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Additive quasiparticle and vortex Hall conductivities in $La_{2-x}Sr_xCuO_4$ and untwinned $YBa_2Cu_3O_{6.93}$

J. M. Harris, N. P. Ong, and P. Matl
Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08544

R. Gagnon and L. Taillefer
Department of Physics, McGill University, 3600 University Street, Montreal, Quebec, Canada H3A 2T8

T. Kimura and K. Kitazawa
Department of Applied Chemistry, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113, Japan

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The flux-flow Hall conductivity σ_{xy} in untwinned $YBa_2Cu_3O_{6.93}$ (YBCO) is determined from Hall and resistivity measurements with the current applied along the a and the b axes. We show that sign reversal results from competition between a positive quasiparticle current ($\sim H$) and a negative vortex-motion term ($\sim 1/H$). The Hall drag coefficient in YBCO is measured to be $\alpha(T) = -4.4\times 10^{-8}(1-T/T_c)$ N s/m^2 ($75<T<93$ K). In single-crystal $La_{2-x}Sr_xCuO_4$, a similar decomposition into quasiparticle and vortex currents (both positive) is possible even though σ_{xy} shows no sign reversal.

The behavior of the flux-flow Hall effect in high-T_c superconductors is a challenging problem in the rich phenomenology of vortex dynamics in type-II superconductors. Of particular interest is the sign reversal observed in the Hall resistivity ρ_{yx} of most of the cuprates.[1,2] Many models have been proposed to explain this anomaly.[3–6] However, until recently, the crucial contribution of quasiparticle excitations to the observed current has not been appreciated.

Dorsey[5] and Kopnin, Ivlev, and Kalatsky[6] independently proposed that the sign reversal could arise if the quasiparticle and vortex Hall currents have opposite signs. Because of the additivity of the two currents, the simplest way to express the quasiparticle contribution is by the conductivity, viz.,

$$\sigma_{xy} = \sigma_{xy}^n + \sigma_{xy}^f , \tag{1}$$

where σ_{xy} is the total conductivity and σ_{xy}^n (σ_{xy}^f) is the Hall conductivity of the quasiparticles (vortices). Following a suggestion by Geshkenbein and Larkin (GL),[7] Harris, Ong, and Yan (HOY)[8] extracted the Hall conductivity of $YBa_2Cu_3O_{6.93}$ (YBCO) measured in oblique fields and found striking agreement with the scaling relationship of GL. Further, HOY found that the vortex term σ_{xy}^f is negative at all fields and tilt angles. The additivity in Eq. (1) has also been tested by Samoilov, Ivanov, and Johansson[9] in $Tl_2Ba_2CaCu_2O_{8+\delta}$, and by Ginsberg and Manson[10] on untwinned YBCO crystals. Harris *et al.*[11] reported that, in high-purity 60-K YBCO crystals, σ_{xy} is negative below 40 K at all fields H up to ~ 24 T with a field dependence that approaches $-1/H$ at high fields. These experiments show that the sign reversal is a direct consequence of adding a positive term that increases with H to a negative term that varies as $-1/H$. A different analysis, based on additivity of the Hall angles, has been proposed by Kunchur *et al.*[12]

We report Hall measurements on two untwinned crystals of YBCO and a crystal of $La_{2-x}Sr_xCuO_4$ (LSCO). In YBCO, the ratio of the resistivities ρ_a and ρ_b, measured with $\mathbf{J}\|\mathbf{a}$ and $\mathbf{J}\|\mathbf{b}$, respectively, varies strongly with H and T in the mixed state. ($\mathbf{J}$ is the current density; $\mathbf{H}\|\mathbf{c}$ in all measurements.) To determine the Hall conductivity σ_{ab}, it is necessary to measure *both* ρ_a and ρ_b. We selected two untwinned, optimally doped $YBa_2Cu_3O_{6.93}$ crystals (A and B) with closely similar electrical properties (caption of Fig. 1). The Hall resistivity ρ_{ba} (ρ_{ab}) and the resistivity ρ_a (ρ_b) were measured simultaneously in crystal A (B). The four quantities allow the Hall conductivity $\sigma_{xy}\equiv\sigma_{ab}$ to be calculated as $\rho_{ba}/[\rho_a\rho_b-\rho_{ab}\rho_{ba}]$. We also checked that the Onsager relation $\rho_{ba}=-\rho_{ab}$ is accurately satisfied. Details of the crystal growth and detwinning are given elsewhere.[13] The $La_{2-x}Sr_xCuO_4$ ($x=0.17$) sample was cut from a large crystal grown using the traveling-solvent-floating-zone method. In LSCO, sign reversal in the Hall resistivity is absent, indicating that σ_{xy}^n and σ_{xy}^f are both positive.

