Model of track formation

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The developed theoretical model of track formation accounts for the influence of initial atomic displacements, momenta, and initial heating of atoms on the plastic deformation of solids near the trajectory of a penetrating fast heavy ion. These initial conditions result from the energy and momentum transfer from the excited electron subsystem to the ion one near the ion trajectory. The theoretical results obtained make it possible to investigate the dependencies of the anisotropic growth of amorphous alloys irradiated with high-energy heavy ions on the inelastic and elastic energy losses of the moving ion and on the irradiated temperature. A good agreement between these theoretical dependencies and existing experimental results was obtained.

I. INTRODUCTION

When a fast heavy particle with energy $E \sim 1$ MeV/amu penetrates into a solid, the energy $\Delta E \sim 1-10$ keV/Å is lost during a short time $t \lesssim 10^{-16}$ s within a small material volume surrounding the ion trajectory. Subsequent energy and momentum transfer in the electron and ion subsystems may create a stable trail of damaged material generated along the ion trajectory (track). These tracks influence subsequent developments of the material defect ensemble and the physico-mechanical property changes of the irradiated materials. The anisotropic growth of the dimensions of amorphous materials under heavy-ion irradiation is also related to the track formation. Therefore soon after the discovery in 1958 of the formation of particle tracks in solids this research field rapidly grew up due to numerous applications in various scientific fields.[1,2]

The total energy loss of a fast charged particle in a solid includes the electronic energy loss, resulting from interaction of the particle with the electron subsystem of the irradiated solid, and the elastic energy loss, due to the elastic scattering of the particle on the material ion core. At the relevant particle energies ($E \sim 1$ MeV/amu), the electronic energy loss constitutes more than 90% of the total energy loss. It was long thought and supported by theoretical arguments[3-12] that particle tracks may be formed in bulk samples of insulating material but not in metals and good conductors. The principal argument was that electronic excitations would be so rapidly and efficiently shared among the continuum electronic states in the conduction band that these excitations would be neither spatially localized nor retained in sufficiently large numbers to produce atomic displacements.[13] The borderline between insulators and electrical conductors has been located at an electrical resistivity ρ of about 2×10^{3} Ω cm in bulk samples[1] and 1×10^{-3} Ω cm in thin films.[8] The experimental situation changed when a new generation of ion accelerators allowed experiments with ions of electronic energy loss much larger than that previously possible. A few years ago it became clear that particle tracks are formed also in metals and alloys with $\rho < 2\times10^{-4}$ Ω cm provided the electronic energy loss is sufficiently high.[14-18] A particularly interesting case is track formation in high-T_c superconductors ($\rho \simeq 10^{-4}$ Ω cm) because in these materials particle tracks are ideal pinning centers for flux lines.[19-21]

These new experimental facts stimulated additional theoretical investigations of the mechanisms which convert electronic excitation energy into atomic motion. Both the ion-explosion spike[22,23] and the thermal spike[24] mechanisms have been revised and modified to account also for the more recent observations. It has been argued[22,23] that the Coulomb repulsion in the ion-explosion spike generates a collective motion of atoms which finally results in atomic rearrangements in amorphous metals and crystalline metals with soft phonon modes. On the other hand, it has been argued[24] that even in metals the electron-phonon interaction might be strong enough to induce local melting if the electronic energy loss is sufficiently high. The model used in Ref. 25 to analyze energy dissipation near the penetrating ion trajectory considered the influence of a pressure gradient of the excited electron gas on the ions as an effect of some volume force. This influence was interpreted as an "instantaneous" blow of electron pressure to the ion subsystem, which may cause a shock wave near the ion trajectory.

However, the physical picture of the track formation in solids irradiated with high-energy heavy ions is now far from complete in spite of quite a number of models of this phenomenon. Moreover, up to now, there exists no treatment which includes the above-mentioned mechanisms at a comparative level. It is the purpose of this paper to make a first step in this direction. Besides that, we investigate the influence of the behavior of the viscosity of the material in the track region and surrounding matrix on the track formation ability.

We demonstrate that a possible plastic flow of the ma-

terial in the region near the particle trajectory is correlated with the momenta of the initial atoms, the initial atom displacements, and the ion heating resulting from the energy and momentum transfer from the excited electron subsystem to the ionic one. An investigation of this correlation allow us to estimate the validity of models such as the "thermal spike," the "Coulomb explosion," and the "electron blow" for a description of the material's plastic deformation in the track. Based on the elaborated model we obtain theoretical dependencies of the anisotropic growth rate of the dimensions of amorphous alloys irradiated with heavy ions on the electronic and ionic energy losses of the penetrating particle as well as on the irradiation temperature, which are in good agreement with those observed in the experiments.[26-31]

II. THE KINETICS OF MATERIAL PLASTIC DEFORMATION NEAR THE HEAVY-ION TRAJECTORY

The excited electrons near the heavy-ion trajectory are thermalized a short time ($t \lesssim 10^{-14}$ s) after the ion penetration.[3,4,7,24] In crystalline metals the energy of the thermalized electron subsystem is transferred to the ionic subsystem via the electron-ion interaction during the characteristic time $t_{ei}^{\Delta E} \sim 10^{-11}-10^{-12}$ s after the particle passes.[3,4,7,24,32] Thus, after a short time $t > t_{ei}^{\Delta E}$ the small region near the particle trajectory, l ($l \cong 10-100$ Å, where l is this region's radius) is characterized by a high temperature, a sharp temperature gradient, and a high defect concentration (due to elastic collision of the penetrating ion with the material's atoms). Moreover, these excited ions get initial displacements and initial momenta in the direction normal to that of the incident particle beam, due to interaction with the charged particle and excited electrons via Coulomb repulsion[6,18] and/or electronic blow.[25]

Any solid can be characterized by its effective viscosity. However, the physical mechanisms creating viscous flow of a material depend on the solid structure, topology, temperature, and loading conditions.

In materials with sharp temperature and defect-concentration dependencies of the viscosity (like amorphous alloys[33-36]), such fast temperature and defect-concentration increases lead to a drastic viscosity decrease during a short time $t \sim t_{ei}^{\Delta E}$ in a small material volume near the fast-ion trajectory. In this case the large temperature gradients, the initial atom displacements, and the initial atom momenta near the particle trajectory can induce plastic flow of the material in the direction normal to that of the incident particle beam. This flow is determined by the high ion temperature in the region near the particle trajectory and not by the irradiation temperature.

