London penetration depth in a tight-binding model of layered narrow-band anisotropic superconductors

Susumu Misawa
Faculty of Human Science, Tokiwa University, Mito, Ibaraki 310, Japan
(Received 20 October 1994; revised manuscript received 20 January 1995)

The London magnetic penetration depth is investigated for the layered narrow tight-binding band anisotropic superconductors. The nearest-neighbor pairing interaction in planes and the tunneling between planes are assumed. The manifestly gauge-invariant expression for the static electromagnetic response kernel is derived within the framework of the ladder-diagram approximation. The out-of-plane penetration depth $\lambda_\perp(T)$ as well as the in-plane one $\lambda_\parallel(T)$ is calculated as a function of hole density n_h and temperature $T(<T_c)$. The resultant $\lambda_\perp(0)$ and $\lambda_\parallel(0)$ depend symmetrically on hole and electron densities, and are almost not affected by the anisotropy of the order parameter. For both of the extended s-wave and d-wave states, the obtained anisotropy ratio $\lambda_\perp(0)/\lambda_\parallel(0)$ as well as $\lambda_\parallel(0)$ and T_c can be in the range of the experimental results for high-T_c superconductors. For the d-wave state, the resultant $\lambda_\perp(T)/\lambda_\perp(0)$ as well as $\lambda_\parallel(T)/\lambda_\parallel(0)$ has T-linear behavior at low temperature, and substantially deviates from those for the extended and usual s-wave states for all $T<T_c$. The in-plane and out-of-plane Ginzburg-Landau (GL) coherence lengths are also calculated by assuming their GL relations with the above penetration depths, and are compared with the experimental results. The case with the out-of-plane tunneling in proportion to carrier density is simply examined.

I. INTRODUCTION

The pairing state of high-T_c superconductors has not been fully understood, though many experimental results supporting the d-wave pairing have been reported recently.[1] The magnetic penetration depth $\lambda_L(T)$ (as a function of temperature T) is one of the most important quantities that give information on the pairing state. Recently, $\lambda_L(T)$ has been calculated on the basis of the tight-binding models for the two-dimensional (2D) anisotropic superconductors[2–4] and the layered superconductors.[5,6] The theoretical investigation of $\lambda_L(T)$ based on the tight-binding model is considered to be worthwhile for the following reasons. In many theoretical studies of high-T_c superconductors, the pairing interaction is assumed to work over the whole region of the narrow band, and various properties are calculated for Bloch electrons in the narrow tight-binding model. Furthermore, in many theoretical studies of the layered superconductors, the tunneling between planes is taken as small, and is treated in the form of a tight-binding band.

In a previous paper,[4] the present author has shown that the ladder-diagram approximation[7,8] is consistent with the gauge invariance in the tight-binding model of the general anisotropic narrow-band superconductors. Within the framework of this approximation, the *manifestly* gauge-invariant expression for the static electromagnetic response kernel has been derived, and $\lambda_L(T)$ for the extended s-wave and d-wave states has been calculated as a function of hole density n_h and temperature $(T<T_c)$ on the basis of the 2D square lattice model. The obtained results have been compared with those of the effective hole- and electron-mass approximations.

However, the previous calculation[4] is for the 2D system and is not sufficient, since $\lambda_L(T)$ is to be considered for the 3D structure. In this work, we extend the previous work[4] to the layered system. Our model does not include the electron-correlation effect (though the transfer is treated in the form of a narrow band). However, most properties due to the symmetry of the pairing state are expected to be examined also in our model.

Some studies have already treated the layered tight-binding model of the high-T_c superconductors. Schneider and Frick[5] have investigated the effect of the anisotropy due to the layered structure on the behavior of $\lambda_L(T)$. Mirsiglio and Hirsch[6] have investigated $\lambda_L(T)$ of the layered hole superconductors with a somewhat different mechanism due to a modulated hopping interaction. However, these works[5,6] have examined only the (extended) s-wave states (though they have taken account of the interlayer pairing) essentially. In this work, we examine the d-wave state also and compare the result with that for the extended s-wave state. We also compare the obtained values for the in-plane and out-of-plane penetration depths [and also for the Ginzburg-Landau (GL) coherence lengths] with the experimental results. We take the nearest-neighbor attractive interaction in planes and the tunneling between planes, but we assume that the attractive interaction between the planes is small enough to be neglected.

This work is organized as follows. In Sec. II, we apply the ladder-diagram approximation to the electromagnetic response of our layered narrow tight-binding band system. In Sec. III, we derive the *manifestly* gauge-invariant expression for the static electromagnetic response kernel, and calculate the out-of-plane penetration depth $\lambda_\perp(T)$ as well as the in-plane one $\lambda_\parallel(T)$ as a function of hole density n_h and temperature $T(<T_c)$. In Sec. IV, we calculate

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the in-plane and out-of-plane GL coherence lengths. Furthermore, the case with the out-of-plane tunneling in proportion to carrier density is simply examined. A conclusion is given in Sec. V.

II. MODEL AND FORMULATION

In this section, we apply the ladder-diagram approximation[7,8] to the electromagnetic response of the layered narrow-band anisotropic superconductor consisting of Bloch electrons. Our starting model Hamiltonian is the following phenomenological layered tight-binding model:

$$H=\sum_{\langle \mathbf{r},\mathbf{r}'\rangle\sigma} t(\mathbf{r}-\mathbf{r}')c^{\dagger}_{\mathbf{r}\sigma}c_{\mathbf{r}'\sigma}+\text{H.c.}-\mu\sum_{\mathbf{r},\sigma} n_{\mathbf{r}\sigma}$$
$$+\sum_{\langle \mathbf{r},\mathbf{r}'\rangle\sigma\sigma'}[V(\mathbf{r}-\mathbf{r}')+V_C(\mathbf{r}-\mathbf{r}')]n_{\mathbf{r}\sigma}n_{\mathbf{r}'\sigma'}\ , \tag{2.1}$$

where $t(\mathbf{r}-\mathbf{r}')$ is the transfer integral and $V(\mathbf{r}-\mathbf{r}')$ the attractive intersite interaction, $V_C(\mathbf{r}-\mathbf{r}')$ the Coulomb interaction; $c_{\mathbf{r}\sigma}$ and $n_{\mathbf{r}\sigma}$ are, respectively, the annihilation and number operators for an electron of spin σ with the chemical potential μ at the $\mathbf{r}$th site. In our model, we assume that the intersite pairing interaction between planes are small enough to be neglected. We take