We first discuss the anisotropy ρ_b/ρ_a in YBCO. As reported by Safar *et al.* and by Kwok *et al.*,[14] the transition to the dissipative state is abrupt in untwinned crystals. In our crystals, the transition width is less than 0.2 K below 5 T but increases to ~ 1 K above 9 T (Fig. 1, inset). In Fig. 1 we compare the resistivities by plotting the ratio ρ_b/ρ_a against T at selected fields. [To correct for a slight difference in T_c's (0.3 K), we form the ratio ρ_b/ρ_a from resistivities measured at the same reduced temperature $t\equiv(T_c-T)/T_c$ in the two crystals.] Starting at low temperatures, the ratio ρ_b/ρ_a rises steeply, signaling the transition to the dissipative state in sample B. When T increases above T_c, the curves converge to the normal-state value[13] $(\rho_b^N/\rho_a^N)=0.45$, which is almost independent of field. Because ρ_b/ρ_a is sensitive to both H and T, the approximation $\sigma_{ab}\sim\rho_{ba}/\rho_a^2$ is unreliable except within a few degrees of T_c. Interestingly, in the mixed state, ρ_b/ρ_a rises to values significantly higher than the normal-state value (before the onset of pinning drives it to zero). The enhancement implies that the anisotropy is weaker in the mixed state than in the normal state. This implies that the

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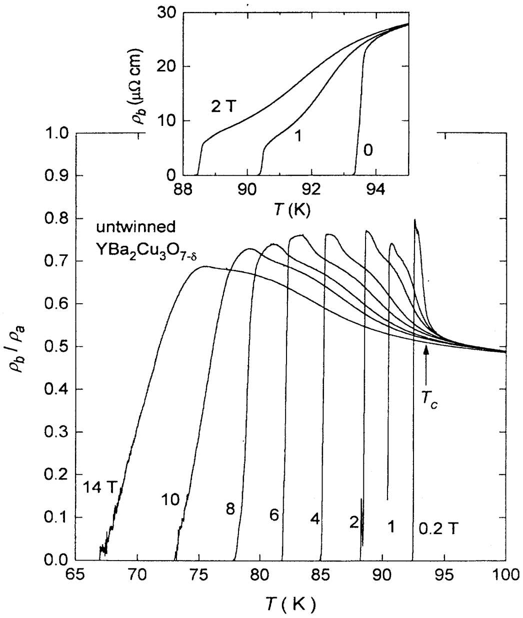

FIG. 1. (Main panel) The ratio of the in-plane resistivities ρ_b/ρ_a in optimally oxygenated $YBa_2Cu_3O_{6.93}$ determined from measurements on two untwinned crystals A and B (with $\mathbf{H}\|\mathbf{c}$ and $J\sim 3$ A/cm^2). The ratio increases below T_c, indicating reduced in-plane anisotropy. Sample dimensions are $1.0\times0.4\times0.073$ mm^3 (sample A) and $0.95\times0.5\times0.055$ mm^3 (sample B). In zero field, T_c equals 93.2 (93.5) K and the transition width ΔT_c is 0.15 (0.21) K in sample A (B). At 100 K, ρ_a (measured in A)=67.5 $\mu\Omega$ cm while ρ_b (in B)=32.7 $\mu\Omega$ cm. The inset shows ρ_b (of sample B) measured in 0, 1, and 2 T.