Thus in amorphous alloys a high plastic deformation can be left near the particle trajectory after cooling of the hot region (at times $t > t_c^i \sim l^2/4\chi_i \sim 10^{-10}-10^{-9}$ s, where χ_i is the ionic temperature conductivity coefficient). This region is not stable due to a compression which is induced by interaction with the surrounding matrix. Therefore after some time this expanded region should relax to the initial state. The characteristic time of this reverse relaxation is determined by the material viscosity at the irradiation temperature. Hence, the lower the irradiation temperature, the larger is the reverse relaxation time in the region where the process of plastic deformation occurred at the hot stage.

In order to describe the plastic deformation of amorphous alloys at times $t > t_{ei}^{\Delta E}$ in the region near the heavy-ion trajectory, we consider these materials as viscoelastic. We suppose also that at times $t_{ei}^{\Delta E} < t < t_c^i$ ("hot" stage) the plastic flow is characterized by the viscosity η_h averaged over the transient temperature T_h at the "hot" stage. Analogously, the reverse viscous flow in the second, "cold" stage at times $t > t_c^i$ is characterized by the effective viscosity η_c depending on the irradiation temperature $T_{\rm irr}$. Since $T_h \gg T_{\rm irr}$ it follows that $\eta_c \gg \eta_h$.

We also suppose that (a) all processes of energy and momentum transfer from the excited electron subsystem to the ionic one had finished during the time $t_{ei}^{\Delta E} \sim 10^{-12}$ s and had resulted in the initial ion displacements, the initial ion momenta, and the initial ion heating; (b) the initial ion temperature increase does not exceed the material melting temperature T_m. The last suggestion restricts the value of the electronic energy loss for our model.

We take the following dependence of the stress tensor for viscoelastic isotropic media with a nonuniform temperature field:[37]

$$\sigma_{ik} = -K\alpha T\delta_{ik} + Ku_{ll}\delta_{ik} + 2\mu\left[u_{ik} - u_{ll}\frac{\delta_{ik}}{3}\right] + \xi\dot{u}_{ll}\delta_{ik} + 2\eta\left[\dot{u}_{ik} - \dot{u}_{ll}\frac{\delta_{ik}}{3}\right], \quad (1)$$

$$u_{ik} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right].$$

Here K is the bulk modulus, μ is the shear modulus, ξ is the bulk viscosity, η is the shear viscosity, $T = T_i - T_{\rm irr}$, T_i is the ion temperature, α is the thermal expansion coefficient, u_{ik} is the strain tensor, $\mathbf{u}$ is the displacement vector in the deformed material, and δ_{ik} is the Kronecker symbol. The Einstein summation rule is assumed in Eq. (1).

Substituting the stress tensor σ_{ik} (1) into the equation of motion for a small material volume, we have

$$\rho(d^2\mathbf{u}_i/dt^2) = \partial\sigma_{ik}/\partial x_k ,$$

where ρ is the material density. Taking into account the shear viscous deformations only, we suppose that the material density during this deformation is constant ($\rho = \text{const}$, $\text{div}\mathbf{u} = 0$). Note that constant material density was observed in the experimental investigations of the anisotropic growth of amorphous materials during irradiation with high-energy heavy ions.[26-31]

Usually the track length is much longer than its radius; hence we can obtain the following equation for the radial

component of the displacement vector u_r in the cylindrical geometry:

$$\rho\ddot{u}_r=-K\alpha\frac{\partial T}{\partial r}+\mu\left[\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}\right]+\eta_h\left[\frac{\partial^2\dot{u}_r}{\partial r^2}+\frac{1}{r}\frac{\partial\dot{u}_r}{\partial r}-\frac{\dot{u}_r}{r^2}\right],\quad t_{ei}^{\Delta E}<t<t_c^i\ . \tag{2}$$

We take initial conditions for Eq. (2) in the form

$$u_r(r,0)=u_0 r\exp(-r^2/R_l^2)\ ,$$

$$\dot{u}_r(r,0)=\dot{u}_0 r\exp(-r^2/R_v^2)\ .$$

Here R_l and R_v are characteristic dimensions of the regions where the initial ion displacements and initial ion momenta appear, respectively; u_0 and $\dot{u}_0$ are constants which can be obtained, e.g., from the Coulomb explosion and the electron blow models.

Taking into account the cylindrical track geometry and the restriction of atom displacements at large distances from the track core, the boundary conditions for Eq. (2) can be written as

$$u_r(0,t)=0,\quad u_r(\infty,t)\text{ remains finite}\ .$$

Equation (2) is supplemented by the equation describing the ion temperature variations in the hot region near the particle track at the times $t_{ei}^{\Delta E}<t$,

$$\frac{\partial T}{\partial t}=\chi_i\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right], \tag{3}$$

with $T=T_i-T_{\text{irr}}$. The initial and boundary conditions for the ion temperature are chosen in the form

$$T(r,0)=T_0\exp(-r^2/R_T^2)\ ,$$

$$T(0,t),T(\infty,t)\text{ remain finite}\ .$$

Here T_0 is the initial ion temperature increase at the track axis, R_T is the effective radius of the hot region, and χ_i is the temperature conductivity of the material at high temperatures. The account of the influence of the high ion temperature in the track region allows us to estimate the applicability of the thermal spike model to investigation of the track formation.

The solution of Eq. (3) with corresponding initial and boundary conditions has the form

$$T(r,t)=\frac{T_0R_T^2}{(R_T^2+4\chi_i t)}\times\exp[-r^2/(R_T^2+4\chi_i t)]\ .$$

Solving Eq. (2) with account of the relation for $T(r,t)$, it is easy to obtain the general solution for the displacement vector component u_r in the inverse space of the Hankel transformation:[38]

$$H_m(f,s)=\int_0^\infty rf(r)J_m(sr)dr\ ,$$

$$f(r)=\int_0^\infty sH_m(f,s)J_m(sr)ds\ .$$

Here J_m is the Bessel function of order m.

