Three-dimensional XY scaling of the resistivity of $YBa_2Cu_3O_{7-\delta}$ single crystals

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The electrical resistivity of a single-crystal sample of $YBa_2Cu_3O_{7-\delta}$ has been measured. The results are consistent with the existence of a critical regime governed by three-dimensional XY critical exponents. Evidence is also found for the divergence of the conductivity along a line $H_g(T)\propto(1-T/T_c)^{2\nu}$, the vortex glass transition line. The vortex glass exponents characteristic of this line are $z_g(\nu_g-1)\approx 5\pm 1$. The results are not consistent with the lowest Landau-level scaling.

The phase transition in conventional superconductors has, for a long time, been considered a paradigm of a continuous phase transition described by mean-field theory. In high-T_c superconductors this is no longer the case. The coherence length is anisotropic, but with an average of about 10 Å, and the region over which the effects of Gaussian fluctuations are thought to be observed is now of the order of 100 K. The temperature range over which critical fluctuations may be observed is thought to be of the order of 10 K.[1–4]

The universality class that describes the critical fluctuations in a superconductor is not known. For an uncharged Bose fluid, such as liquid ^{4}He, we expect to observe three-dimensional (3D) XY critical behavior with the exponent for the correlation length $\nu\approx 0.669$. The evidence is mounting that this is also the universality class for high-T_c superconductors within the experimentally accessible temperature range near T_c. Very close to T_c the effect of fluctuations in the vector potential, present because Cooper pairs carry charge, are expected. But we emphasize that in an extreme type-II superconductor this region of fluctuations in the vector potential is probably experimentally inaccessible, and so we expect to observe critical behavior in a region around T_c governed by 3D XY critical exponents similar to the liquid ^{4}He.

The mean-field phase diagram of a type-II superconductor in an applied magnetic field H exhibits a line $H_{c2}(T)$ at which the amplitude of the superconducting order parameter goes continuously to zero. Beyond mean-field theory, the theoretical situation is rather uncertain. Within the lowest-Landau-level (LLL) approximation, which assumes that the order parameter is negligibly small, one finds that various physical properties can be expressed in a scaling form.[11–15] The LLL approximation is probably valid near a line in the phase diagram which can be regarded as a renormalized version of the mean-field $H_{c2}(T)$,[11,12] and scaling behavior in such a region is indeed found in conventional type-II materials. Whether a similar region exists in the phase diagrams of high-T_c materials is not known. However, this scaling behavior is not associated with a phase transition. The superconducting coherence length remains finite at $H_{c2}(T)$, and no singular behavior is apparent either in the specific heat or in the electrical conductivity. The scaling behavior associated with the lowest-Landau level is of a different kind from critical point scaling, and the regions, if they exist, where these two types of behavior occur, cannot overlap. Thus, we might expect to see critical behavior, governed by the 3D XY critical exponents, in a region of small t and H, near $T=T_c$ and $H=0$ and LLL scaling in a region which surrounds $H_{c2}(T)$ but stops short of the critical region.

A number of groups have reported what they believe to be scaling behavior in a magnetic field typical of the LLL region.[7,8] Welp *et al.*[8] report observing LLL scaling, which appears to improve at higher fields ($H>40$ kOe), in the magnetization and resistivity. However, they do *not* see LLL scaling in their analysis of the *specific heat* data of Inderhees *et al.*[16] and have to introduce an *arbitrary* exponent for the magnetic field in order to see poor scaling over a very limited range of reduced temperature, t. On the other hand, other groups have reported 3D XY behavior typical of the critical regime.[1,3,4,9,10] We have carried out a study of the electrical resistivity and of the specific heat[4] of a single crystal of Y-Ba-Cu-O in a magnetic field in order to see if there is a consistent picture of the phase diagram. In this paper we concentrate on the electrical resistivity data.

Within both the LLL and critical regimes the $H_{c2}(T)$ line no longer marks a line of phase transitions. There is no region associated with the $H_{c2}(T)$ line where there are singularities in the specific heat, the magnetization or the resistivity[2,5] in the presence of a magnetic field. Fisher, Fisher, and Huse[2] have also pointed out that, while there is no transition near $H_{c2}(T)$ in the presence of a magnetic field there is the possibility of what they call a vortex glass transition along a line $H_g(T)$. This line should be observable from the electrical conductivity data as a divergence to a zero resistivity state.

