Effective lattice actions for correlated electrons

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We present an exact, unconstrained representation of the electron operators in terms of operators of opposite statistics. We propose a path-integral representation for the t-J model and introduce a parameter controlling the semiclassical behavior. We extend the functional approach to the Hubbard model and show that the mean-field theory is equivalent to considering, at Hamiltonian level, the Falikov-Kimball model. Connections with a bond-charge model are also discussed.

The investigation of the nature of the ground-state and low-lying excitations of strongly correlated electron systems is a fundamental problem of the modern many-body theory. The standard model of correlated electrons on a lattice is the Hubbard Hamiltonian

$$H_H = -t \sum_{i,j,s} \Lambda_{ij} c_{i,s}^\dagger c_{j,s} + U \sum_i n_{i,\uparrow} n_{i,\downarrow}, \qquad (1)$$

where Λ_{ij} is the symmetric adherence matrix connecting the nearest neighbor (NN) sites of a hypercubic lattice of M sites, $c_{i,s}^\dagger$ and $c_{i,s}$ are creation and annihilation operators for electrons with spin projection $s = \uparrow, \downarrow$, respectively, and $n_{i,s} = c_{i,s}^\dagger c_{i,s}$. Henceforth we denote the states at site i with the notation

$$|1,\Uparrow\rangle_i = c_{i,\uparrow}^\dagger |\nu\rangle, \qquad |0,\Uparrow\rangle_i = |\nu\rangle_i,$$
$$|1,\Downarrow\rangle_i = c_{i,\downarrow}^\dagger |\nu\rangle, \qquad |0,\Downarrow\rangle_i = c_{i,\downarrow}^\dagger c_{i,\uparrow}^\dagger |\nu\rangle, \qquad (2)$$

where $|\nu\rangle = \bigotimes_i |\nu\rangle_i$. The essential behavior of H_H in the strong-coupling regime $U/|t| \gg 1$ below half-filling is commonly investigated[1] by means of the effective Hamiltonian $H^{\text{eff}} = H_{tJ} + H^{(3)}$ acting in the subspace without doubly occupied states $|0,\Downarrow\rangle_i$, where

$$H_{tJ} = -t \sum_{i,j,s} \Lambda_{ij} \tilde{c}_{i,s}^\dagger \tilde{c}_{j,s} + J \sum_{\langle i,j \rangle} (\vec{S}_i \cdot \vec{S}_j - \tfrac{1}{4} N_i N_j) \qquad (3)$$

is the t-J model and $H^{(3)}$, not explicitly displayed,[1] is the three-site term. Here $\langle i,j \rangle$ denotes the summation over the NN sites, $J = 4t^2/U$, $\tilde{c}_{i,s} = c_{i,s}(1 - n_{i,\bar{s}})$, with $\bar{s}$ denoting the opposite spin projection, are projected electron operators, and $\vec{S}_i = \frac{1}{2} \sum_{s,p} c_{i,s}^\dagger \vec{\sigma}_{sp} c_{i,p}$ and $N_i = \sum_s \tilde{c}_{i,s}^\dagger \tilde{c}_{i,s}$ are the spin and number operators, respectively, with $\vec{\sigma}_{sp}$ the Pauli matrices.

To approach the problem of strong correlation, slave-boson (-fermion) methods[1,2] have been widely employed, because they give a physically intuitive way to work in the subspace without doubly occupied states and allow the introduction of mean-field approximations by assuming condensation of the bosons. In the present investigation we provide a path-integral description of models (1) and (3), starting from the observation that the decomposition of the electron operators via operators of opposite statis-

tics can be achieved in terms of exact operator identities. Hence, contrary to the slave-particle methods, in our approach constraints among the operators are absent. In fact, the states (2) can be generated by means of canonical spinless fermions $f_i^\dagger$ and Pauli operators $\vec{\sigma}_i$,