For times $t_{ei}^{\Delta E}\ll t\sim t_c^i=4R_T^2/\chi_i$ (t_c^i is the characteristic time of cooling of the hot region) the backward transformation of the general solution is easily performed and gives at times $t_{ei}^{\Delta E}\ll t<t_c^i$

$$u_r(r,t)=\left\{u_0 r\exp\left[-\frac{r^2}{R_l^2}\right]+\dot{u}_0\frac{\rho R_v^4}{4\eta_h r}\left[1-\exp\left[-\frac{r^2}{R_v^2}\right]\right]\right\}+\frac{T_0K\alpha R_T^2}{2\eta_h}\frac{1}{r}\left[1-\exp\left[-\frac{r^2}{R_T^2}\right]\right]t\ . \tag{4}$$

The second stage of the track formation (cold stage) is characterized by viscoelastic flow at the constant irradiation temperature T_{irr} and the cold viscosity $\eta_c(\eta_c\gg\eta_h)$. The equation describing variation of the radial component of the displacement vector in this case is ($t>t_c^i$)

$$\rho\ddot{u}_r=\mu\left[\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}\right]+\eta_c\left[\frac{\partial^2\dot{u}_r}{\partial r^2}+\frac{1}{r}\frac{\partial\dot{u}_r}{\partial r}-\frac{\dot{u}_r}{r^2}\right]\ . \tag{5}$$

The initial and boundary conditions for Eq. (5) are defined by the values of the radial component of the hot displacement vector and its variation rate near the penetrating particle trajectory at the cooling time t_c^i:

$$u_r^c(r,0)=u_r^h(r,t_c^i),\quad \dot{u}_r^c(r,0)=0\ ,$$

$$u_r^c(0,t)=0,\quad u_r^c(\infty,t)\text{ remains finite}\ .$$

Here $u_r^h(r,t_c^i)$ is defined by the expression (4) at $t=t_c^i=4R_T^2/\chi_i$:

$$u_r^h(r,t_c)=u_0 r\exp\left[-\frac{r^2}{R_l^2}\right]+\dot{u}_0\frac{\rho R_v^4}{4\eta_{hr}}\left[1-\exp\left[-\frac{r^2}{R_v^2}\right]\right]+\frac{T_0K\alpha R_T^4}{8\chi_i\eta_h}\frac{1}{r}\left[1-\exp\left[-\frac{r^2}{R_T^2}\right]\right]\ .$$

Solving Eq. (5) and retaining the principal terms in the solution obtained, we get the expression for the displacement vector variations during the viscoelastic relaxation at the second stage of the track formation ($t>t_c^i$):

$$u_r(r,t)=\left\{u_0 r\exp\left[-\frac{r^2}{R_l^2}\right]+\dot{u}_0\frac{\rho R_v^4}{4\eta_h r}\left[1-\exp\left[-\frac{r^2}{R_v^2}\right]\right]+\frac{T_0}{8\chi_i}\frac{K\alpha R_T^4}{\eta_h}\frac{1}{r}\left[1-\exp\left[-\frac{r^2}{R_T^2}\right]\right]\right\}\exp(-\mu t/\eta_c)\ . \tag{6}$$

Without detailed analysis of the energy and momentum transfer from the electron subsystem to the ionic one, it is impossible to make an unequivocal statement about the mechanisms causing the plastic deformation: either the influence of the large initial ion velocities and displacements or the effect of the high temperature gradients in the region near the penetrating particle trajectory. But it is clear from Eq. (6) that a large difference between the material viscosities at the hot and cold stages as well as a possible viscosity decrease created by irradiation significantly facilitate the tract formation. It should be noted that the viscosity of amorphous solids can considerably decrease under irradiation.[39]

III. THE ANISOTROPIC GROWTH OF AMORPHOUS SOLIDS UNDER IRRADIATION WITH HIGH-ENERGY HEAVY IONS

When a beam of fast ions with a kinetic energy $E > 1$ MeV/amu penetrates amorphous samples with thicknesses much smaller than the projected ranges of the ions, a drastic anisotropy increase of sample dimensions in the direction normal to that of the incident particle beam takes place if the irradiation temperature is sufficiently low ($T_{\rm irr} < 100$ K).[26–31] This phenomenon has the following features.

(1) Large anisotropic variations of the sample dimensions are produced by irradiation. The anisotropy is induced by the incident particle beam (the growth direction is normal to the beam direction).

(2) These dimensional changes start from incubation dose B, reach 10%, and do not saturate with irradiation dose.

(3) The effect strongly depends on the irradiation temperature.

(4) The effect is observed both in amorphous metallic alloys and in covalently bonded amorphous solids.

(5) A correlation between the magnitude of the effect and the electronic energy loss $\langle S_e \rangle$ has been observed. In amorphous alloys this dependence is nonlinear at low $\langle S_e \rangle$ ($\langle S_e \rangle \lesssim 20$ KeV/nm for amorphous $Pd_{80}Si_{20}$) and degenerates to the linear one at higher electronic energy loss ($\langle S_e \rangle \gtrsim 20$ keV/nm).

In order to describe the total material deformation as the sum of the deformations from all beam tracks, let us introduce $\mathbf{u}(\mathbf{r}-\mathbf{r}';t-t')$ as the displacement of the amorphous material at an arbitrary depth z ($-0.5h < z < 0.5h$) in a plane point $\mathbf{r}$ at time t resulting from the heavy-ion track created in a point $\mathbf{r}'$ at time t'. The total displacement $\mathbf{U}(r,t)$ in the point $\mathbf{r}$ at time t in the irradiated material is defined as

$$\mathbf{U}(r,t)=\int_S dS' \int_0^t dt' \mathbf{u}(\mathbf{r}-\mathbf{r}';t-t')j(\mathbf{r}',t') \ . \tag{7}$$

Here S is the sample surface area and $j(\mathbf{r}',t')$ is the particle flux in the point $\mathbf{r}'$ at the time t'.

The total strain tensor ε_{ik} results from all beam track contributions and is defined in the usual way:

$$\varepsilon_{ik}=\int_S dS' \int_0^t dt' u_{ik}(\mathbf{r}-\mathbf{r}';t-t')j(\mathbf{r}',t') = \frac{1}{2}\left[\frac{\partial U_i}{\partial x_k}+\frac{\partial U_k}{\partial x_i}\right] .$$

It has been suggested[33–36] that the temperature dependence of the metal glass viscosity can be taken in the form

$$\eta^{-1}=(\eta_0 T_i)^{-1}\exp(-G/T_i) \ . \tag{8}$$

Here $\eta_0=(\gamma^2\Omega n\nu)^{-1}$; n is the atomic concentration of the flow defects, Ω is the flow-defect volume; ν is the frequency of possible defect jumps, T_i is the ion temperature, and γ is the strain per one defect jump.