$$t(\mathbf{r}-\mathbf{r}')=\begin{cases}-t_{\parallel} & (\mathbf{r},\mathbf{r}' \text{ nearest neighbors in the plane})\ ,\\ t_{\perp} & (\mathbf{r},\mathbf{r}' \text{ nearest neighbors out of plane})\ ,\\ 0 & (\text{otherwise})\ ,\end{cases} \tag{2.2}$$

and

$$V(\mathbf{r}-\mathbf{r}')=\begin{cases}-V & (\mathbf{r},\mathbf{r}' \text{ nearest neighbors in the plane})\ ,\\ 0 & (\text{otherwise})\ .\end{cases} \tag{2.3}$$

By using the Nambu notation[7] and by dropping some nonessential terms, we rewrite Eq. (2.1) into the Fourier transformed form

$$H=\sum_{\mathbf{k}}(\epsilon_{\mathbf{k}}-\mu)\Psi^{\dagger}_{\mathbf{k}}\tau_3\Psi_{\mathbf{k}}$$
$$+\frac{1}{2N}\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}}\tilde{V}(\mathbf{q})\Psi^{\dagger}_{\mathbf{k}+\mathbf{q}/2}\tau_3\Psi_{\mathbf{k}-\mathbf{q}/2}\Psi^{\dagger}_{\mathbf{k}'-\mathbf{q}/2}\tau_3\Psi_{\mathbf{k}'+\mathbf{q}/2}\ . \tag{2.4}$$

Here, $\Psi^{\dagger}_{\mathbf{k}}=[c^{\dagger}_{\mathbf{k}\uparrow},c_{-\mathbf{k}\downarrow}]$; $\tau_i(i=1,2,3)$ are the Pauli matrices, and N is the total number of the lattice sites. The wave-number summations are restricted within the unit cell of the reciprocal lattice.

In our model, the single-particle energy is expressed as

$$\epsilon_{\mathbf{k}}=-2t_{\parallel}[\cos(k_xa)+\cos(k_ya)]+2t_{\perp}\cos(k_zd)\ , \tag{2.5}$$

where a and d denote the in-plane and out-of-plane lattice spacings, respectively. We consider, in our model, only the in-plane pairing, and the interaction in Eq. (2.4) is expressed as

$$\tilde{V}(\mathbf{q})=V(\mathbf{q}_{\parallel})+V_C(\mathbf{q})\ , \tag{2.6}$$

with the pairing interaction

$$V(\mathbf{q}_{\parallel})=-2V[\cos(q_xa)+\cos(q_ya)]\ , \tag{2.7}$$

and the Coulomb interaction

$$V_C(\mathbf{q})=\frac{2\pi e^2}{q_{\parallel}a^2}\left[\frac{\sinh q_{\parallel}d}{\cosh q_{\parallel}d-\cos q_{\perp}d}\right]\ , \tag{2.8}$$

in the limit $a\to 0$. Here, $\mathbf{q}_{\parallel}$ denotes a 2D in-plane wave vector. We consider $V_C(\mathbf{q})$, however, only for the vacuum-polarization process as in the usual non-Bloch treatment,[8] and we will see that this $V_C(\mathbf{q})$ does not contribute to the static electromagnetic response.

We consider only the singlet superconducting state. The temperature Green's function in the Hartree-Fock approximation is given by

$$G(\mathbf{k},i\omega_n)\equiv-\int_0^{\beta}d\tau\, e^{i\omega_n\tau}\langle\Psi_{\mathbf{k}}(\tau)\Psi^{\dagger}_{\mathbf{k}}(0)\rangle$$
$$=\{i\omega_n-(\epsilon_{\mathbf{k}}-\mu)\tau_3-\Delta_{\mathbf{k}_{\parallel}}\tau_1\}^{-1}\ , \tag{2.9}$$

where $\omega_n=(2n+1)\pi T$ with T the temperature and n an integer, $\beta=1/T$, and $\langle\ \rangle$ denotes the thermal average of the system under no field. The order parameter $\Delta_{\mathbf{k}_{\parallel}}=\sum_{\mathbf{q}}V(\mathbf{k}_{\parallel}-\mathbf{q}_{\parallel})\langle c_{-\mathbf{q}\downarrow}c_{\mathbf{q}\uparrow}\rangle/N$ is chosen to be real and is determined without the Coulomb interaction (the effect of which is assumed to be small and is neglected here as in the usual non-Bloch treatment[8]) as

$$\Delta_{\mathbf{k}_{\parallel}}\tau_1=-\frac{T}{N}\sum_{n\mathbf{q}}V(\mathbf{k}_{\parallel}-\mathbf{q}_{\parallel})\tau_3G(\mathbf{q},i\omega_n)\tau_3$$
$$=-\frac{1}{N}\sum_{\mathbf{q}}\frac{V(\mathbf{k}_{\parallel}-\mathbf{q}_{\parallel})\Delta_{\mathbf{q}_{\parallel}}}{2E_{\mathbf{q}}}\tanh\frac{E_{\mathbf{q}}}{2T}\tau_1\ , \tag{2.10}$$

with $E_{\mathbf{k}}=\sqrt{(\epsilon_{\mathbf{k}}-\mu)^2+\Delta^2_{\mathbf{k}_{\parallel}}}$. This expression is in the form of the self-energy, to which precisely the following contribution to correct $\epsilon_{\mathbf{k}}$ is to be added:

$$\delta\epsilon_{\mathbf{k}}\tau_3=-\frac{1}{2N}\sum_{\mathbf{q}}V(\mathbf{k}_{\parallel}-\mathbf{q}_{\parallel})n_{\mathbf{q}}\tau_3\ , \tag{2.11}$$

where $n_{\mathbf{q}}$ is given below by Eq. (2.14). In the usual treatments, this correction is neglected. We retain the notation $\epsilon_{\mathbf{k}}$ for the corrected energy $\tilde{\epsilon}_{\mathbf{k}}\equiv\epsilon_{\mathbf{k}}+\delta\epsilon_{\mathbf{k}}$.