$$|1,\Uparrow\rangle_i = |v\rangle_i, \qquad |0,\Uparrow\rangle_i = f_i^\dagger |v\rangle,$$
$$|1,\Downarrow\rangle_i = \sigma_{i,-} |v\rangle, \qquad |0,\Downarrow\rangle_i = f_i^\dagger \sigma_{i,-} |v\rangle, \qquad (4)$$

where $\sigma_{i,\pm} = (\sigma_{i,x} \pm i\sigma_{i,y})/2$ and $|v\rangle = \bigotimes_i |v\rangle_i$ is the reference vacuum annihilated by the f_i's and $\sigma_{i,+}$'s. We observe that the representation is closed, since no new states can be generated by applying $f_i^\dagger$ and/or $\sigma_{i,-}$, so that the operators of a given basis can be expressed in terms of the others,

$$f_i^\dagger = \bar{n}_{i,\downarrow} c_{i,\uparrow} - n_{i,\downarrow} c_{i,\uparrow}^\dagger, \qquad \sigma_{i,-} = c_{i,\downarrow}^\dagger (c_{i,\uparrow} + c_{i,\uparrow}^\dagger), \qquad (5)$$
$$c_{i,\uparrow}^\dagger = \gamma_{i,+} f_i - \gamma_{i,-} f_i^\dagger, \qquad c_{i,\downarrow}^\dagger = \sigma_{i,-} (f_i + f_i^\dagger), \qquad (6)$$

where $\bar{n}_{i,s} = 1 - n_{i,s}$ and $\gamma_{i,\pm} = (1 \pm \sigma_{i,z})/2$. Hence, it is possible to construct nonlinear combinations of the electron operators satisfying $\{f_i^\dagger, f_j\} = \delta_{ij}$, $[\sigma_{i,\alpha}, \sigma_{j,\beta}] = 2i \delta_{ij} \epsilon^{\alpha\beta\gamma} \sigma_{j,\gamma}$ or, relevant for us, one sees that the anticommutation relations $\{c_{i,s}^\dagger, c_{j,p}\} = \delta_{ij} \delta_{sp}$ can be fulfilled via operators of opposite statistics.

For the moment, it is useful to avoid reference to a particular Fock representation. To this aim we say that model (1) [or (3)] acts on "objects"[3] $|\xi, \sigma\rangle_i$ of four (or three) different species, placed exactly one to a site, which we label by defining the "grade" $\pi(\xi) = (-)^\xi = \pm 1$ ($\xi = 0,1$), i.e., by dividing the species into even and odd, and the "isospin" $q_z = \pm\frac{1}{2}$ (pictorially $\sigma = \Uparrow, \Downarrow$) quantum numbers. The unitary transformations of the species assigned to each site are given by the Hubbard operators[1,2] $X_{i\xi'\sigma'}^{\xi\sigma} = |\xi,\sigma\rangle_i\langle\xi',\sigma'|_i$, which form the fundamental representation of the su(2|2) graded algebra.[1,4] The special character is due to the completeness relation $\sum_{\xi\sigma} N_{i,\xi\sigma} = 1$, where $N_{i,\xi\sigma} = X_{i\xi\sigma}^{\xi\sigma}$ are the local densities, expressing the one-to-one correspondence between objects and sites. With our notation, the anticommuting, or "odd," operators are those changing the grade [i.e., with $\pi(\xi) = -\pi(\xi')$], henceforth denoted

$$\chi_{i,\sigma} = X_{i1\sigma}^{0\Uparrow}, \quad \chi_i^\sigma = X_{i0\Uparrow}^{1\sigma}, \quad \tau_{i,\sigma} = X_{i1\sigma}^{0\Downarrow}, \quad \tau_i^\sigma = X_{i0\Downarrow}^{1\sigma}, \qquad (7a)$$

so that $\chi_i^\sigma = \chi_{i,\sigma}^\dagger$ and $\tau_i^\sigma = \tau_{i,\sigma}^\dagger$. For the commuting, or "even," operators [i.e., those with $\pi(\xi) = \pi(\xi')$], we define the linear combinations