High-energy heavy-ion irradiation significantly influences the flow-defect concentration due to creation of new displaced atoms. Moreover, new low-activation-energy channels of plastic flow are perhaps opened by irradiation in the excited material region near to the penetrating particle trajectory.[39] As local material excitations disappear with time, these low-energy channels of plastic flow should degenerate when the irradiation is over.

Under this assumption the plastic deformation of the irradiated amorphous alloy results from a superposition of different plastic flow channels:

$$\eta^{-1}=\sum_k \eta_k^{-1}=\sum_k \gamma_k^2\Omega_k n_k \nu_k T_i^{-1}\exp(-G_k/T_i) \ . \tag{9}$$

Here n_k is the concentration of defects supporting the k channel of plastic flow; γ_k, Ω_k, ν_k, and G_k are this defect strain per jump, volume, jump frequency, and migration barrier, respectively. Naturally, the lowest migration barrier defines the main channel of plastic flow.

The unstable material state near the penetrating ion trajectory should relax with time. Hence the high concentration of flow defects continuously decreases in this region, leading to a sharp increase of the effective viscosity of the amorphous metal. Hence after some characteristic time τ_v fast plastic flow of the excited material becomes impossible.

Experimental investigation of the influence of structural relaxation on the viscosity of the amorphous alloy $Pd_{77.5}Cu_6Si_{16.5}$ has shown that below T_g ($T < T_g$) the viscosity increase due to the flow-defect concentration decrease can be described by the "bimolecular" equation for the flow-defect concentration[34]

$$\frac{dn}{dt}=-\frac{n^2}{\tau_0} \ , \tag{10}$$

where τ_0 is the characteristic time of flow-defect recombination. We suppose that a fast channel of the material viscous flow opens during the incubation dose B when a threshold concentration of flow defects is created in the sample due to atomic displacement caused by the elastic collisions of the penetrating ion with atoms of the material. Hence we take the following initial condition for Eq. (10):

$$n(t=0)=n_0=\langle P \rangle B \ , \tag{11}$$

where

$$\langle P\rangle=\int_{E_d}^{E_m}\nu(e)\frac{d\sigma}{dE}dE\ ,$$

$\nu(E)$ is the cascade function, $(d\sigma/dE)(E',E)$ is the differential cross section of energy transfer E from a penetrating particle with the initial energy E' to a target atom, E_m is the maximum transfer energy, and E_d is the threshold energy for the atom displacement. Solving Eq. (10) with initial condition (11) we obtain

$$n=n_0(1+t/\tau_n)^{-1}\ , \tag{12}$$

where $\tau_n=\tau_0/n_0$ is the characteristic time for the flow-defect concentration decrease.

For the characteristic value $n_0\sim10^{-4}$,[31] and for the smallest possible value of the characteristic time of the flow defect recombination, $\tau_0\sim10^{-12}$ s, the defect concentration remains constant ($n=\langle P\rangle B$) during the time $t<(\tau_0/B\langle P\rangle)\simeq10^{-8}$ s. Hence, during the hot stage of track formation $t<t_c^i\le10^{-9}$ s, the flow-defect concentration does not change ($n=\langle P\rangle B=n_0$).

Introducing Eq. (12) into Eqs. (6)–(8) we obtain the space and time distribution of the displacement field for constant penetrating ion flux ($j=$const, $t>t_c^i$, $t\gg\tau_n$):

$$\mathbf{U}=jt\Psi(\mathbf{r})\exp\left[-\frac{\mu\Omega\nu\gamma^2}{T_{\rm irr}}\tau_0\exp(G/T)\right]\ , \tag{13}$$

where

$$\Psi(\mathbf{r})=\int_S dS'\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}\Phi(|\mathbf{r}-\mathbf{r}'|)\ ,$$

$$\Phi(r)=u_0 r\exp(-r^2/R_l^2)+\dot{u}_0\frac{\rho R_v^4}{4\eta_{hr}}r\left[1-\exp\left[-\frac{r^2}{R_v^2}\right]\right]+\frac{T_0K\alpha R_T^4}{8\chi_i\eta_h}\frac{1}{r}\left[1-\exp\left[-\frac{r^2}{R_T^2}\right]\right]\ .$$

Irradiation-induced variation Δb of the sample dimension along the symmetry axis OX is defined by ($t>t_c^i$, $t\gg\tau_n$, $r=b/2$)

$$\Delta b=2jt\psi_x(b/2)\exp\left[-\frac{\mu\tau_0}{T_{\rm irr}\eta_{0c}}\right]\ , \tag{14}$$

where

$$\eta_{0c}=\exp(G/T)/\Omega\nu\gamma^2\ .$$

Here b is the sample dimension perpendicular to the beam direction and Δb is the increase of this dimension during irradiation.

The experimental dependence of $\Delta b/b$ is described by the following expression:

$$\frac{\Delta b}{b}=A(jt-B)\quad(jt>B)\ , \tag{15}$$

where A is the irradiation growth rate

$$A=j^{-1}\frac{d}{dt}\left[\frac{\Delta b}{b}\right]\ . \tag{16}$$

Using Eqs. (14) and (16), we obtain that A is described by ($t\gg\tau_n$)

$$A=(2/b)\psi_x(b/2)\exp\left[-\frac{\mu\tau_0}{T_{\rm irr}\eta_{0c}}\right]\ . \tag{17}$$

Substituting (13) into (17) and shifting the center of the polar coordinate system to the point $(b/2,0)$ we obtain for $t\gg\tau_n$

$$A=\left\{\frac{\dot{u}_0\rho R_v^2}{2\eta_h}+\frac{T_0K\alpha R_T^4}{2\chi_i\eta_h}\right\}\exp[-(\mu\tau_0/\eta_{0c})]+\frac{\sqrt{\pi}R_l}{b}\left\{4u_0R_l^2-\frac{\dot{u}_0\rho R_v^3}{2\eta_hR_l}-\frac{T_0K\alpha R_T^5}{2\chi_i\eta_hR_l}\right\}\times\exp[-(\mu\tau_0/\eta_{0c})]\ . \tag{18}$$