In our model, the order parameter has the following forms for the possible extended s-wave and d-wave states: $\Delta^{\alpha}_{\mathbf{k}_{\parallel}}=\Delta_{\alpha}w^{\alpha}_{\mathbf{k}_{\parallel}}$ $(\alpha=es,d)$ with $w^{es}_{\mathbf{k}_{\parallel}}=\cos(k_xa)+\cos(k_ya)$, $w^{d}_{\mathbf{k}_{\parallel}}=\cos(k_xa)-\cos(k_ya)$, and Δ_{α} determined by

$$1=\frac{V}{N}\sum_{\mathbf{k}}\frac{(w^{\alpha}_{\mathbf{k}_{\parallel}})^2}{2E_{\mathbf{k}}}\tanh\frac{E_{\mathbf{k}}}{2T}\quad(\alpha=es,d)\ , \tag{2.12}$$

along with $E_{\mathbf{k}}=\sqrt{(\epsilon_{\mathbf{k}}-\mu)^2+\Delta^2_{\alpha}(w^{\alpha}_{\mathbf{k}_{\parallel}})^2}$. Here, we note that our pairing potential can be rewritten as

$$V(\mathbf{k}_{\parallel}-\mathbf{q}_{\parallel})=-V[w^{es}_{\mathbf{k}_{\parallel}}w^{es}_{\mathbf{q}_{\parallel}}+w^{d}_{\mathbf{k}_{\parallel}}w^{d}_{\mathbf{q}_{\parallel}}+w^{p_x}_{\mathbf{k}_{\parallel}}w^{p_x}_{\mathbf{q}_{\parallel}}$$
$$+w^{p_y}_{\mathbf{k}_{\parallel}}w^{p_y}_{\mathbf{q}_{\parallel}}]\ , \tag{2.13}$$

with $w^{p_i}_{\mathbf{k}_{\parallel}}=\sqrt{2}\sin(k_ia)$ $(i=x,y)$. We determine μ by

$$n_e \equiv 2-n_h=\frac{1}{N}\sum_{\mathbf{k}} n_{\mathbf{k}} \equiv \frac{1}{N}\sum_{\mathbf{k}}\left[1-\frac{\epsilon_{\mathbf{k}}-\mu}{E_{\mathbf{k}}}\tanh\frac{E_{\mathbf{k}}}{2T}\right], \tag{2.14}$$

where n_e and n_h are the electron and hole densities, respectively.

The current carried by Bloch electrons under the weak long-wavelength vector-potential field $A_i(\mathbf{q})$ are expressed as

$$j_i(\mathbf{q})=j_i^p(\mathbf{q})+j_i^d(\mathbf{q}) , \tag{2.15}$$

where the paramagnetic- and diamagnetic-current densities are given by

$$j_i^p(\mathbf{q})=-\frac{e}{\hbar}\sum_{\mathbf{k}}\Psi^\dagger_{\mathbf{k}-\mathbf{q}/2}\frac{\partial\epsilon_{\mathbf{k}}}{\partial k_i}\tau_0\Psi_{\mathbf{k}+\mathbf{q}/2} , \tag{2.16}$$

and

$$j_i^d(\mathbf{q})=-\frac{e^2}{c\hbar^2 N}\sum_{\mathbf{k}\mathbf{q}'}\sum_j\frac{\partial^2\epsilon_{\mathbf{k}}}{\partial k_i\partial k_j}\Psi^\dagger_{\mathbf{k}-\mathbf{q}/2+\mathbf{q}'/2}\tau_3 \times\Psi_{\mathbf{k}+\mathbf{q}/2-\mathbf{q}'/2}A_j(\mathbf{q}') , \tag{2.17}$$

respectively.[4] In Eq. (2.16), τ_0 is the 2×2 unit matrix. If we concentrate on the linear response to the weak field $A_i(\mathbf{q})$, the Fourier transform of the expectation value of the current density is expressed [the perturbing term $H_p=-(1/cN)\sum_{\mathbf{q}}\sum_i j_i(-\mathbf{q})A_i(\mathbf{q})$ is added to our Hamiltonian (2.4)] as

$$J_i(\mathbf{q},\omega)=-\frac{c}{4\pi}\sum_j K_{ij}(\mathbf{q},\omega)A_j(\mathbf{q},\omega) , \tag{2.18}$$

where the electromagnetic response kernel $K_{ij}(\mathbf{q},\omega)$ is given by

$$K_{ij}(\mathbf{q},\omega)=K_{ij}^d(\mathbf{q},\omega)+K_{ij}^p(\mathbf{q},\omega) . \tag{2.19}$$

Here, the diamagnetic term is obtained as

$$K_{ij}^d(\mathbf{q},\omega)=\frac{4\pi e^2}{c^2\hbar^2 N}\sum_{\mathbf{k}}\frac{\partial^2\epsilon_{\mathbf{k}}}{\partial k_i\partial k_j}n_{\mathbf{k}} . \tag{2.20}$$

The paramagnetic term is described by

$$K_{ij}^p(\mathbf{q},\omega)=\frac{4\pi}{c^2}P_{ij}(\mathbf{q},\omega+i\delta) , \tag{2.21}$$

with

$$P_{ij}(\mathbf{q},i\omega_m)\equiv-\frac{1}{N}\int_0^\beta d\tau\, e^{i\omega_n\tau}\langle j_i^p(\mathbf{q},\tau)j_j^p(-\mathbf{q},0)\rangle . \tag{2.22}$$