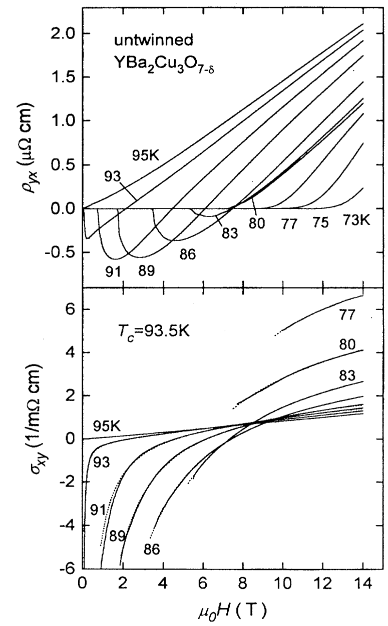

FIG. 2. The Hall resistivity $\rho_{yx}=\rho_{ab}$ vs H in YBCO (sample B) observed with $\mathbf{H}\|\mathbf{c}$ and $\mathbf{J}\|\mathbf{b}$ (upper panel). The Hall resistivity ρ_{ba} taken in sample A ($\mathbf{H}\|\mathbf{c}$, $\mathbf{J}\|\mathbf{a}$) is closely similar. The Hall data ρ_{yx} and the two resistivity curves ρ_a and ρ_b are used to compute the Hall conductivity $\sigma_{xy}=\sigma_{ab}$ which is displayed as solid lines in the lower panel. σ_{xy} is monotonic at all T. In low fields, σ_{xy} diverges as $-1/H$ while, in high fields, it approaches the quasiparticle value σ^n_{xy}. Dashed lines are fits to Eq. (2).

chain conductance (which is responsible for the anisotropy above T_c) becomes less important relative to the conductance within the layers in the mixed state.

As in the resistivity, the onset of the Hall resistivity $\rho_{yx}=\rho_{ba}$ is much sharper than observed in heavily twinned crystals, especially at temperatures above 84 K (Fig. 2, upper panel). In weak fields, ρ_{yx} initially decreases to a minimum, then changes sign before increasing linearly with H at high fields. This nonmonotonic behavior immediately simplifies when we convert ρ_{yx} into the Hall conductivity (Fig. 2, lower panel). In contrast to ρ_{yx}, σ_{xy} is always monotonic in field in YBCO.[8] The resistivities ρ_{xx} and ρ_{yx} decrease abruptly to zero when the vortex lattice is pinned, but σ_{xy} does not. In weak fields, σ_{xy} diverges in the negative direction while in high fields σ_{xy} asymptotically approaches a straight line with positive slope in H.

The viscous-drag (η) and Hall-drag (α) coefficients are defined by the equation of motion $\eta\mathbf{v}_L+\alpha\mathbf{v}_L\times\mathbf{z}=\phi_0\mathbf{J}\times\mathbf{z}$ ($\mathbf{v}_L$ is the line velocity, ϕ_0 the flux quantum, and $\mathbf{z}=\mathbf{B}/B$). In terms of η and α, the vortex conductivity elements are given by $\sigma^f_{xx}=\eta/B\phi_0$ and $\sigma^f_{xy}=\alpha/B\phi_0$. Vinokur *et al.*[15] have argued, on general grounds, that α should be independent of field, so that σ^f_{xy} varies as $1/B$. In weak fields the quasiparticle term σ^n_{xy} is linear in B. Hence, we expect the total Hall conductivity σ_{xy} to equal c_1B-c_3/B (with $c_1,c_3>0$). However, we find systematic deviation from this simple fit at low T and high fields.

An attractive explanation for the deviation is the scattering of quasiparticles (qp) by the vortices (at $T>70$ K, d-wave or anisotropic s-wave pairing implies a large qp population *outside* the core). Microwave experiments in zero field suggest that the quasiparticle lifetime in YBCO increases sharply below T_c.[16,17] This strong T dependence represents an inelastic scattering process that is very likely electronic in origin. In the mixed state, vortices act as "impurities" in the sea of quasiparticles. In addition to degrading the qp lifetime (described by the transport cross section $\sigma_{\rm tr}$), the vortices also cause asymmetric scattering (described by the transverse cross-section $\sigma_\perp$).[18] The latter generates the qp Hall current. We may express the mean free path (mfp) in a field $l_{\rm tot}$ by $l^{-1}_{\rm tot}=l^{-1}=l^{-1}_\nu$ where l is the mfp in zero field and $l^{-1}_\nu=\sigma_{\rm tr}|B|/\phi_0$. Recalculating the quasiparticle current with $l_{\rm tot}$ in place of l, we obtain for the total Hall conductivity[19]

$$\sigma_{xy}=c_1l^2H(1+l\sigma_{\rm tr}|B|/\phi_0)^{-2}-c_3/H, \qquad (2)$$

where $c_1=Y(T)(e^2k_F/2\pi\hbar)\sigma_\perp/\phi_0$. The function $Y(T)=(1/\pi k_F)\int ds_{\mathbf{k}}\int d\varepsilon(-\partial f_0/\partial E_{\mathbf{k}})$, with qp energy $E_{\mathbf{k}}=\sqrt{[\varepsilon^2_{\mathbf{k}}+\Delta(s_{\mathbf{k}})^2]}$, describes the T dependence of the qp population [we assumed d-wave symmetry for the gap function $\Delta(s_{\mathbf{k}})$ in the fit]. The values of $\sigma_{\rm tr}$ and l obtained from the fit (inset of Fig. 3) are quantitatively consistent with the physical picture underlying Eq. (2).[20] The cross section $\sigma_{\rm tr}$ shows a weakly T-dependent value $\sim$30Å, or about twice the in-plane coherence length ξ_{ab}. More interestingly, the