The second term in (18) is of the order of $(R_l/b)<10^{-2}$ ($R_T\simeq R_v\simeq R_l$); thus it can be neglected and hence the term resulting from the initial atom displacements near the penetrating particle trajectory ($\sim u_0R_l^2$) is negligible. For $t\gg\tau_n$ the irradiation growth rate of amorphous alloys has the form

$$A=\left\{\frac{u_0\rho R_v^2}{2\eta_h}+\frac{T_0K\alpha R_T^4}{2\chi_i\eta_h}\right\}\exp[-(\mu\tau_0/\eta_{0c})]\ . \tag{19}$$

IV. THE TEMPERATURE DEPENDENCE OF THE IRRADIATION GROWTH RATE OF AMORPHOUS SOLIDS

The irradiation temperature dependence of $A(T_{\rm irr})$ is given by

$$A(T_{\rm irr})\propto\eta_h^{-1}(T_{\rm irr}+T_0)\exp\{-[\mu\tau_0/\eta_{0c}(T_{\rm irr})]\}\ . \tag{20}$$

Retaining the main term we obtain

$$\frac{A}{A_{\max}}\propto\exp\{-[\mu\tau_0/\eta_{0c}(T_{\rm irr})]\}=\exp\left[-\frac{T^*}{T_{\rm irr}}\exp(-G/T_{\rm irr})\right]\ , \tag{21}$$

where $A_{\max}=A(T_{\rm irr}=0)$ and $T^*=\mu\tau_0\Omega\nu\gamma^2$.

If $T_{\rm irr}\to0$ then $\exp(-G/T_{\rm irr})\to0$. The relaxation of the deformations is suppressed and the effect of anisotropic growth is at maximum. When the irradiation temperature increases the function

$$\exp\left[-\frac{T^*}{T_{\rm irr}}\exp(-G/T_{\rm irr})\right]$$

sharply decreases and the effect vanishes. For parameter values $T^*=4024$ K and $G=530$ K the theoretical curve for the temperature dependence of the irradiation growth rate is in good agreement with the experimental one obtain for $Pd_{80}Si_{20}$ metallic glass[31] (see Fig. 1).

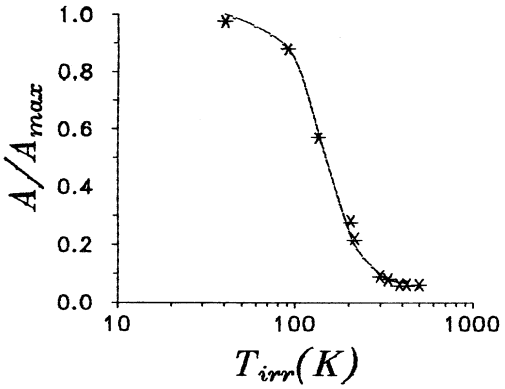

FIG. 1. Experimental (Ref. 31) (*) and theoretical (——) dependencies of the deformation rate of $Pd_{80}Si_{20}$ versus irradiation temperature. The normalization value is $A_{\max}=5.5\times10^{-15}$ cm^2 for $Pd_{80}Si_{20}$.

V. THE DEPENDENCE OF IRRADIATION GROWTH RATE ON THE ELECTRONIC ENERGY LOSS

We assume that the effect is determined by the initial temperature increase only, whereas the influence of the initial atom momenta on the irradiation growth is neglected:

$$(T_0 K\alpha R_T^4/\chi_i)\gg \dot{u}_0\rho R_v^2 .$$

In this case the electronic energy loss enters into Eq. (19) through the initial ion temperature.

The temperature dependence of the ionic temperature conductivity (χ_i) is[40]

$$\chi_i=\frac{\Lambda_i c}{3} . \tag{22}$$

Here Λ_i is the mean free path of phonons, $\Lambda_i=a(20/\Gamma_i^2)(T_m/T_i)$, T_m is the melting temperature, Γ_i is the ionic Grüneisen coefficient ($\Gamma_i\simeq2$), a is the interatomic distance, and c is the sound velocity.

Supposing that the effective viscosity at the hot stage of the track formation is defined by the initial ion temperature in the track axis and substituting (22) and (8) into (19), we obtain ($t\gg\tau_n$)

$$\frac{A}{\langle P\rangle}=A_p\frac{R_T^4 T_0}{T_m}\exp[-G/(T_{\rm irr}+T_0)] ,$$
$$A_p=\frac{3K\alpha\nu\Omega\gamma^2 B}{10ac}\exp\left[-\frac{T^*}{T_{\rm irr}}\exp(-G/T_{\rm irr})\right] . \tag{23}$$

We suppose that T_m does not depend on temperatures T_0 and T_e and, hence, on the electronic energy loss S_e. The value R_T^2 defines the region of initial ion heating at the characteristic time of the energy transfer from the excited electron subsystem to the ionic one $t_{ei}^{\Delta E}$:

$$R_T^2=R_0^2+4\chi_e t_{ei}^{\Delta E}\cong 4\chi_e t_{ei}^{\Delta E} , \tag{24}$$

where R_0^2 is the dimension of the region of the initial electron excitations and $\chi_e\gg\chi_i$, $\chi_e t_{ei}^{\Delta E}\gg R_0^2$.

Taking into account the Gaussian form of the initial ion temperature and a constant value of the high-temperature limit of the ion specific heat ($c=3n_{\rm at}$ for $T_i>h\nu_D$, where ν_D is the Debye frequency and $n_{\rm at}$ the atomic density), we obtain the following expression for the ion temperature increase at the track axis:

$$T_0=\frac{\langle S_e\rangle}{3\pi n_{\rm at}R_T^2} . \tag{25}$$

For high temperatures the characteristic time of the energy transfer from the excited electron subsystem to the ionic one, $t_{ei}^{\Delta E}$, has the form[3–5]

$$t_{ei}^{\Delta E}\propto(T_e-T_i)^{-1}\sim T_e^{-1},\quad T_e\gg T_i . \tag{26}$$

The dependence of the electron temperature conductivity on the electron temperature $\chi_e(T_e)$ is nonmonotonic.[41,42] This dependence has a deep flat minimum ($\chi_e\sim1$ cm^2/s) at temperatures $\varepsilon_f<T_e<10\varepsilon_j$ (where ε_j is the Fermi energy) and χ_e increases practically linearly for higher T_e (see Fig. 2).