$$S_{i,z} = \frac{1}{2}X_{i1\Uparrow}^{1\Uparrow} - \frac{1}{2}X_{i1\Downarrow}^{1\Downarrow}, \qquad S_{i,+} = X_{i1\Downarrow}^{1\Uparrow},$$

$$S_{i,t} = \frac{1}{2}X_{i1\Uparrow}^{1\Uparrow} + \frac{1}{2}X_{i1\Downarrow}^{1\Downarrow}, \qquad S_{i,-} = X_{i1\Uparrow}^{1\Downarrow}, \qquad (7\text{b})$$

$$L_{i,z} = \frac{1}{2}X_{i0\Uparrow}^{0\Uparrow} - \frac{1}{2}X_{i0\Downarrow}^{0\Downarrow}, \qquad L_{i,+} = X_{i0\Downarrow}^{0\Uparrow},$$

$$L_{i,t} = \frac{1}{2}X_{i0\Uparrow}^{0\Uparrow} + \frac{1}{2}X_{i0\Downarrow}^{0\Downarrow}, \qquad L_{i,-} = X_{i0\Uparrow}^{0\Downarrow}. \qquad (7\text{c})$$

For graded (i.e., supersymmetric) algebras, the grading of the states is a convention of purely formal nature, because the grading of the operators does not depend on the choice used. Physically, this freedom corresponds to the fact that the *particle-hole* transformation $|0,\sigma\rangle_i \leftrightarrow |1,\sigma\rangle_i$ leaves su(2|2) unaltered, i.e.,

$$(S_{i,t}, \vec{S}_i) \leftrightarrow (L_{i,t}, \vec{L}_i),$$

$$(\chi_i^\Uparrow, \chi_i^\Downarrow, \tau_i^\Uparrow, \tau_i^\Downarrow) \leftrightarrow (\chi_{i,\Uparrow}, \tau_{i,\Uparrow}, \chi_{i,\Downarrow}, \tau_{i,\Downarrow}), \qquad (8)$$

where $S_{i,\pm} = S_{i,x} \pm iS_{i,y}$ and $L_{i,\pm} = L_{i,x} \pm iL_{i,y}$. Hereafter we shall always use the correspondence (2), so that $\vec{S}_i$ and $\vec{L}_i$ are identified as the local spin and pseudospin operators, respectively. Henceforth we also define the isospin vector as the sum of spin and pseudospin: $\vec{Q}_i = \vec{S}_i + \vec{L}_i$. The total numbers of species-(ξ,σ) objects, $N_{\xi\sigma} = \sum_i N_{i,\xi\sigma}$, are related to the total numbers of spin-s electrons, henceforth denoted N_s, through the self-evident relations $N_\uparrow = N_{1\Uparrow} + N_{0\Downarrow}$ and $N_\downarrow = N_{1\Downarrow} + N_{0\Uparrow}$, from which we obtain the equivalent expressions

$$S_z = \frac{1}{2}(N_{1\Uparrow} - N_{1\Downarrow}), \qquad S_z = \frac{1}{2}(N_\uparrow - N_\downarrow),$$

$$L_z = \frac{1}{2}(N_{0\Uparrow} - N_{0\Downarrow}), \qquad L_z = \frac{1}{2}(M - N_\uparrow - N_\downarrow), \qquad (9)$$

where $\vec{S} = \sum_i \vec{S}_i$ and $\vec{L} = \sum_i \vec{L}_i$. The operators S_z and L_z are conserved because $N_\uparrow$ and $N_\downarrow$ are constant; however, from Eq. (9) we might also say that conservation follows because even and odd objects, when not conserved, are both created and/or destroyed in pairs. The latter is a useful way to visualize the less intuitive properties of the pseudospin. The picture is sharpened by observing that spin and pseudospin always act on $N_{\xi\sigma}$ as raising-lowering operators, thus changing $N_{\xi\sigma}$ by one unit in states with a definite number of objects. We have $[N_{0\Uparrow}, L_\pm] = \pm L_\pm$ and $[N_{0\Downarrow}, L_\pm] = \mp L_\pm$, as well as the commutators where $N_{0\sigma} \to N_{1\sigma}$ and $L_\pm \to S_\pm$.