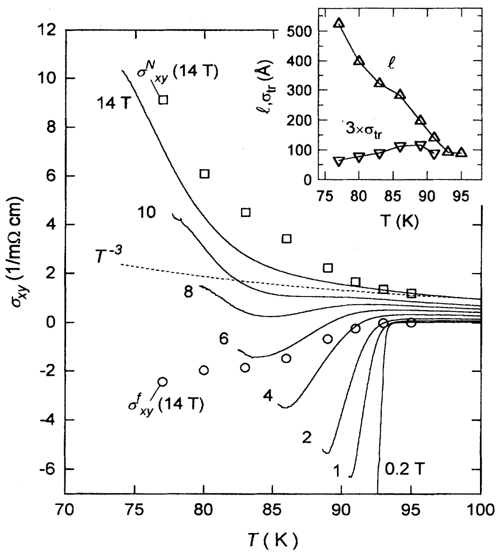

FIG. 3. (Main panel) The total Hall conductivity σ_{xy} of untwinned YBCO vs temperature at fixed field (solid lines). Using Eq. (2), we separate out σ_{xy} at 14 T into σ^n_{xy} (squares) and $\sigma^f_{xy}=-c_3/H$ (at 14 T). By fitting to a straight line, we may express σ^f_{xy} as $-(2.1\times10^7)t/B$ $(\Omega\mathrm{m})^{-1}$, which gives the value of $\alpha(T)$ discussed in the text. Below T_c, σ^n_{xy} increases much more rapidly than the T^{-3} behavior in the normal state (dashed line). The inset shows l (mfp in zero field) and σ_{tr} (transport cross section) obtained from the fit to Eq. (2).

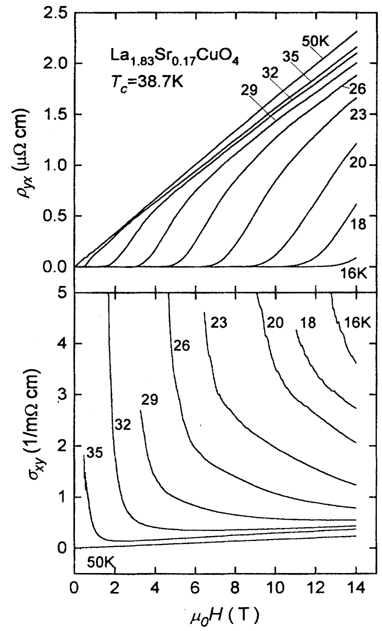

FIG. 4. The Hall resistivity ρ_{yx} vs H of $La_{2-x}Sr_xCuO_4$ $(x=0.17)$ measured with $\mathbf{H}\|\mathbf{c}$ (upper panel). The crystal (size $1.34\times0.94\times0.18$ mm^3) displays a zero-field transition at $T_c=38.7$ K with a width $\Delta T_c=2$ K. The in-plane resistivity is linear in T, with a value 416 $\mu\Omega$ cm at 300 K. The lower panel shows the field dependence of the Hall conductivity σ_{xy} computed from ρ_{yx} and the in-plane resistivity. Near T_c (curves at 29, 32, and 35 K), σ_{xy} decreases to a minimum, and then increases slowly to approach a line parallel with the curve for σ^n_{xy} at 50 K. This behavior shows that σ_{xy} is the sum of a positive, monotonically decreasing term σ^f_{xy} and a positive σ^n_{xy} that increases linearly with H.

steep increase in (the zero field) l below T_c is in striking agreement with the surface resistance results of Bonn *et al.*[16] who inferred a qp conductivity that rises steeply to a peak near 40 K. The strong suppression of the inelastic scattering rate results in a qp lifetime enhancement that more than offsets the decrease in $Y(T)$, so that σ^n_{xy} increases.[19]