The non-Ohmic resistivity has been extensively investigated[9,10] providing evidence for the existence of a glass transition line $H_g(T)$ along which the Ohmic resistivity vanishes. Within the one parameter critical scaling regime the Ohmic conductivity is conjectured[2] to have the scaling form

$$\sigma_{\rm fl}=\xi^{2+z-d}S_{\pm}(B\xi^2)\ , \tag{1}$$

where $\xi \propto |T-T_c|^{-\nu}$ is the zero-field coherence length, and where $S_{\pm}(y)$ are the scaling functions that hold above and below T_c. The scaling variable is $B\xi^2$ but since high-T_c superconductors are extreme type-II superconductors, then $B\approx\mu_0 H$, where H is the applied field so that to a large extent H and B are interchangeable—this approximation gets worse as we approach closer to the Meissner phase.

The dynamical universality class of the 3D XY model is not clear. For superfluid liquid helium, although the order parameter is not a conserved quantity because of phase fluctuations, the charge density is conserved and so helium is considered to exhibit model F dynamics[11] with $z=3/2$. However, Fisher, Fisher, and Huse[2] argue the number of Cooper pairs in a superconductor are not conserved, because of the presence of plasma fluctuations, so that both the magnitude and phase are fully relaxational and model A dynamics are appropriate with $z\approx 2$.

It is more convenient to use a temperaturelike scaling variable x, and if we take $z=2$ with $d=3$, then

$$\sigma_{\rm fl}B^{1/2}=s(t/B^{1/2\nu})\ . \tag{2}$$

In this form there is only one branch of the scaling function. Although we do not know the form of $s(x)$ over the whole critical regime, we do know its behavior in certain regions. For $T>T_c$ the conductivity must remain finite as $B\rightarrow 0$ and so $s(x\rightarrow+\infty)=x^{\nu}$. At T_c the conductivity must remain finite except at $B=0$ and so $s(0)=$const. When pinning gives rise to a vortex glass the Ohmic conductivity should diverge along a line given by $x=-x_g$ and the behavior close to this line will be governed by the glass coherence length[2] $\xi_g(T)\propto(x+x_g)^{-\nu_g}$ which leads to the prediction

$$\sigma_{\rm fl}H^{1/2}=s(x)\approx s_g(x+x_g)^{-z_g(\nu_g-1)}\ , \tag{3}$$

where z_g and ν_g are exponents governing the glass transition. The glass transition line is then given by $B_g(T)=[(1-T/T_c)/x_g]^{2\nu}$. All these features are shown in Fig. 1.

Within the LLL regime, at high fields, a different scaling behavior to the critical scaling is predicted[5,7,8]

$$\sigma_{\rm fl}=\left[\frac{T^2}{H}\right]^{1/2}F\left[\frac{T-T_c(H)}{TH^{2/3}}\right] \tag{4}$$

for the field perpendicular to the current. If this holds at all it will hold close to the $H_{c2}(T)$ line but outside the critical regime as shown in Fig. 1.

The crystal used for the resistivity measurements was the same crystal Y8 as used in the paraconductivity data published earlier.[12] It is also the same crystal for which we published specific-heat data.[4] A four-probe dc method was used to measure the Ohmic resistivity in the a-b plane with the magnetic field either longitudinal or transverse and in the a-b plane or with the field along the c axis. The $V(I)$ curves were measured at various temperatures to confirm that the resistivity measurements were taken in the Ohmic regime. The current density used for the measurements was 4×10^4 Am^{-2} and a Keithley 181 nanovoltmeter used to measure the voltage across the sample. Figure 2 shows the broadening of the resistivity in a magnetic field for the current in the a-b plane and the field along the c axis, while the inset shows the data for the field transverse to the current and in the a-b field.