Hubbard operators are useful for discussing symmetry properties and by using them it is easily seen that *any* transformation of the electron operators can be exactly rephrased in the basis (4). In particular, in the representations (2) and (4) the particle-hole transformation (8) is achieved by letting $c_{i,\uparrow}^\dagger \leftrightarrow c_{i,\uparrow}$ and $f_i^\dagger \leftrightarrow f_i$, respectively. The realization of Eq. (7) in the basis (4) is

$$\chi_i^\Uparrow = \gamma_{i,+}f_i, \quad \tau_i^\Uparrow = \sigma_{i,+}f_i, \quad S_{i,t} = \frac{1}{2}(1 - n_i),$$

$$\chi_i^\Downarrow = \sigma_{i,-}f_i, \quad \tau_i^\Downarrow = \gamma_{i,-}f_i, \quad \vec{S}_i = \frac{1}{2}\vec{\sigma}_i(1 - n_i), \qquad (10)$$

where $n_i = f_i^\dagger f_i$, and the operators not explicitly displayed are easily obtained by means of Eq. (8). The local densities are $N_{i,0\Uparrow} = \gamma_{i,+}n_i$, $N_{i,0\Downarrow} = \gamma_{i,-}n_i$, $N_{i,1\Uparrow} = \gamma_{i,+}(1 - n_i)$, $N_{i,1\Downarrow} = \gamma_{i,-}(1 - n_i)$, and automatically satisfy $\sum_{\xi\sigma} N_{i,\xi\sigma} = 1$. For the realization of Eq. (7) with electron operators we refer instead to Refs. 4, 5.

We now consider the auxiliary model[6] acting by permutation of the four species of objects $|\xi,\sigma\rangle_i$,

$$H^{(2|2)} = \sum_{\langle i,j\rangle} \left[-tP_{ij}^{01} + \frac{J}{2}(P_{ij}^{11} - P_{ij}^{00})\right], \qquad (11)$$

where $P_{ij}^{11} = 2(\vec{S}_i\vec{S}_j + S_{i,t}S_{j,t})$ and $P_{ij}^{00} = 2(\vec{L}_i\vec{L}_j + L_{i,t}L_{j,t})$ are the spin and pseudospin permutators acting nontrivially on the odd and even objects, respectively, and $P_{ij}^{01} = \sum_\sigma(\chi_i^\sigma\chi_{j,\sigma} + \tau_i^\sigma\tau_{j,\sigma}+\text{H.c.})$ is the operator permuting pairs of objects of opposite grade. Equation (11), henceforth referred to as the extended model, is the case $x = y = 2$ of the class of Hamiltonian $H^{(y|x)}$ permuting objects of y odd and x even different species, introduced by Sutherland[3] for the special case $J = \pm2|t|$ in studying exactly solvable systems in one dimension. The spectrum of $H^{(2|2)}$ contains that of $H^{(2|1)}$ by construction, and noting that the latter is actually the t-J model, we see that information about the Hamiltonian (3) can be obtained from the model (11) by imposing the *conserved* constraint $N_{0\Downarrow} = 0$. Using Eq. (10), $H^{(2|2)}$ becomes

$$H_{tJ}^{\text{ex}} = t\sum_{i,j} \Lambda_{ij}P_{ij}f_i^\dagger f_j + \frac{J}{2}\sum_{\langle i,j\rangle} \Delta_{ij}(P_{ij} - 1), \qquad (12)$$