A different perspective on σ_{xy} (versus T in fixed field) is shown in Fig. 3. The fan-out pattern illustrates the competition between the two currents of opposite sign that diverge with decreasing T. In low fields, the vortex Hall conductivity σ^f_{xy} dominates, so that σ_{xy} swings to large negative values. With increasing fields, however, σ^n_{xy} grows while $|\sigma^f_{xy}|$ decreases. Thus, above 10 T, σ_{xy} increases monotonically with decreasing T. Using Eq. (2), we have separated out the two components at 14 T. With falling T, the quasiparticle σ^n_{xy} (squares) rises because of the increase in l. [Since σ^n_{xy} rises much faster than the T^{-3} normal-state behavior (broken line), it is incorrect to assume that the latter holds within the mixed state.[12]] The vortex part σ^f_{xy} increases in magnitude linearly with the reduced temperature t as $\sigma^f_{xy}=-2.1\times10^7 t/B$ $(\Omega m)^{-1}$ (open circles). From this, we derive the temperature-dependent Hall-drag coefficient $\alpha(T)=-4.4\times10^{-8}(1-T/T_c)$ Ns/m^2, $(75<T<93$ K). It is interesting to compare $\alpha(T)$ with the Hall-drag coefficient α_∞ expected in the superclean regime where the vortex core moves (anti-)parallel with the applied current $\mathbf{J}$.[21] Setting $\mathbf{v}_L=\mathbf{J}/n_s e$, we obtain $\alpha_\infty=\pm n_s e\phi_0$ which has the value $\pm1.4\times10^{-7}(m^*/m_0)$ Ns/m^2. (n_s is the superconducting electron density and m_0 the free mass; the in-plane penetration length λ is taken to be 1500 Å.) Extrapolated to $T\ll T_c$, $\alpha(T)$ seems to imply a low-temperature value $\alpha(0)$ an order of magnitude smaller than α_∞ (taking $m^*/m_0\sim2$). However, we expect $\alpha(T)$ to deviate strongly from linear-t dependence when the Hall angle diverges below $\sim$50 K, as observed in 60-K YBCO.[11] [For comparison, in 60-K YBCO, $\alpha\sim-1.3\times10^{-8}$ Ns/m^2 at 37 K, while $\alpha_\infty=-1.5\times10^{-7}(m^*/m_0)$ Ns/m^2.]

Flux-flow Hall measurements on single-crystal $La_{2-x}Sr_xCuO_4$ $(x=0.17)$ provide an interesting confirmation of Eq. (1) even though no sign-reversal anomaly occurs. The Hall resistivity ρ_{yx} (Fig. 4, upper panel) is positive (holelike) in fields up to 14 T and at temperatures down to 16 K. In comparison with the data for YBCO, its onset with field is much more gradual. When the resistivity is converted to the conductivity tensor (Fig. 4, lower panel), we find that in high fields σ_{xy} again approaches an asymptote that represents σ^n_{xy}. However, in contrast to YBCO, the vortex term σ^f_{xy} is positive. We have confirmed that σ^f_{xy} is positive at all angles including the orientation with $\mathbf{H}\perp\mathbf{c}$. Both σ^f_{xy} and σ^n_{xy} are clearly visible down to $T=26$ K, while at lower temperatures σ^f_{xy} becomes dominant. The magnitude of σ^f_{xy} decreases with H just as in YBCO.

The data for the two cuprates, taken together, provide persuasive evidence for the additivity of the quasiparticle and vortex Hall currents, as expressed in Eq. (1). The simple expedient of associating opposite signs to the two terms provides a rather natural explanation of the sign-reversal anomaly of the in-plane Hall response in YBCO. Similarly, the absence of sign reversal in LSCO also fits naturally within this explanation. The conductivity curves in Fig. 4 demonstrate clearly that σ_{xy}^f adds to a linearly increasing σ_{xy}^n, and that both are positive. These tests, together with verification of the GL scaling relations in a tilted-field experiment[8] and the low-temperature measurement showing that $\sigma_{xy} \sim -1/H$,[11] bolster the case for additivity of the Hall currents in the cuprates. Moreover, the high-field fits allow the determination of two important quasiparticle quantities, l and σ_{tr}. The factors that fix the sign of σ_{xy}^f are not understood. Although a few microscopic calculations have addressed this issue,[22] there is considerable uncertainty about why σ_{xy}^f is negative while σ_{xy}^n is positive in YBCO. This difficulty is compounded by our observation that the two quantities are positive in LSCO.

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