(\omega_n) = 1 + \lambda_i + \Gamma_{\text{imp}}^{ik}\varphi_k \ . \tag{26}$$

Here $i,k = \{\alpha,\beta\}, i \neq k$; the amplitudes Γ_{imp}^{ik} describe a nonmagnetic interband scattering. We assume $\lambda_\beta = 0$.

Strictly speaking, in this case both equations contain the terms Γ_M^i describing the magnetic scattering. Only in the special case when $\Gamma_M^\alpha \ll \varepsilon_\alpha$ (at the same time $\Gamma_M^\beta < \varepsilon_\beta$ and the term Γ_M^β should not be neglected), one can neglect the term $\Gamma_{M\varphi\alpha}^\alpha$.

In the general case, one can obtain the following equations at $\omega=0$:

$$(1+\lambda_\alpha)\Delta^\alpha\Delta^\beta+\Gamma_{\rm imp}^{\alpha\beta}\Delta^\alpha+\Gamma_M^\alpha\Delta_\beta=\Delta_\alpha^{\rm ph}\Delta_\beta\ , \tag{27}$$

$$\Delta^\alpha\Delta^\beta+\Gamma_{\rm imp}^{\beta\alpha}\Delta^\beta+\Gamma_M^\beta\Delta^\alpha=\Delta_\beta^{\rm ph}\Delta_\alpha\ . \tag{28}$$

The densities of states are defined by Eq. (11). One can see that the critical value $\Gamma_M^c=\varepsilon_\beta\cong(\lambda_{\beta\alpha}/\lambda_\alpha)\varepsilon_\alpha$ corresponds to the appearance of gapless superconductivity. In this case there is also no drastic change in T_c. Indeed, T_c is determined by the equations, similar to (19), but with the replacement $\Gamma^{\beta\alpha}\rightarrow\Gamma_{\rm imp}^{\beta\alpha}$. Then the shift $\Delta T_c/T_c\approx\gamma_M^c\gamma_{\rm imp}^{\alpha\beta}\gamma_{\rm imp}^{\beta\alpha}$, where $\gamma_M^c\cong\varepsilon_\beta/\pi T_c$, $\gamma_{\rm imp}^{ik}=\Gamma_{\rm imp}^{ik}/\pi T_c$. Therefore, the presence of the two-gap structure leads to an expansion of the gapless region.

If we are dealing with the case of large energy-gap anisotropy (such a situation may occur even in the case of single band) that is,the case when some region of momentum space corresponds to a small value of the energy gap, then the pairing can be described by the set of equations similar to those in the two-gap case, and, therefore the large anisotropy is also characterized by an extension of the gapless region relative to the isotropic case.

III. DISCUSSION: GAPLESS BEHAVIOR IN THE CUPRATES

A. $YBa_2Cu_2O_x$ (YBCO)

The compound YBCO is the most studied of the cuprates but is certainly not the easiest to understand because of the existence of two conducting (and thus superconducting below T_c) subsystems, the CuO_2 planes and the CuO chains that in many respects make this compound appear like an intrinsic proximity-effect structure. In fact, our model of induced superconductivity and multigap structure, which was described in detail in Ref. 1 was motivated by the properties of YBCO. We will not repeat the complete description of the properties of YBCO in this article but will point out a number of crucial factors. We have modeled this compound as intrinsically superconducting planes and intrinsically normal chains. Thus the superconductivity in the chains is indeed induced by the presence of the superconductivity in the neighboring layers. Thus the density of states of this compound in the superconducting state has two gaplike structure (peaks) but in reality the thermodynamic behavior is dominated by the lower energy peak, which is the actual energy gap. Thus measurements of the low-temperature surface resistance, penetration depth, Knight shift, etc., will be dominated by the induced energy gap ε_β. Measurements of the spectroscopy should in fact be sensitive to the double-peak structure and, in fact, evidence for the two peaks has been observed in tunneling[18] and infrared reflectivity.[19] This compound has its maximum transition temperature when the oxygen content x is over 6.9 but is superconducting down to $x=6.4$. For samples with oxygen contents less than the stoichiometric 7, there is evidence[20] that intrinsic magnetic impurities appear on the chains (and may ultimately appear on the planes as well). These magnetic impurities act like pair breakers (see above, Sec. II) and rapidly suppress the induced gap so that at oxygen content around 6.9 the samples are already gapless. This gapless behavior is manifested in many experiments. One of the most convincing is the measurements of the low-temperature specific heat.[21] A linear term at low temperature is observed almost universally, which indicates the existence of a significant number of unpaired quasiparticles at low temperatures. This magnitude of this linear term has been shown to correlate with the size of a Shottky anomaly also observed at low temperatures, which signals the presence of a significant number of magnetic species. Furthermore, in nearly stoichiometric films of YBCO, a small gap can be inferred from penetration depth and simultaneous surface-resistance measurements.[22] In fact these microwave experiments[22] also demonstrated that in oxygen-reduced samples the exponential low-temperature behavior characteristic of a real gap was replaced by a power law indicative of a nonvanishing density of unpaired quasiparticles. Of course experimentally the transition temperature for such an oxygen content is still around 90 K so that the suppression of the transition temperature is minimal even in the gapless regime. In this article (see above, Sec. II) we have now derived the expression for the suppression of T_c and estimate it to be about 1% which is very consistent with the data.