where $P_{ij} = \frac{1}{2}(1 + \vec{\sigma}_i\cdot\vec{\sigma}_j)$ and $\Delta_{ij} = (1 - n_i - n_j)$, and where we have added a constant to ensure that Eq. (12) reduces to the standard definition (3) of the t-J model for $N_{0\Downarrow} = 0$. We consider the generalization (11) useful[7] because it leads in H_{tJ}^{ex} both (i) to the isotropic contribution P_{ij} in the hopping term, similarly to the generalization of Khaliullin,[8] and (ii) to the quadratic contribution Δ_{ij} in the magnetic term. In this respect, Eq. (12) is easier to study than the original t-J model, as for, e.g., the magnetic term of H_{tJ} in the basis (4) has a four-fermion interaction $M_{ij} = (1 - n_i)(1 - n_j)$.

For the time being the state $|0,\Downarrow\rangle_i$ will be interpreted as a fictitious "polarization state of the hole," and not as the physical doubly occupied state, because we are interested in Eq. (12) only as an auxiliary description of Eq. (3). From this interpretation it follows that the conserved quantity $G = \sum_\sigma N_{1\sigma}$ always corresponds to the number of electrons, $N_{\text{el}} = \sum_s N_s$, and $\delta = \sum_\sigma N_{0\sigma}/M = \sum_i n_i$ to the doping. The extended model commutes with $\vec{S}$ and $\vec{L}$, and from Eq. (9) one easily sees that for any given $G = N_{\text{el}}$ the condition $N_{0\Downarrow} = 0$ projects onto the sector where pseudospin L_z and total pseudospin L attain their maximum allowed value $L_z = L = (M - N_{\text{el}})/2$.

In a grand-canonical approach the partition function of the model (12) is

$$Z = \text{Tr}\exp\{-\beta H_\mu\}, \qquad H_\mu = H_{tJ}^{\text{ex}} - \sum_\sigma \mu^{0\sigma}N_{0\sigma}, \qquad (13)$$

where $\mu^{0\sigma}$ are the chemical potentials for the two species

of "holes." In the basis (4) a path-integral representation for Z can be built up immediately by using Grassmann variables η_i, η_i^* for the spinless fermions and *standard* SU(2) coherent states $|\vec{\Omega}_i\rangle$ for the isospin vectors. In fact, $\vec{Q}_i = \vec{S}_i + \vec{L}_i$ is the sum of two *reducible* operators, for which vector addition does not apply. The only eigenvalues of $Q_{i,\alpha}$ are $q_z = \pm\frac{1}{2}$ and indeed in the basis (4) we can write the isospin vector as a pure spin-$\frac{1}{2}$ operator: $\vec{Q}_i = \frac{1}{2}\vec{\sigma}_i$. Moreover, this property allows us to introduce an expansion parameter, denoted q, by enlarging the dimensionality of the SU(2) representation of $\vec{Q}_i$. For consistency we enlarge the whole even sector (7b) and 7(c) of su(2|2), setting $S_{i,t} = q(1-n_i)$ and $L_{i,t} = qn_i$ in Eq. (10). Following the spin-wave approach adopted in Ref. 6, the generalization of H_μ at arbitrary q is thus achieved by letting

$$(P_{ij} - 1) \rightarrow P_{ij}^{(-)} = 2(\vec{Q}_i\vec{Q}_j - q^2), \tag{14}$$

$$P_{ij} \rightarrow P_{ij}^{(+)} = 2(\vec{Q}_i\vec{Q}_j + q^2), \quad \gamma_{i,\pm} \rightarrow q \pm Q_{i,z}.$$