The scaled fluctuation conductance data are shown in Fig. 3. To calculate the fluctuation conductance we have subtracted a normal-state conductance, assuming $\rho\propto aT$, from the measured conductance and scaled the data according to Eq. (2). The scaling is not sensitive to the exact value of a or whether we assume that $\rho\propto aT+b$. While experiments which investigate the Gaussian fluc-

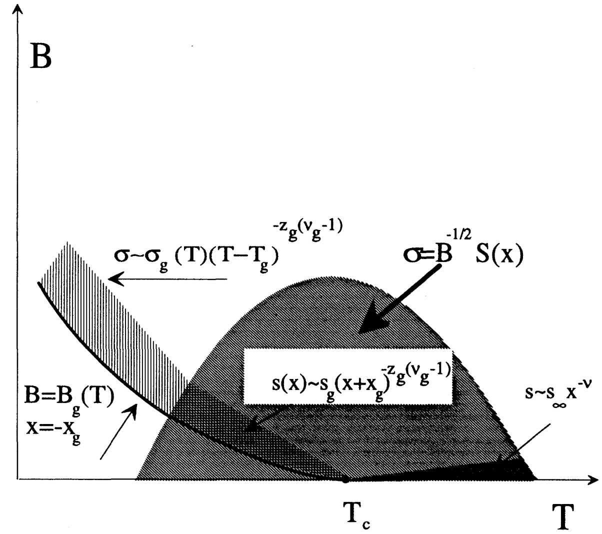

FIG. 1. A *schematic* diagram of the H-T plane for a superconductor indicating the region in the vicinity of the multicritical point T_c where one parameter scaling is valid. The line $B=B_g(T)$ is the vortex glass transition. Two "crossover lines" are also shown to indicate the region close to the vortex glass transition line and the region as $x\rightarrow\infty$ where the form for $s(x)$ should be well approximated by power laws.

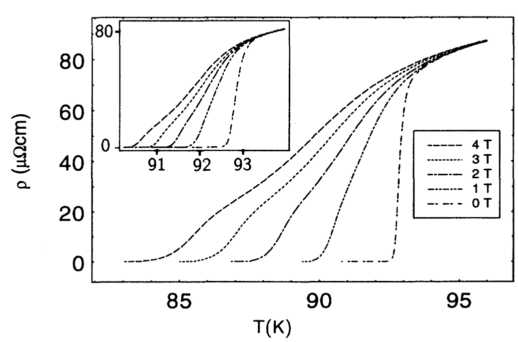

FIG. 2. The resistivity vs temperature for sample Y8 with the magnetic field along the c axis and the current in the a-b plane (perpendicular). The inset shows data for the transverse case with the field in the a-b plane.

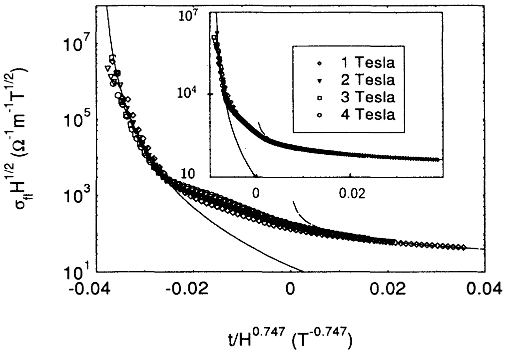

FIG. 3. The scaled conductivity data with $z=2$. The fluctuation contribution is obtained from the measured conductivity by subtracting the normal background term $[(0.77\ \mu\Omega\,\text{cm}\,\text{K}^{-1})T]^{-1}$ from the data. The dotted line is the divergence predicted by the vortex-glass transition and the solid line is the expected asymptotic behavior $\approx x^{-\nu}$. The magnetic field is along the c axis. The inset is data for the transverse case.