B. La-Sr-Cu-O

In many respects the doped alloy cuprate $La_{2-x}Sr_xCuO_4$ is one of the simpler materials from the standpoint of its structure. The unit cell has two CuO_2 planes separated by La (Sr) layers. The doping is usually provided by replacing La with Sr which adds carriers to the CuO planes which become metallic, then metallic and superconducting, and then metallic and normal as the carrier concentration is increased. At the same time the antiferromagnetic behavior at low doping involving the magnetic Cu^{2+} disappears as the carrier concentration is increased but magnetic excitations and local magnetic moments remain well into the superconducting state.[23] The presence of these magnetic moments can lead to the appearance of a gapless state (see above, Sec. II).

Let us now discuss some relevant experimental data. Neutron-scattering measurements of the magnetic response as a function of temperature and specific-heat measurements at low temperature on several crystals of $La_{1.86}Sn_{0.14}Sr_{0.14}CuO_4$ with T_c's of 35 K (Ref. 23) determined that this compound was gapless and had a concentration of local magnetic moments sufficient to produce gapless behavior. According to the authors,[23] the only strange feature of these results was the small depression in the transition temperature relative to the maximum that can be obtained at slightly higher levels of doping. This observation is in strong agreement with our state-

ment (see above, Sec. II) that energy-gap anisotropy leads to an extension of the gapless region. In fact reasonable choices for the parameters [see discussion following Eq. (28)] can explain the results of the neutron-scattering paper,[23] i.e., that this compound could become gapless at a critical temperature of 35 K.

This clear result of gaplessness was indicated in earlier measurements of specific heat which had low-temperature linear terms.[24] Of course these conclusions provide extremely strong support for our calculations above which clearly show that for strongly anisotropic superconductors with magnetic impurities, the conventional isotropic results for gapless superconductivity have to be modified and can result in a significantly expanded region of gapless behavior.

C. Bi-Sr-Ca-Cu-O

This compound has several forms with differing numbers of intrinsically superconducting CuO_2 planes (mainly 1, 2, or 3) but with conducting or nearly conducting double layers of BiO_2 with an incommensurate modulation of the lattice.[25] Thus these compounds can also be characterized by our theory. Unfortunately the amount of data on these materials is not as comprehensive as the YBCO and we thus cannot make any quantitative comparison. However, our picture is that the BiO layers are induced into the superconducting state because of their proximity to the CuO_2 planes. The density of states in the BiO planes depends on the oxygen and Bi contents and thus value of the induced gap as well as the height of the peak in the BiO part of the density of states is quite variable, whereas the transition temperature which is dominated by the CuO_2 planes is to first order unchanged. This picture can be used to interpret photoemission data[26] from which it is very clear that there is a large energy-gap anisotropy due to the multiple bands and the angular dependence of the interaction. In fact, very recent results[27] show that the degree of anisotropy is correlated with the oxygen content in the BiO planes. Since the density of state in the BiO layers varies with the oxygen content it is clear from our model that both the induced gap and the size of the BiO peak in the density of states should correlate with the composition. This picture can also explain the mystery of why a gap can be observed using photoemission in Bi-Sr-Ca-Cu-O but not in YBCO. We know from transport data in YBCO that the chain density of states is large and in fact for nearly stoichiometric (x near 7) samples can be larger than the plane density of states.[1] We also know that the induced gap on the chains is quite small.[22] Thus photoemission which has an energy resolution at best of 5 meV would not be able to distinguish the peak due to the chains and thus would not see a shift in the Fermi energy within their sensitivity. On the other hand, the density of states (and induced gap) for the BiO layers is at best much smaller than the density of states in the CuO_2 planes[28] and thus the photoemission experiments would only be able to observe the much larger peak from the CuO_2 planes at the much larger gap energy. Thus they would infer a large gap for the Bi-Sr-Ca-Cu-O samples although the true gap was much smaller but could not be observed because of the very low-energy of the small peak (below the resolution of the photoemission instruments).

D. Ti-Ba-Ca-Cu-O

This series of compounds also comes in a variety of compositions.[25] The number of CuO_2 planes can vary between 1 and 3 and the TlO planes can be single or double. In many respects they are very similar to the Bi-Sr-Ca-Cu-O compounds in that both the CuO_2 layers, the BiO, and the TlO layers can be conducting and thus induced in the superconducting state. The very interesting feature of the Tl compounds is the degree to which they can be overdoped. There is very strong evidence that in the overdoped region these compounds become gapless.[29] In fact, the gapless behavior starts at relatively weak overdoping where the T_c is only slightly suppressed. As the doping is increased the density of superconducting states gets smaller as the overall density of states increases.[29] Again this behavior can be understood in the framework of our theory in the following way. We assume that the interaction responsible for superconductivity in these compounds is strongly anisotropic and thus the energy gap will be strongly anisotropic. Furthermore we associate overdoping with the appearance of intrinsic magnetic impurities (this can be checked but we are not aware of systematic experiments that have done this). Thus the overdoped region is one where we see a very expanded gapless region starting at very modest suppression of the maximum transition temperature. If the number of magnetic impurities scales with the overdoping then the decreased pair density is consistent with this assumption.