Due to the presence of Δ_{ij} in Eq. (12), the Grassmann integration over the variables η_i^* and η_i entering Z can be performed *exactly* and we obtain $Z = \int \mathcal{D}\vec{\Omega} \exp\{-S_{\text{eff}}\}$, where $\mathcal{D}\vec{\Omega}$ denotes the usual[9] integration over the classical unit vectors $\vec{\Omega}_i(\tau)$ and the effective lattice action is

$$S_{\text{eff}} = iq\sum_i \int_0^\beta d\tau \vec{A}_i \partial_\tau \vec{\Omega}_i + 4q^2\frac{J}{2}\sum_{\langle ij\rangle} \int_0^\beta d\tau \mathcal{P}_{ij}^{(-)} \tag{15}$$

$$-\text{Tr}\ln\{[\partial_\tau - 4q^2(\mu_i + \mu_-\Omega_{i,z})]\delta_{ij} + 4q^2 t\Lambda_{ij}\mathcal{P}_{ij}^{(+)}\},$$

with $\vec{A}_i$ the Dirac potential[9] for a monopole of unit strength, $\mathcal{P}_{ij}^{(\pm)} = \frac{1}{2}(\vec{\Omega}_i\vec{\Omega}_j \pm 1)$, and

$$\mu_\pm = \frac{1}{2q}\frac{\mu^{0\Uparrow} \pm \mu^{0\Downarrow}}{2}, \quad \mu_i = \mu_+ + \frac{J}{2}\sum_k \Lambda_{ik}\mathcal{P}_{ik}^{(-)}. \tag{16}$$

The model (12) behaves as a correlated hopping, and in fact μ_i enters S_{eff} as a rotationally invariant local chemical potential correlated to the background. In dealing with the extended model H_{tJ}^{ex} itself, the most natural choice is to treat the two species of holes equally, thus setting $\mu_- = 0$, whereas when considering the t-J model one has to impose the condition $\partial \ln Z/\partial\mu^{0\Downarrow} = 0$. As expected, in the latter case, the rotational symmetry in isospin space is explicitly broken, as μ_- is coupled to $\Omega_{i,z}$ and acts as an effective magnetic field. However, as discussed in Ref. 6, in the static limit $t = 0$ the ground-state energies of H_{tJ}^{ex} and H_{tJ} are degenerate and we expect the degeneracy to remain for a physically sensible range of the parameters $J/|t|$ and δ. Hence, in this case, one can investigate the zero-temperature properties of the t-J model directly with the extended Hamiltonian H_{tJ}^{ex}, thus working with the simpler condition $\mu_- = 0$.

The doping is accounted for by the third term on the right-hand side (rhs) of Eq. (15). This contribution is ineffective at zero doping, and because when $\delta = 0$ spin and isospin coincide (as for $\vec{L}_i = 0$), S_{eff}

correctly reduces to the standard lattice action of a quantum antiferromagnet.[9] We also notice that, contrary to other proposed functional integrals[10] for the t-J model, the imaginary part of Eq. (15) is a sum of standard Berry phases, so that the quantization condition on the monopole flux[9] is satisfied for any δ. We remark that the classical field $\vec{\Omega}_i(\tau)$ is the expectation value of the operator $\vec{Q}_i/q$; hence for $\delta \neq 0$ *physical* correlation functions can be evaluated only by differentiating Z with respect to suitable source terms, e.g., $H_s = \sum_i \vec{h}_i\vec{S}_i \equiv \sum_i \vec{h}_i(1-n_i)\vec{Q}_i$, added to H_μ. Allowing for these modifications of Eq. (13), the effective action is modified as well. However, the resulting Eq. (15) remains a purely classical expression, for which a gradient expansion[11] controlled by the parameter q allows extraction of the relevant low-energy behavior.