Taking into account this behavior of χ_e we will investigate the following limiting cases.

(1) High electron temperatures

$$T_e\gg\varepsilon_j,\quad \chi_e\propto T_e .$$

In this case we have from Eqs. (24) and (26)

$$R_T^2=4\chi_e t_{ei}^{\Delta E}=\text{const},\quad \rightarrow T_0\propto S_e \tag{27}$$

and the irradiation growth rate is defined by

$$\frac{A}{\langle P\rangle}=A_e S_e\exp[-G_n/(T_n+S_e)] ,$$
$$A_e=\frac{K\alpha R_T^2\nu\Omega^2\gamma^2}{10\pi acT_m S^*}\exp\left[-\frac{T^*}{T_{\rm irr}}\exp(-G/T_{\rm irr})\right] , \tag{28}$$
$$G_n=G(3\pi nR_T^2),\quad T_n=T_{\rm irr}(3\pi nR_T^2) .$$

According to the experimental results for the $Pd_{80}Si_{20}$ amorphous alloy,[31]

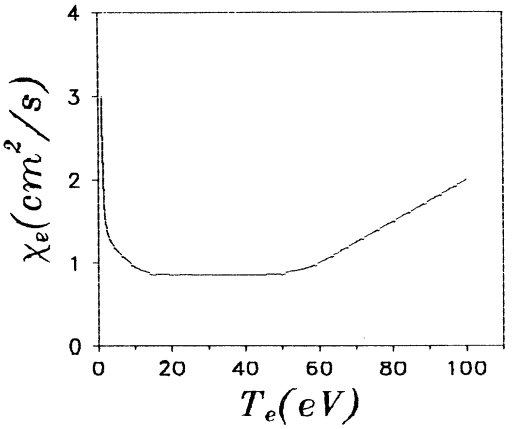

FIG. 2. Electron temperature conductivity as a function of the electron temperature $\chi_e(T_e)$ (Ref. 42).

$$\left[\frac{A}{\langle P\rangle}\right]_{\text{expt}} = 45(S_e - 2.45) , \tag{29}$$

where S_e is measured in keV/Å.

For large S_e ($S_e \gg T_n$, $S_e > G_n$) the theoretical dependence (28) is also linear:

$$\frac{A}{\langle P\rangle} = A_e(S_e - G_n) . \tag{30}$$

Hence, using the experimental values for A_e and G_n ($A_e = 45$, $G_n = 2.45$ keV/Å), we can estimate the parameter R_T. The value R_T is defined by

$$R_T = \left[\frac{G_n}{3\pi n G}\right]^{1/2} . \tag{31}$$

Substituting in (31) the values of $G = 530$ K and $G_n = 2.45$ keV/Å as characteristic for the $Pd_{80}Si_{20}$ amorphous alloy, we obtain

$$R_T \simeq 3.5 \times 10^{-6} \text{ cm} .$$

This value of R_T coincides with its estimation $R_T^2 = 4\chi_e t_{ei}^{\Delta E}$ for $t_{ei}^{\Delta E} = 3.0 \times 10^{-12}$ s ($\chi_e = 1$ cm^2/s). Using these parameter values we present the comparison between the theoretical dependence (30) and the experimental results for the $Pd_{80}Si_{20}$ amorphous alloy in Fig. 3.

(2) For $\varepsilon_j < T_e < 10\varepsilon_j$, $\chi_e = \text{const}$ and from Eqs. (24) and (25) we obtain

$$R_T^2 \propto T_e^{-1}, \quad \Longrightarrow T_0 \propto S_e T_e . \tag{32}$$

As the atom ionization time ($t_{\text{ion}} \sim 10^{-16}$ s) is much smaller than the characteristic time of electron thermalization ($t_e^T \sim 10^{-14}$ s), we investigate the ionization equilibrium between the excited electron plasma in the valence zone of the material and localized electrons in the ion shells to obtain the electron temperature dependence on the electronic energy loss.

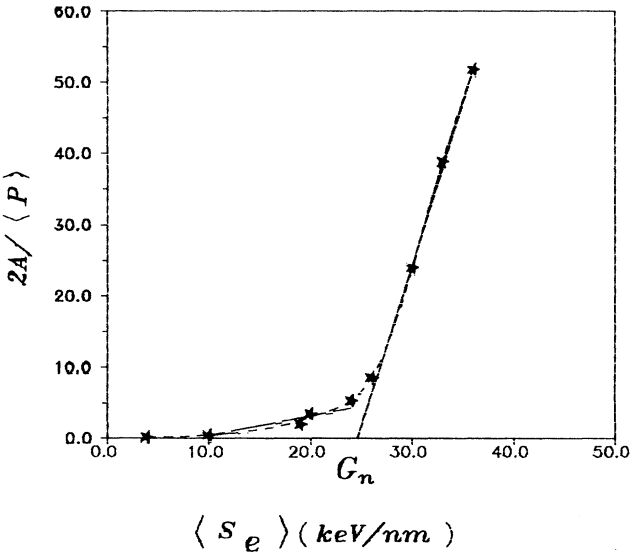

FIG. 3. Experimental (Ref. 31) (*) and theoretical dependencies (——) of the deformation rate of $Pd_{80}Si_{20}$ normalized to the total displacement cross section $\langle P\rangle$ as a function of the electron energy loss $\langle S_e\rangle$ (Ref. 31).