IV. SUMMARY

The main results can be summarized as follows:

(1) The presence of different lower dimensional structural units leads to the formation of a two-gap spectrum such as has been observed in YBCO. If magnetic impurities are introduced, at some critical concentration a gapless state appears. If we are dealing specifically with YBCO and we introduce magnetic impurities only on the chains (as with oxygen depletion) then the gapless behavior extends to both the chains and the planes because of charge transfer between them.

(2) The gapless state for a two-gap or highly anisotropic system is not accompanied by a significant shift in T_c, that is there is an extension of the gapless region relative to conventional isotropic one-gap superconductors.

(3) Deviations from ideal stoichiometry in the cuprates, in general, lead to the gapless state because of the mixed valence of the copper. This greatly affects the spectroscopy of these compounds.

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[1]V. Kresin and S. Wolf, Phys. Rev. B **46**, 6458 (1992); Physica C **198**, 328 (1992).
[2]V. Kresin, H. Morawitz, and S. Wolf, *Mechanisms of Conventional and High-T_c Superconductivity* (Oxford, New York, 1993).
[3]A. Abrikosov and I. Gor'kov, Sov. Phys. JETP **12**, 1243 (1961).
[4]S. Skalski, O. Betbeder, and P. Weiss, Phys. Rev. **136**, A1500 (1964).
[5]P. De Gennes, Phys. Condens. Matter **3**, 79 (1964); *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966), Chap. VIII.
[6]K. Maki, in *Superconductivity,* edited by R. Parks (Marcel Dekker, New York, 1969), p. 1035.
[7]D. Saint-James, G. Sarma, and E. Thomas, *Type-II Superconductors* (Pergamon, Oxford, 1969).
[8]J. Clem, Am. Phys. **40**, 268 (1966).
[9]A. Shepelev, Sov. Phys. Usp. **96**, 217 (1968).
[10]G. Binning, A. Baratoff, H. Hoenig, and G. Bednorz, Phys. Rev. Lett. **45**, 1352 (1980).
[11]W. McMillan, Phys. Rev. **175**, 537 (1968).
[12]V. Kresin, Phys. Rev. B **25**, 157 (1982).
[13]A. Kaizer and M. Zuckermann, Phys. Rev. B **1**, 229 (1970).
[14]H. Suhl, B. Mattiass, and L. Walker, Phys. Rev. Lett. **3**, 552 (1959); V. Moskalenko, Fiz. Met. Metalloved. **8**, 503 (1959).
[15]B. Geilikman, V. Kresin, and R. Zaitsev, Sov. Phys. Solid State **9**, 542 (1966); V. Kresin and S. Wolf, Physica C **169**, 476 (1990).
[16]C. Owen and D. Scalapino, Physica **55**, 691 (1971).
[17]V. Kresin, H. Gutfreund, and W. Little, Solid State Commun. **51**, 339 (1984); V. Kresin, Phys. Lett. **122**, 234 (1987).
[18]J. Geerk, X. X. Xi, and G. Linker, Z. Phys. B **73**, 329 (1988); M. Gurvich *et al.*, Phys. Rev. Lett. **63**, 10 098 (1989).
[19]Z. Schlesinger *et al.*, Phys. Rev. Lett. **65**, 801 (1990); Physica C **57**, 185 (1991).
[20]S. C. Sanders *et al.*, Appl. Phys. Lett. (to be published).
[21]N. Phillips *et al.*, Phys. Rev. Lett. **65**, 357 (1990); N. E. Phillips and R. A. Fisher, *High Temperature Superconductivity,* edited by J. Ashkenazi, Fulin Zuo, Gary Vezzoli, and Barry M. Klein (Plenum, New York, 1991).
[22]N. Klein *et al.*, Phys. Rev. Lett. **71**, 3355 (1993).
[23]T. E. Mason *et al.*, Phys. Rev. Lett. **71**, 919 (1993).
[24]R. A. Fisher, J. E. Gordon, and N. E. Phillips, J. Supercond. **1**, 231 (1988).
[25]B. Raveau, C. Michel, M. Hervieu, and D. Groult, *Crystal Chemistry of High-T_c Superconducting Copper Oxides* (Springer-Verlag, Berlin, 1991).
[26]Z. X. Shen *et al.*, Phys. Rev. Lett. **70**, 1553 (1993).
[27]H. Ding *et al.* (unpublished); M. Onellion *et al.* (private communication).
[28]W. Pickett, Rev. Mod. Phys. **61**, 433 (1989).
[29]Ch. Neidermayer *et al.*, J. Supercond. **7**, 165 (1994).