In the ladder-diagram approximation[7,8] (the diagram is almost the same as in Ref. 4), the paramagnetic term is expressed as

$$P_{ij}(\mathbf{q},i\omega_m)=\frac{e^2T}{N}\sum_{n,\mathbf{p}}\mathrm{Tr}[\gamma_i(p_-,p_+) \times G(p_+)\Gamma_j(p_+,p_-)G(p_-)] , \tag{2.23}$$

where $p_\pm\equiv(\mathbf{p}\pm\mathbf{q}/2,i\nu_n\pm i\omega_m/2)$; the free vertex $\gamma_i(p_-,p_+)$ is given by

$$\gamma_i(p_-,p_+)=\frac{1}{\hbar}\frac{\partial\epsilon_{\mathbf{p}}}{\partial p_i}\tau_0 , \tag{2.24}$$

and the vertex function $\Gamma_i(p_+,p_-)$ satisfies the linear integral equation

$$\Gamma_i(p_+,p_-)=\gamma_i(p_+,p_-)-\frac{T}{N}\sum_{l,\mathbf{k}}\tau_3G(k_+)\Gamma_i(k_+,k_-)G(k_-)\tau_3\tilde V(\mathbf{p}-\mathbf{k})+\tilde V(\mathbf{q})\tau_3\frac{T}{N}\sum_{l,\mathbf{k}}\mathrm{Tr}[\tau_3G(k_+)\Gamma_i(k_+,k_-)G(k_-)], \tag{2.25}$$

with $k_\pm\equiv(\mathbf{k}\pm\mathbf{q}/2,i\nu_l\pm i\omega_m/2)$. The last term on the right-hand side of Eq. (2.25) expresses the vacuum-polarization correction.[7,8] In our scheme, the effect of the Coulomb interaction $V_C(\mathbf{p}-\mathbf{k})$ in the second term of Eq. (2.25) is considered to be small. Hence, below, we will take $\tilde V(\mathbf{p}-\mathbf{k})$ in this term as $V(\mathbf{p}_\parallel-\mathbf{k}_\parallel)$ (as in the usual non-Bloch treatment[8]) so as to be consistent with Eq. (2.10).

III. RESPONSE KERNEL AND PENETRATION DEPTH

Here, by using the ladder-diagram approximation given in Eq. (2.25), we derive the *manifestly* gauge-invariant expression for the electromagnetic response kernel K_{ij} $(i,j=x,y,z)$ at zero frequency, and calculate the penetration depth for our layered system, in the same way as for the 2D case in Ref. 4. We also note that $\Delta^\alpha_{\mathbf{k}_\parallel}$ and $w^\alpha_{\mathbf{k}_\parallel}$ depend only on 2D vectors $\mathbf{k}_\parallel$. We assume that the solution $\Gamma_i(p_+,p_-)$ $(i=x,y,z)$ to Eq. (2.25) has the form

$$\Gamma_i(p_+,p_-)=\gamma_i(\mathbf{p})+\sum_{j=0}^{3}\sum_\beta w^\beta_{\mathbf{p}_\parallel}X^\beta_{ij}(\mathbf{q},i\omega_m)\tau_j + Y_{i3}(\mathbf{q},i\omega_m)\tau_3 , \tag{3.1}$$

for the superconducting α-wave state ($\alpha=es$ or d). After some manipulation, we can obtain the expressions for $X^\beta_{ij}(\beta=es,d,p_x,p_y)$ and Y_{i3}. At $\omega=0$, the analytically continued forms of X^β_{i1}, X^β_{i3}, and Y_{i3} vanish. We expand Eq. (2.25) [written in terms of X^β_{i2} and X^β_{i0}] up to the order of q^2. We can solve this equation by using Eq. (2.12), integration by parts, and some symmetries. In the following, we treat the energy correction $\delta\epsilon_{\mathbf{k}}$ given by Eq. (2.11), in a self-consistent symmetric way, as

$$\delta\epsilon_{\mathbf{k}}=w^{es}_{\mathbf{k}_\parallel}\frac{V}{2N}\sum_{\mathbf{q}}w^{es}_{\mathbf{q}_\parallel}n_{\mathbf{q}} \ . \tag{3.2}$$

Then the corrected quasiparticle energy has the same symmetrical property as the bare one, and the following result with respect to the symmetry coincides with that obtained by using the bare energy as usual.

By appropriate symmetrical considerations, we can see that $X^\beta_{i0}(\beta=es,d)$ and $X^\beta_{i2}(\beta=p_x,p_y)$ vanish. For the α-wave state, when we take the limit $q\rightarrow 0$, only X^α_{i2} has the essential contribution terms of the order $O(q^{-1})$ as

$$X^\alpha_{i2}(\mathbf{q},\omega=0)\cong-\frac{2\Delta_\alpha\sum_j R_{ij}q_j}{\sum_{l,m}q_lR_{lm}q_m}\sqrt{-1} \ , \tag{3.3}$$

where R_{ij} is given by

$$R_{ij}=\frac{2}{\hbar^2N}\sum_{\mathbf{k}}\frac{\Delta^\alpha_{\mathbf{k}_\parallel}}{E^2_{\mathbf{k}}}\frac{\partial\epsilon_{\mathbf{k}}}{\partial k_i}\left[\Delta^\alpha_{\mathbf{k}_\parallel}\frac{\partial\epsilon_{\mathbf{k}}}{\partial k_j}-(\epsilon_{\mathbf{k}}-\mu)\frac{\partial\Delta^\alpha_{\mathbf{k}_\parallel}}{\partial k_j}\right]\times\left[\frac{\partial f(E_{\mathbf{k}})}{\partial E_{\mathbf{k}}}+\frac{1}{2E_{\mathbf{k}}}\tanh\frac{E_{\mathbf{k}}}{2T}\right] , \tag{3.4}$$

with $f(x)=1/[\exp(x/T)+1]$. We have finite values of the order $O(1)$ for $X^{p_i}_{i0}(i=x,y)$. However, these coefficients only correct nonessentially $\gamma_i(\mathbf{k})=\hbar^{-1}(\partial\epsilon_{\mathbf{k}}/\partial k_i)\tau_0$ in Γ_i, corresponding to that correction [given in Eq. (2.11) or (3.2)] of $\epsilon_{\mathbf{k}}$ by the self-energy which we have neglected in Eq. (2.10), as in the 2D case.[4] So, we neglect these coefficients, and our solution reduces to that for the pure α-wave potential $V(\mathbf{p}_\parallel-\mathbf{k}_\parallel)=-Vw^\alpha_{\mathbf{p}_\parallel}w^\alpha_{\mathbf{k}_\parallel}$. It should be noted that the Coulomb potential has no effect on the vertex at $\omega=0$. The rather complicated expression (3.4) can be rewritten as

$$R_{ij}=\frac{1}{\hbar^2N}\sum_{\mathbf{k}}\left[2\frac{\partial\epsilon_{\mathbf{k}}}{\partial k_i}\frac{\partial\epsilon_{\mathbf{k}}}{\partial k_j}\frac{\partial f(E_{\mathbf{k}})}{\partial E_{\mathbf{k}}}+\frac{\partial^2\epsilon_{\mathbf{k}}}{\partial k_i\partial k_j}n_{\mathbf{k}}\right] . \tag{3.5}$$