For the expected temperatures ($T_e > \varepsilon_j$) the electron plasma of the valence zone is described as a Boltzmann electron gas with weak interaction between particles ($n_e\lambda^3 \ll 1$, where λ is the de Broglie electron wavelength, $n_e l_c^3 \ll 1$, $l_c = e^2/T_e$, and n_e is the electron density in the valence zone).[43] We take the following form for the Gibbs energy of the system of localized and delocalized electrons:

$$G = G_e^{\text{id}} + G_e^{\text{int}} + G_{\text{sh}}$$
$$= N_e\mu_e^{\text{id}} - \frac{e^2 N_e}{2r_D} + \sum_{i=1}^{z} N_i\mu_i^0 - T_e \ln\frac{N_{\text{at}}!}{\prod_{i=1}^{z} N_i!} , \tag{33}$$

where G_e^{id} is the Gibbs energy of the ideal electron gas, G_e^{int} is the Gibbs energy of the interaction between electrons in the electron gas, G_{sh} is the Gibbs energy of electrons in the ion shells, N_{at} is the total number of atoms, N_e is the total number of electrons in the valance zone, and N_i is the number of i-ionized atoms, z is the maximum ion charge, and μ_e^{id} is the electron chemical potential in the ideal electron gas.

$$\mu_e^{\text{id}} = T_e \ln(N_e/V) - \tfrac{3}{2}T_e \ln T_e - T_e \ln\left[2\left[\frac{m^*}{2\pi\hbar}\right]^{3/2}\right] ,$$

where V is the system volume, $r_D = \{T_e/[4\pi(N_e/V)e^2]\}$ is the screening radius,

$$\mu_k^0 = -T_e \ln\left[\sum_s g_s \exp(-E_s/T_e)\right]$$

is the chemical potential of the k-ionized isolated shells (k ion), E_s are the electron energy levels of this k ion, and g_s is the degeneracy of these levels.

Using the equation of ionization equilibrium[43,44]

$$\mu_i = \mu_{i+1} + \mu_e , \tag{34}$$

we obtain

$$\frac{C_i}{C_{i+1}C_e} = (\mathfrak{T}_0/T_e)^{3/2} \exp\left\{\frac{\mu_{i+1}^0 + \mu_i^0}{T_e} - \frac{e^2}{2T_e r_D}\right\} ,$$
$$\mathfrak{T}_0 = \frac{2\pi\hbar^2}{2^{2/3} m^* v^{2/3}} . \tag{35}$$

Here $C_i = N_i/N_{\text{at}}$, $C_e = N_e/N_{\text{at}}$, and m^* is the effective electron mass in the valence zone.

Summing in (35) over i, using the equation of particle number conservation and supposing that $\mu_{i+1}^0 - \mu_i^0 = J = \text{const}$, we obtain

(a) for $T_e > J$ ($C_e \propto T_e^{3/2}$)

$$T_e \propto (S_e)^{2/5} ; \tag{36}$$

(b) in the opposite case $T_e \lesssim J$ ($C_e \propto T_e$)

$$T_e \propto (S_e)^{1/2} . \tag{37}$$

Thus combining Eqs. (36) and (37) we obtain that in the broad region of S_e variation the dependence of the electron temperature on the inelastic energy loss of the penetrating particle has the form

$$T_e \simeq (S_e)^{1/2} K \ , \tag{38}$$

where K does not depend on the electronic energy loss.

Substituting Eqs. (32) and (38) into Eq. (23), we obtain the following expression for the irradiation growth rate for this electron temperature region $\varepsilon_j < T_e < 10\varepsilon_j$ ($\chi_e = \text{const}$):

$$\frac{A}{\langle P \rangle} \sim S_e^{1/2} \exp\left[-G/(T_{\text{irr}} + K S_e^{3/2}) \right] \ . \tag{39}$$

The function

$$\frac{A}{\langle P \rangle} = 6.89 S_e^{1/2} \exp[-G/(T_{\text{irr}} + 141.14 S_e^{3/2})] \tag{40}$$

gives the best fitting of the experimental data[31] for $Pd_{80}Si_{20}$ for $1 \le S_e \le 2.4$ keV/Å (see Fig. 3).

In the high ion temperature limit, when $T_i \gtrsim T_m$, the plastic deformation is defined by the melted region that can exist close to the moving ion trajectory. In such a region, the material viscosity η_h and the defect concentration n_j correspond to their characteristic values for the melted material and do not depend on the elastic energy loss of the penetrating particle. Therefore no dependence of the deformation near the trajectory of the penetrating heavy ion on the elastic energy loss $\langle S_n \rangle$ is manifested in this case.

The radius R_m of the cylindrical melted region can be estimated with the help of the expression

$$R_m^2 = \frac{\langle S_e \rangle - \langle S_e \rangle_m}{\pi(3nT_m + Q)} \ , \tag{41}$$

where Q is the specific heat of melting of the material and

$$\langle S_e \rangle_m = 3\pi n_{\text{at}} R_T^2 T_m \ .$$

Supposing that in this case the rate of the anisotropic growth is proportional to the total melted material volume

$$\dot{\varepsilon} \sim R_m^2 \ ,$$

we obtain a linear dependence of this value on the electronic energy loss. Note that such behavior is characteristic for irradiated amorphous SiO_3.[28,29]

This reasoning is insensitive to the type of solid and depends only on the correlation between T_i and T_m in the small hot region near the particle trajectory.

VI. CONCLUSION

(1) A theoretical model of plastic flow of materials irradiated with high-energy heavy ions is presented. This model is based on the viscoelastic behavior of the material and a drastic viscosity decrease in the small material volume near the ion trajectory for a short time after the ion penetration.

(2) The model takes into account the effect of initial atom displacements, and momenta and initial ion temperature gradients arising due to interaction of the excited electron subsystem with the ionic one in the small region near the particle trajectory on the material plastic flow in the track.

(3) The theoretical model developed of the plastic flow of the material in the tracks has been applied for investigation of anisotropic irradiation growth of amorphous solids irradiated with high-energy heavy ions. This growth was described as resulting from superposition of the material plastic flow in the tracks. The obtained theoretical dependencies of the anisotropic growth rate of amorphous alloys on the electron energy loss $\langle S_e \rangle$ and elastic energy loss $\langle P \rangle$ of the penetrating heavy particle, as well as on the irradiation temperature, are in good agreement with experimental results.

[1] R. L. Fleischer, P. B. Price, and R. M. Walker, *Nuclear Tracks in Solids* (University of California Press, Los Angeles, 1975).

[2] R. Spohr, *Ion Tracks and Microtechnology* (Vieweg, Braunschweig, 1990).

[3] V. L. Ginzburg and V. P. Shabanskiy, Dokl. Akad. Nauk. SSSR **100**, 445 (1955).

[4] M. I. Kaganov, I. M. Lifshitz, and L. V. Tanatarov, Zh. Eksp. Teor. Fiz. **31**, 232 (1956) [Sov. Phys. JETP **4**, 173 (1957)].