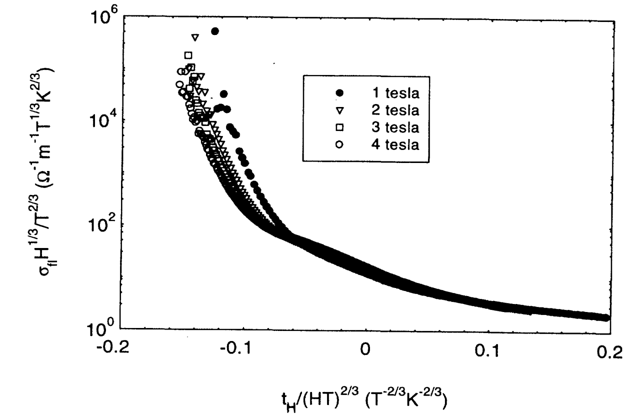

FIG. 4. The scaled conductivity data using the LLL scaling of Eq. (4), and the field along the c axis, with $dB_{c2}/dT=-2$ T/K.

tuations well above T_c are very sensitive to the exact choice of the normal-state resistivity here we are looking at the fluctuation conductivity, close to and below T_c, which is diverging over several orders of magnitude.

In both figures the data scale very well. The solid line shown on the curves is $s(x)\propto x^{-\nu}\propto x^{-0.669}$, the expected asymptotic behavior at low fields. The dotted line represents the behavior expected from Eq. (3) with $z_g(\nu_g-1)\approx 5\pm 1$. These results are consistent with a divergence in the Ohmic conductivity along a line $B_c(T)\propto(1-T/T_c)^{1.34}$ similarly to previous results.[9,13] The divergence takes plane along a line $x=x_g=(75\pm 4\ \text{T})^{-0.747}$ with the field along the c axis.

The scaling analysis of the perpendicular and transverse data shows that the region of small t and B close to T_c is the 3D XY critical regime and further supports the view that there is a divergence to a zero resistance state along some vortex glass transition line.

There has been some controversy as to whether LLL scaling or critical scaling is appropriate for the field and temperature regimes that have been explored experimentally. Some groups show data which exhibit convincing scaling for the resistivity and magnetization using LLL scaling[7,8] and other groups show equally good scaling for the resistivity using 3D XY critical scaling.[1] So, in Fig. 4 we show the conductivity scaled according to Eq. (4) for the LLL approximation. We see that the scaling of the data is not as good as that using the critical scaling form of Eq. (2). To obtain this scaling we have had to use a T_c of 92.5 K and $dB_{c2}/dT=-2$ T/K. It is difficult to choose a value of dB_{c2}/dT; Welp *et al.*[8] used -1.7 T/K for their LLL analysis, while Welp *et al.*[14] obtained -1.9 T/K from magnetization measurements. Our value was estimated from the shift in the midpoint of the rise in the specific heat.[4] It is possible to improve the LLL scaling but only by using values of dB_{c2}/dT as low as -1 T/K.

This indicates that the range of fields and temperatures we have investigated lie in critical regime. The extent of the regions in which critical or LLL scaling should hold is hard to determine theoretically. It has been argued[5–8] that the LLL approximation should be valid in a region where $H\xi^2/\Phi_0\gg 1$, but this criterion must be treated with caution. If ξ is interpreted as the zero-field coherence length, then the LLL region would overlap considerably with the critical region which, as noted above, cannot be the case.

In the Hartree approximation, the quantity $\tilde{\alpha}=\xi(T,B)^{-2}$ satisfies a constraint equation of the form[5,6,16]

$$\tilde{\alpha}=\xi_0^{-2}t+2\pi B/\Phi_0+(\kappa^2kT/\Phi_0^2)(B/\Phi_0)^{1/2}f(\tilde{\alpha}\Phi_0/B)\ , \tag{5}$$

where ξ_0 is a characteristic length, of the order of the zero-temperature, zero-field coherence length, and the function f involves a sum over all Landau levels. This function is well approximated by retaining only the lowest level when $\tilde{\alpha}\Phi_0/B$ is very small, in which case the constraint reads

$$\tilde{\alpha}=\xi_0^{-2}t+2\pi B/\Phi_0+(\kappa^2kT/\Phi_0^2)(B/\Phi_0)f_0\tilde{\alpha}^{-1/2}\ , \tag{6}$$

where f_0 is a constant. It is this equation, together with the requirements that $\tilde{\alpha}\Phi_0/B$ be small and the order parameter be negligible, which determines the extent of the LLL region.[15] The $H_{c2}(T)$ line corresponds to $\xi_0^{-2}t+2\pi B/\Phi_0=0$. In the mean-field approximation, where $f=0$, this is indeed the line along which the order parameter vanishes. While the Hartree approximation provides qualitative guidance on the nature of the phase diagram, Lawrie[15] has attempted to estimate the crossover field from the critical to the LLL regimes. He finds it to be very sensitive to κ, but estimates it be of the order of 10 T for Y-Ba-Cu-O, beyond the field range investigated in this work.