Equation (3.5) can be easily transformed (by integrating the second term by parts) into its original form (3.4). By using Eq. (3.3) and Eqs. (2.19)–(2.25), we obtain the gauge-invariant form for the static total K_{ij} as

$$K_{ij}(\mathbf{q},0)=\frac{4\pi e^2}{c^2}\left[R_{ij}-\frac{\left[\sum_l R_{il}q_l\right]\left[\sum_m R_{jm}q_m\right]}{\sum_{l,m}q_lR_{lm}q_m}\right] . \tag{3.6}$$

This form is easily seen to satisfy the gauge-invariance condition at zero frequency: $\sum_j K_{ij}q_j=0$ (K_{ij} becomes purely transverse). Furthermore, it is considered to be the generalized form of the hydrodynamical kernel[9] for the singlet superconducting state; R_{ij} can be interpreted as the superfluid density per effective mass.[4] If R_{ij} has the simple form $R_{ii}\delta_{ij}$ as in our case, it follows that K_{ij} is of the form

$$K_{ij}(\mathbf{q},0)=\frac{4\pi e^2}{c^2}R_{ii}\left[\delta_{ij}-\frac{R_{jj}q_iq_j}{\sum_l R_{ll}q_l^2}\right] . \tag{3.7}$$

For $\mathbf{q}\|\hat{\mathbf{y}}$, in the limit of $q\rightarrow 0$, we have

$$K_{iy}(\mathbf{q},0)=K_{yi}(\mathbf{q},0)=0 \quad (i=x,y,z) \ , \tag{3.8}$$

and

$$K_{ij}(\mathbf{q},0)=\frac{4\pi e^2}{c^2}R_{ii}\delta_{ij} \quad (i,j=x,z) \ . \tag{3.9}$$

By considering the case of the applied magnetic field $\mathbf{H}\|\hat{\mathbf{z}}$, we obtain the in-plane penetration depth (the depth of the field penetration screened by the supercurrent flow parallel to the plane, in the x direction here) as

$$\lambda_\parallel(T)^{-2}=\frac{4\pi e^2}{c^2a^2d}R_{xx}(T) \ ,$$
$$=\frac{4\pi e^2}{c^2a^2d\hbar^2N}\sum_{\mathbf{k}}\left[2\left[\frac{\partial\epsilon_{\mathbf{k}}}{\partial k_x}\right]^2\frac{\partial f(E_{\mathbf{k}})}{\partial E_{\mathbf{k}}}+\frac{\partial^2\epsilon_{\mathbf{k}}}{\partial k_x^2}n_{\mathbf{k}}\right] . \tag{3.10}$$

Similarly, by considering the case $\mathbf{H}\|\hat{\mathbf{x}}$, we obtain the out-of-plane penetration depth (the depth of the field penetration screened by the supercurrent flow perpendicular to the plane) as

$$\lambda_\perp(T)^{-2}=\frac{4\pi e^2}{c^2a^2d}R_{zz}(T) \ ,$$
$$=\frac{4\pi e^2}{c^2a^2d\hbar^2N}\sum_{\mathbf{k}}\left[2\left[\frac{\partial\epsilon_{\mathbf{k}}}{\partial k_z}\right]^2\frac{\partial f(E_{\mathbf{k}})}{\partial E_{\mathbf{k}}}+\frac{\partial^2\epsilon_{\mathbf{k}}}{\partial k_z^2}n_{\mathbf{k}}\right] . \tag{3.11}$$

These expressions coincide with those obtained for the transverse field in the Hartree-Fock approximation. Here, however, Eq. (3.6) or (3.7) guarantees that the expressions (3.10) and (3.11) are really gauge invariant.

We calculate numerically $\lambda_\perp(T)$ as well as $\lambda_\parallel(T)$ as a function of n_h and T. The chemical potential μ is determined by Eq. (2.14). At $T=0$ K, only the second terms in Eqs. (3.10) and (3.11) contribute, and the resultant $\lambda_\perp(0)$ and $\lambda_\parallel(0)$ as functions of n_h are almost unaffected by the anisotropy of $\Delta^\alpha_{\mathbf{k}_\parallel}$ as shown in Fig. 1. They depend symmetrically on hole and electron densities. If we take $t_\parallel=0.15$ eV, $d=7.7$ Å and $a=3.85$ Å, almost following Schneider and Frick,[5] we obtain the value of the unit used in Fig. 1 as $\sqrt{m_{h\parallel}c^2a^2d/4\pi e^2}\simeq 743$ Å. Here, $m_{h\parallel}\equiv\hbar^2/2t_\parallel a^2$ is the effective in-plane hole mass when $n_h\rightarrow 0$. Then, for $n_h=0.2$, we have $\lambda_\parallel(0)\simeq 1783$ Å, which is little larger than the typical experimental value 1400 Å for high-T_c superconductors.[10] The resulting maximum T_c for $V/t_\parallel=2$ and $t_\perp/t_\parallel=0.1$ shown in Fig. 1 is estimated as 261 K for the extended s-wave state and 696 K for the d-wave state, and is too large compared with the experimental values. However, we can obtain the more realistic value by using smaller V almost without the changes of $\lambda_\parallel(0)$ and $\lambda_\perp(0)$ as shown later in Fig. 4 and in Sec. IV.