[5] J. E. Geguzin, M. I. Kaganov, and I. M. Lifshitz, Fiz. Tverd. Tela **15**, 2425 (1973) [Sov. Phys. Solid State **15**, 1612 (1973)].

[6] R. L. Fleischer, P. B. Price, and R. M. Walker, J. Appl. Phys. **36**, 3645 (1965).

[7] M. I. Kaganov, I. M. Lifshitz, and L. V. Tanatarov, Reactor Sci. **12**, 69 (1960).

[8] L. T. Chadderton and I. McC. Torrens, *Fission Damage in Solids* (Methuen, London, 1969).

[9] R. H. Ritchie and C. Claussen, Nucl. Instrum. Methods Phys. Res. **198**, 133 (1982).

[10] C. C. Watson and T. A. Tombrello, Radiat. Eff. **89**, 263 (1985).

[11] T. A. Tombrello, Nucl. Instrum. Methods Phys. Res. Sect. B **2**, 555 (1984).

[12] N. Ifohand and T. Nakayama, Nucl. Instrum. Methods Phys. Res. Sect. B **13**, 550 (1986).

[13] W. L. Bronson, in *Beam-Solid Interactions and Phase Transformations*, edited by H. Kurz, G. L. Olson, and J. M. Poate, MRS Symposia Proceedings No. 51 (Materials Research Society, Pittsburgh, 1986), p. 53.

[14] A. Barbu, A. Dunlop, D. Lessueur, and R. S. Averback, Europhys. Lett. **15**, 37 (1991).

[15] A. Barbu, A. Dunlop, D. Lessueur, R. S. Averback, R. Spohr, and J. Vetter, Nucl. Track Radiat. Meas. **19**, 35 (1991).

[16] J. Henry, A. Barbu, B. Leridon, D. Lessueur, and A. Dunlop, Nucl. Instrum. Methods Phys. Res. Sect. B **67**, 390 (1992).

[17] C. Trautmann, S. Andler, W. Bruchle, R. Spohr, and M. Toulemonde, Radiat. Eff. Defects Solids **126**, 207 (1993).

[18] A. Dunlop and D. Lessueur, Radiat. Eff. Defects Solids **126**, 123 (1993).

[19] B. Hensel, B. Roas, S. Henke, R. Hopfingurtner, M. Lippest,

J. P. Strobel, M. Vildic, G. Saemann-Ischenko, and S. Klaumunzer, Phys. Rev. B **42**, 4135 (1990).
[20]L. Krusin-Elbaum, Y. Sun, J. R. Clem, F. Holtzberg, L. Civale, A. D. Marwick, T. K. Worthington, M. A. Kirk, and J. R. Thompson, Phys. Rev. Lett. **67**, 648 (1991).
[21]W. Gerhauser, G. Ries, H. W. Neumuller, W. Schmidt, O. Eibl, G. Saemann-Ischenko, and S. Klaumünzer, Phys. Rev. Lett. **68**, 879 (1992).
[22]S. Klaumünzer, Ming-dong Hou, and G. Schumacher, Phys. Rev. Lett. **57**, 850 (1986).
[23]D. Lessueur and A. Dunlop, Radiat. Eff. Defects Solids **126**, 163 (1993).
[24]M. Toulemonde, C. Dufour, and E. Paumier, Phys. Rev. B **46**, 14 362 (1992).
[25]I. P. Borin, Fiz. Tverd. Tela **20**, 2222 (1978) [Sov. Phys. Solid State **20**, 1284 (1978)].
[26]S. Klaumünzer, Chagrin Li, and G. Schumacher, Appl. Phys. Lett. **51**, 97 (1987).
[27]S. Klaumünzer *et al.*, Radiat. Eff. Defects Solids **108**, 131 (1989).
[28]S. Klaumünzer *et al.*, Nucl. Instrum. Methods Phys. Res. Sect. B **39**, 665 (1989).
[29]Ming-dong Hou, S. Klaumünzer, G. Schumacher, Nucl. Instrum. Methods Phys. Res. Sect. B **19/20**, 16 (1987).
[30]Ming-dong Hou, S. Klaumünzer, and G. Schumacher, Phys. Rev. B **41**, 1144 (1990).
[31]S. Klaumünzer, Int. J. Radiat. Appl. Instrum. D **19**, 91 (1991).
[32]F. F. Komarov, A. P. Novikov, and An-Tuan Khoang, Vestsi Akad. Nauk. BSSR (5), 81 (1987).
[33]F. Spaepen, Acta Metall. **26**, 407 (1977).
[34]S. S. Tsao and F. Spaepen, Acta Metall. **27**, 881 (1985).
[35]Y. Limoge and G. Brebec, Acta Metall. **36**, 665 (1988).
[36]Yv. Limoge, in *Diffusion in Materials,* edited by A. L. Lascar *et al.* (Kluwer, Dordrecht, 1990).
[37]L. D. Landau and E. M. Lifshitz, *Theory of Elasticity* (Nauka, Moscow, 1965).
[38]M. A. Lavrentiev and B. V. Shabat, *Theoretical Methods in Function of the Complex Variations* (Nauka, Moscow, 1965).
[39]A. Barbu, M. Bifole, R. Le Hazif, S. Bouffard, and J. C. Ranillon, J. Nucl. Mater. **165**, 217 (1989).
[40]J. M. Ziman, *Principles of the Theory of Solids* (Cambridge University Press, Cambridge, England, 1972).
[41]I. A. Baranov, Yu. V. Martynenko, S. O. Tchepelevich, and Yu. N. Yavlinskiy, Usp. Fiz. Nauk **156**, 178 (1988) [Sov. Phys. Usp. **31**, 880 (1988)].
[42]Yu. V. Martynenko and Yu. N. Yavlinskiy, Dokl. Akad. Nauk SSSR **270**, 88 (1983) [Sov. Phys. Dokl. **28**, 391 (1983)].
[43]W. Ebeling, W. D. Kraeft, and D. Kremp, *Theory of Bound States and Ionization Equilibrium in Plasmas and Solids,* Ergenbinsse der Plasmaphysik und der Gaseelectronic Vol. 5 (Akademie-Verlag, Berlin, 1976).
[44]L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Nauka, Moscow, 1976).