It is also important to note that Welp *et al.*[8] could not get the specific-heat data of Inderhees *et al.*[16] and Salamon *et al.*[1] to scale at all with the LLL model, but Salamon *et al.*[1] achieve good 3D XY scaling for the same data, while Overend, Howson, and Lawrie[4] achieve good 3D XY scaling for their specific-heat data. More recently

Junod *et al.*[17] have measured the specific heat in fields up to 20 T and also find they cannot scale the data using the LLL model although the scaling did appear to improve at the higher fields consistent with the crossover from critical to LLL scaling behavior above 10 T. Recent measurements of the penetration depth[3] also show the critical region around T_c to be about 10 K wide and consistent with the 3D XY universality class.

The general smoothness and the fact that the scaling functions monotonically increase with decreasing t make it difficult to distinguish between the LLL and critical scaling. This difficulty is most probably because of the similarity between the scaling variable [for LLL, $(T-T_c(H))/(TH)^{1.5}$ and for critical $(T-T_c)/(T_cH^{1.43})$ *and* the magnetic-field exponent (for LLL, $H^{0.333}$ and for critical with model F dynamics, $H^{0.5}$)].

The difference between $H^{-0.01}$ for critical scaling and H^0 for LLL scaling in the specific heat, and the fact that specific heat is not a simple monotonically increasing function of t but has a peak, allows us to distinguish clearly between the two regimes: it is clear from the data of Overend, Howson, and Lawrie[4] and others that LLL scaling does not work, for the field and temperature range investigated.

To summarize we have presented a scaling analysis of the electrical resistivity of a single crystal of Y-Ba-Cu-O and shown the data to be consistent with 3D XY critical scaling. Previous measurements of the specific heat[16,8,4] also show clearly that for the fields and temperatures used here we are within the 3D XY critical regime. With the present data we are not able to distinguish between model A and F universality classes, however, we have carried out our analysis assuming $z=2$. Within the 3D XY critical scaling analysis of the conductivity data we were able to present some evidence for a transition line consistent with the vortex-glass transition.

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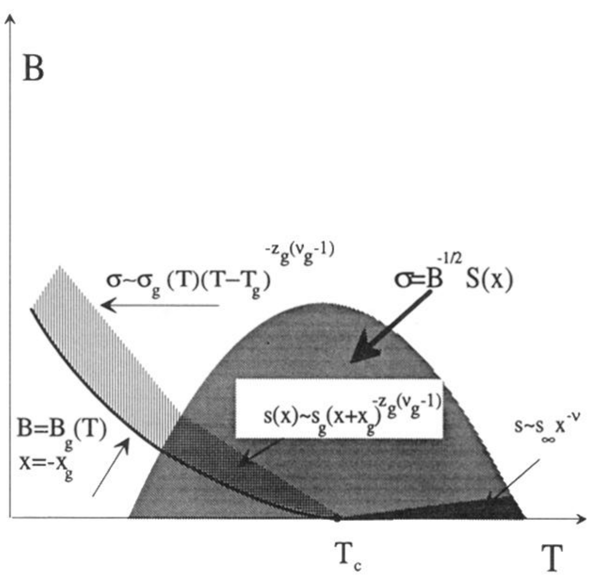

FIG. 1. A *schematic* diagram of the *H*-*T* plane for a superconductor indicating the region in the vicinity of the multicritical point T_c where one parameter scaling is valid. The line $B = B_g(T)$ is the vortex glass transition. Two "crossover lines" are also shown to indicate the region close to the vortex glass transition line and the region as $x \to \infty$ where the form for $s(x)$ should be well approximated by power laws.