By adopting the lattice spacing ratio $d/a=2.0$, the resultant anisotropy ratio $\lambda_\perp(0)/\lambda_\parallel(0)$ is obtained as about 5 for most n_h with $t_\perp/t_\parallel=0.1$ as seen in Fig. 1. This value is reasonable with the experimental results for

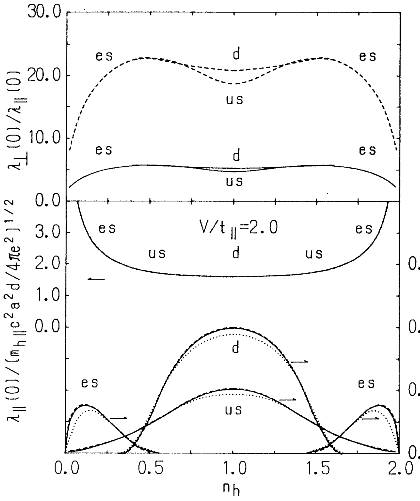

FIG. 1. The in-plane penetration depth $\lambda_\parallel(0)$, the anisotropy ratio $\lambda_\perp(0)/\lambda_\parallel(0)$ and the transition temperature T_c as functions of hole density n_h for $t_\perp/t_\parallel=0.025$ (dashed), 0.1 (solid), 0.3 (dotted but only shown for T_c). The resulting $\lambda_\parallel(0)$ (as well as T_c) for $t_\perp/t_\parallel=0.025$ almost coincides with that for $t_\perp/t_\parallel=0.1$. The result for the usual s-wave state denoted by us is added for comparison.

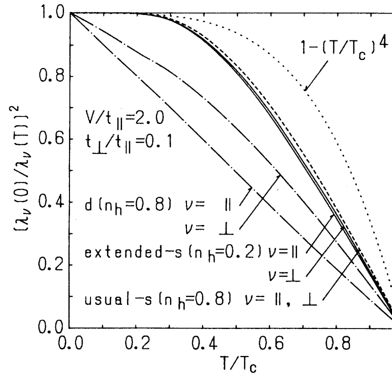

FIG. 2. The reduced in-plane penetration depth $\lambda_\parallel(T)/\lambda_\parallel(0)$ and the reduced out-of-plane penetration depth $\lambda_\perp(T)/\lambda_\perp(0)$ as functions of reduced temperature T/T_c. The results for the usual s-wave state and the empirical two-fluid model are added for comparison.

$YBa_2Cu_3O_{7-\delta}$.[10,11] However, according to the recent experiment[12] on the GL coherence length in $La_{2-x}Sr_xCuO_4$, the anisotropy ratio $\xi_{GL\parallel}(0)/\xi_{GL\perp}(0)$, which is considered to coincide with $\lambda_\perp(0)/\lambda_\parallel(0)$, is significantly larger than 5 and is rather reasonable with the value 20 (which can be obtained for most n_h with $t_\perp/t_\parallel=0.025$ in our model as seen in Fig. 1). These results are not changed for the more realistic case with smaller V, as seen in the next section. In the limit $n_h\to 0$, this ratio in Fig. 1 approximately coincides with that in the effective hole-mass approximation

$$\frac{\lambda_\perp(0)}{\lambda_\parallel(0)}=\left[\frac{t_\parallel a^2}{t_\perp d^2}\right]^{1/2}=\left[\frac{m_{h\perp}}{m_{h\parallel}}\right]^{1/2}, \tag{3.12}$$

which is estimated as 1.58 for $t_\perp/t_\parallel=0.1$, and 3.33 for $t_\perp/t_\parallel=0.025$ ($m_{h\perp}\equiv\hbar^2/2t_\perp d^2$ is the effective out-of-plane hole mass when $n_h\to 0$).

The behavior of $\lambda_\perp(T)/\lambda_\perp(0)$ as well as $\lambda_\parallel(T)/\lambda_\parallel(0)$ as a function T/T_c is affected by the anisotropy of the order parameter. For the d-wave state, the resulting $\lambda_\perp(T)/\lambda_\perp(0)$ as well as $\lambda_\parallel(T)/\lambda_\parallel(0)$ has the T-linear dependence in the low-temperature region, and substantially deviates from the extended and usual s-wave ones for all $T<T_c$ as shown Fig. 2 (where the results for the usual s-wave state and the empirical two-fluid model are added for comparison). In our layered system, the d-wave order parameter retains nodes on the Fermi surface.

IV. COHERENCE LENGTH

In this section, we calculate the GL coherence length $\xi_{GL}(0)$ by using their GL relation with the penetration depth. The critical field H_c is determined by

$$\frac{H_c^2(T)}{8\pi}=\frac{1}{Na^2d}(F_n-F_s)\,,$$
$$\cong\frac{1}{Na^2d}(\Omega_n-\Omega_s)\,, \tag{4.1}$$

where F_s (F_n) and Ω_s (Ω_n) are the free energy and the thermodynamic potential in the superconducting (normal) state, respectively. By the same treatment as the usual BCS one, for the α-wave state, we have

$$\frac{H_c^2(T)}{8\pi}=-\frac{1}{Na^2d}\int_0^V\frac{dV'}{V'}\langle H_{V'}\rangle\,,$$
$$=-\frac{1}{a^2d}\left[\frac{\Delta_\alpha^2}{V}-\frac{1}{N}\sum_{\mathbf{q}}(E_{\mathbf{q}}-|\epsilon_{\mathbf{q}}-\mu|)-\frac{2T}{N}\sum_{\mathbf{q}}\ln\left[\frac{1+e^{-\beta E_{\mathbf{q}}}}{1+e^{-\beta|\epsilon_{\mathbf{q}}-\mu|}}\right]\right]\,, \tag{4.2}$$

where H_V is the second term in our Hamiltonian (2.4) without $V_C(\mathbf{q})$, and we have used the equation $\langle H_V\rangle=-\Delta_\alpha^2/V$ and Eq. (2.12). The in-plane GL coherence length $\xi_{GL\parallel}(T)$ and the out-of-plane one $\xi_{GL\perp}(T)$ are obtained by the GL relations[6]

$$\xi_{GL\nu}(T)=\frac{\phi_0}{2\sqrt{2}\pi H_c(T)\lambda_\nu(T)}\quad(\nu=\parallel,\perp)\,, \tag{4.3}$$

where $\phi_0=\pi\hbar c/|e|$ is the flux quantum. These connections are only exact near T_c. We will proceed nonetheless with GL analysis, following Mirsiglio and Hirsch,[6] and use Eq. (4.3) to calculate the GL coherence length at $T=0$ K. The numerical result for $\xi_{GL\nu}(0)$ ($\nu=\parallel,\perp$) as a function of n_h is shown in Fig. 3. The anisotropy ratio $\xi_{GL\parallel}(0)/\xi_{GL\perp}(0)$ coincides with $\lambda_\perp(0)/\lambda_\parallel(0)$ as seen from Eq. (4.3).

To examine the above treatment itself, we calculate the spatial extent of the pair wave function, as a BCS-like coherence length $\xi_r(0)$, by following Mirsiglio and Hirsch:[6]

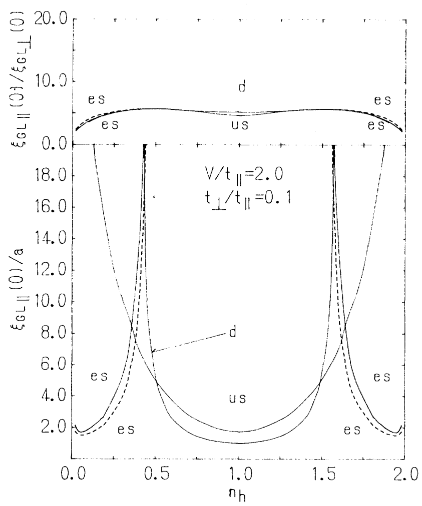

FIG. 3. The in-plane GL coherence length $\xi_{GL\parallel}(0)$ and the anisotropy ratio $\xi_{GL\parallel}(0)/\xi_{GL\perp}(0)$ as functions of hole density n_h (solid). Also are shown the in-plane BCS-like coherence length (the spatial extent of the pair wave function) $\xi_{r\parallel}(0)$ and the anisotropy ratio $\xi_{r\parallel}(0)/\xi_{r\perp}(0)$ (dashed but only shown for the extended s-wave state).

$$\xi_{r\nu}^2(T)\equiv\langle r_\nu^2\rangle\equiv\frac{\sum_{\mathbf{r}} g^*(\mathbf{r})r_\nu^2 g(\mathbf{r})}{\sum_{\mathbf{r}} g^*(\mathbf{r})g(\mathbf{r})} \quad (\nu=\parallel,\perp)\ , \tag{4.4}$$

where

$$g(\mathbf{r})=\frac{\langle c_{\mathbf{r}\uparrow}c_{0\downarrow}\rangle}{\langle c_{0\uparrow}c_{0\downarrow}\rangle}\ , \tag{4.5}$$

with

$$\langle c_{\mathbf{r}\uparrow}c_{0\downarrow}\rangle=\frac{1}{N}\sum_{\mathbf{k}}\frac{\Delta_{\mathbf{k}_\parallel}^\alpha}{2E_{\mathbf{k}}}\tanh\frac{E_{\mathbf{k}}}{2T}e^{i\mathbf{k}\mathbf{r}}\ . \tag{4.6}$$

At $T=0$ K, a combination of Eqs. (4.4)–(4.6) yields

$$\xi_{r\nu}^2(0)=\frac{(1/N)\sum_{\mathbf{k}}|\nabla_{\mathbf{k}}[\Delta_{\mathbf{k}_\parallel}^\alpha/2E_{\mathbf{k}}]|_\nu^2}{(1/N)\sum_{\mathbf{k}}[\Delta_{\mathbf{k}_\parallel}^\alpha/2E_{\mathbf{k}}]_\nu^2}\ . \tag{4.7}$$

As seen in Fig. 3, the difference between the resultant $\xi_{r\parallel}(0)/\xi_{r\perp}(0)$ and $\xi_{GL\parallel}(0)/\xi_{GL\perp}(0)$ as well as the difference between $\xi_{r\parallel}(0)$ and $\xi_{GL\parallel}(0)$ is small.

From Figs. 1 and 3, we can see that the out-of-plane tunneling $t_\perp$ in our model affect the result as follows. (i) It lowers T_c but almost does not narrow the superconducting region of n_h. (ii) It decreases (increases) $\lambda_\perp(0)$ $[\xi_{GL\perp}(0)]$ but it almost does not affect $\lambda_\parallel(0)[\xi_{GL\parallel}(0)]$.

The result in Fig. 3 is only for $t_\perp/t_\parallel=0.1$ and the resulting anisotropy ratio is estimated as about 5. However, as stated in Sec. III, according to the recent experiment[12] on the GL coherence length, the anisotropy ratio $\xi_{GL\parallel}(0)/\xi_{GL\perp}(0)$ is significantly larger than 5 and is rather reasonable with the value 20. This situation can be realized by setting $t_\perp/t_\parallel=0.025$ as seen in Fig. 1, but the result there has been obtained for the penetration depth. The result obtained here for the GL coherence length is shown in Fig. 4, but with $V/t_\parallel=1.0$ to realize the realistic T_c. By taking the same values $t_\parallel=0.15$ eV, $d=7.7$ Å, and $a=3.85$ Å as in Sec. III, we obtain the maximum T_c as 38.28 K for the extended s-wave state and 241.86 K for the d-wave state. We have $\xi_{GL\parallel}(0)\simeq 38.5\sim 154$ Å for most n_h in the extended s-wave state and $\xi_{GL\parallel}(0)=10.1\sim 154$ Å for most n_h in the d-wave state. The experimental result[12] is $\xi_{GL\parallel}(0)=31\sim 52$ Å for $T_c=32\sim 18$ K. Our $\xi_{GL\parallel}(0)$ has a reasonable value around the maximum T_c. For small T_c, however, it becomes much larger than the experimental result.[12] The result for $\xi_{GL\parallel}(0)$ obtained in Ref. 6 based on the kinetic-pairing mechanism has a reasonable value for small T_c; around the maximum T_c, however, it becomes rather smaller than the experimental result.[12] In both our result and the result of Ref. 6, $\xi_{GL\parallel}(0)$ increases with n_h more sharply than that in the recent experiment.[12] The value of the $\lambda_\parallel(0)$ for $n_h=0.2$ is almost not changed from 1783 Å obtained for $V/t_\parallel=2.0$ in Sec. III, and can be in the range of the experimental values.[10,11]

Furthermore the recent experimental result[12] for the anisotropy ratio $\xi_{GL\parallel}(0)/\xi_{GL\perp}(0)$, exhibits a sharp decrease (from about 50 to about 10) as carrier density increases in the low carrier-density region. This cannot be explained in our above model without any change. Here, we examine the case with the out-of-plane tunneling $t_\perp$ in proportion to carrier density, admitting that increasing carrier density may facilitate the electron conduction along the c axis.[12] We change the constant $t_\perp$ in Eq. (2.5) into $t_\perp\equiv\rho_0 t_\parallel n_h$ for the extended s-wave state with some constant ρ_0. We have some difficulty in choosing such form for the d-wave state. If we take $t_\perp\equiv\rho_0 t_\parallel n_h$, we cannot expect the sharp decrease of the anisotropy ratio as n_h increases in the d-wave region. Here, we choose

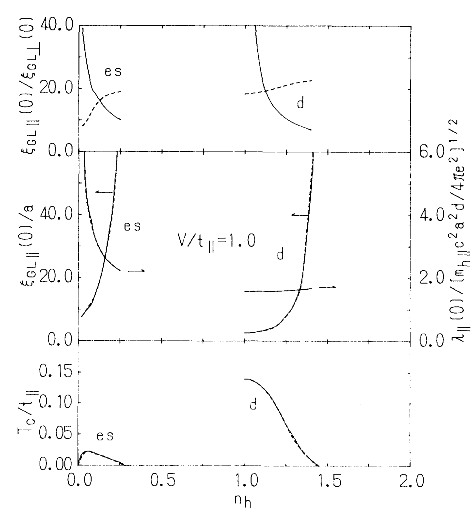

FIG. 4. The same as Figs. 1 and 3 but with $V/t_\parallel=1.0$ and with $t_\perp/t_\parallel=0.2n_h$ for the extended s-wave state (solid) and $t_\perp/t_\parallel=0.2(n_h-1)$ for the d-wave state (solid). Also is shown the result for $V/t_\parallel=1.0$ and $t_\perp/t_\parallel=0.025$ (dashed). The resulting $\xi_{GL\parallel}(0)$ [as well as $\lambda_\parallel(0)$ and T_c] for $t_\perp/t_\parallel=0.025$ almost coincides with that for $t_\perp/t_\parallel=0.2n_h$ or $t_\perp/t_\parallel=0.2(n_h-1)$.

$t_\perp \equiv \rho_0 t_\parallel (n_h - 1)$ for the d-wave state; in those recent treatments[13] of the strong correlation limit which support d-wave superconductivity, the carrier density or the doping fraction is considered to be $n_h - 1(\equiv 1 - n_e)$ rather than n_h. We retain, however, the unrenormalized $t_\parallel$ in this work. Our result [the sharp decrease of $\xi_{\mathrm{GL}\parallel}(0)/\xi_{\mathrm{GL}\perp}(0)$ from about 40 to about 10] can explain the experimental result[12] to some extent as seen in Fig. 4 with $\rho_0 = 0.2$, though not completely. In our treatment, the increasing tunneling $t_\perp$ also almost does not narrow the superconducting region of carrier density, though it lowers T_c by a small amount.

V. CONCLUSION AND DISCUSSION

In this work, we have investigated the London magnetic penetration depth in the layered narrow tight-binding band anisotropic superconductors with the nearest-neighbor in-plane pairing interaction and the nearest-neighbor out-of-plane tunneling. The *manifestly* gauge-invariant expression for the static electromagnetic response kernel has been derived within the framework of the ladder-diagram approximation. The results for $\lambda_\perp(0)$ and $\lambda_\parallel(0)$ (as functions of the hole density n_h) depend symmetrically on hole and electron densities, and are almost not affected by the anisotropy of the order parameter. The obtained anisotropy ratio $\lambda_\perp(0)/\lambda_\parallel(0)$ as well as $\lambda_\parallel(0)$ and T_c for the extended s-wave and d-wave states can be in the range of the experimental results for high-T_c superconductors.[10–12] For the d-wave state, the resultant reduced penetration depth $\lambda_\perp(T)/\lambda_\perp(0)$ (as a function of temperature T) as well as $\lambda_\parallel(T)/\lambda_\parallel(0)$ has T-linear behavior at low temperature and substantially deviates from those for the extended and usual s-wave states for all $T < T_c$.

We have also calculated the in-plane GL coherence length $\xi_{\mathrm{GL}\parallel}(0)$ and the out-of-plane one $\xi_{\mathrm{GL}\perp}(0)$ by assuming their GL relations with the penetration depths. The resultant anisotropy ratio $\xi_{\mathrm{GL}\parallel}(0)/\xi_{\mathrm{GL}\perp}(0)$ $[\equiv \lambda_\perp(0)/\lambda_\parallel(0)]$ as well as T_c for the extended s-wave and d-wave states can be in the range of the experimental results.[12] Our $\xi_{\mathrm{GL}\parallel}(0)$ has a reasonable value around the maximum T_c. For small T_c, however, it becomes much larger than the experimental result.[12] Our $\xi_{\mathrm{GL}\parallel}(0)$ increases with n_h more sharply than the recent experimental result.[12]

The out-of-plane tunneling $t_\perp$ in our model affect the result as follows. (i) It lowers T_c but almost does not narrow the superconducting region of n_h. (ii) It decreases (increases) $\lambda_\perp(0)[\xi_{\mathrm{GL}\perp}(0)]$ but it almost does not affect $\lambda_\parallel(0)[\xi_{\mathrm{GL}\parallel}(0)]$. If we introduce the tunneling $t_\perp$ in proportion to carrier density, the resultant anisotropy ratio $\xi_{\mathrm{GL}\parallel}(0)/\xi_{\mathrm{GL}\perp}(0)$ exhibits a sharp decrease as carrier density increases in the low carrier-density region, in agreement with the recent experiment.[12] In our treatment, the increasing tunneling $t_\perp$ also almost does not narrow the superconducting region of carrier density, though it lowers T_c by a small amount.

In this work, we have treated the transfer $t_\parallel$ and the tunneling $t_\perp$ as small and in the form of a narrow band. We have explained the experimental results for the high-T_c oxides to some extent. However, the electron-correlation effect is not included correctly. The effect of impurities and the strong coupling to the certain bosons have also been neglected. Including these effects remains a future problem.

ACKNOWLEDGMENT

The author would like to express sincere thanks to Professor S. Takada for helpful discussions and encouragement.

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