The basis (4) is suited to define an effective action also for the Hubbard model. To this aim one only needs to insert Eq. (6) into Eq. (1). However, before performing this straightforward calculation, we divide the lattice into two sublattices A and B, and perform the transformation $c_{i,s}^\dagger \rightarrow e^{i\theta_i}c_{i,s}^\dagger$, where $\theta_i = 0$ for $i \in A$ and $\theta_i = \frac{\pi}{2}$ for $i \in B$, so that H_H is mapped into an equivalent Hamiltonian $\hat{H}_H$, whose hopping matrix satisfies $t\hat{\Lambda}_{ij} = -t\hat{\Lambda}_{ji} = t\hat{\Lambda}_{ji}^*$. We make this change because it is known[12] that the operator $\vec{M}$, with components $M_- = \sum_i c_{i,\downarrow}^\dagger[(-)^{m_i}c_{i,\uparrow}^\dagger + c_{i,\uparrow}]$, $M_+ = M_-^\dagger$, and $M_z = \sum_i(\frac{1}{2} - c_{i,\downarrow}^\dagger c_{i,\downarrow})$, where $m_i = 0$ (1) for $i \in A$ (B), commutes with $H = H_H + UL_z$. By changing the phases we have $\vec{M} \rightarrow \vec{Q}$, because the alternating sign $\epsilon_i = (-)^{m_i}$ cancels, and $H_H \rightarrow \hat{H}_H$, so that $[\hat{H}_H + UL_z, \vec{Q}] = 0$. Hence, this choice of the phases leads to the representation of the Hubbard model

$$\hat{H}_H = -t\sum_{i,j} \hat{\Lambda}_{ij}\left[P_{ij}f_i^\dagger f_j + (P_{ij} - 1)C_{ij}\right] + H_{\text{at}}, \tag{17}$$

where $C_{ij} = \frac{1}{2}(f_if_j - f_j^\dagger f_i^\dagger)$ and $H_{\text{at}} = U\sum_i \gamma_{i,-n_i}$, with a manifestly isospin-invariant hopping. In Eq. (17) one easily identifies a term $\hat{H}_0$ conserving the number of objects of each species, and a term $\hat{H}_I$ leading instead to transitions $\Delta G = \pm 2$. Setting $\hat{\Lambda}_{ij} \rightarrow -\Lambda_{ij}$ via the transformation $f_i^\dagger \rightarrow e^{i\theta_i}f_i^\dagger$, we have $\hat{H}_0 \rightarrow \tilde{H}_0 = H_{\text{at}} + t\sum_{i,j} \Lambda_{ij}P_{ij}f_i^\dagger f_j$, and confronting this expression with the results in Ref. 5 one sees that $\tilde{H}_0$ is also the particular bond-charge model $H_{\text{bc}} = H_H + t\sum_{i,j,s} \Lambda_{ij}c_{i,s}^\dagger c_{j,s}(n_{i,\bar{s}} + n_{j,\bar{s}})$. The equivalence of these different looking models[13] is found because both have the same expression $\tilde{H}_0 = H_{\text{at}} - t\sum_{\langle i,j\rangle} P_{ij}^{01}$ in terms of abstract Hubbard operators, and holds because a global phase change in one basis leads instead to site-dependent transformations when rephrased in the other, e.g.,

$$f_i^\dagger \rightarrow e^{i\vartheta}f_i^\dagger \quad \leftrightarrow \quad c_{i,s}^\dagger \rightarrow \exp\{i\vartheta(2n_{i,\bar{s}} - 1)\}c_{i,s}^\dagger. \tag{18}$$

This is actually the main reason why, *before* making use of Eq. (6), we have brought H_H to a form with an antisymmetric adherence matrix. By replacing directly Eq.

(6) into H_H and $\vec{M}$, one obtains messy representations obscuring the result $[H_H + UL_z, \vec{M}] = 0$.

Using Eq. (14) and following the discussion leading to Eq. (15), the Hamiltonian $\hat{H}_\mu = \hat{H}_H - \mu N_{\mathrm{el}}$ can be extended at arbitrary q and its partition function represented as the path-integral $Z = \int \mathcal{D}\vec{\Omega} \exp\{-S_{\mathrm{eff}}^H\}$,

$$S_{\mathrm{eff}}^H = -4q^2\beta\mu_0 M + iq \sum_i \int_0^\beta d\tau \vec{\mathcal{A}}_i \partial_\tau \vec{\Omega}_i - \frac{1}{2}\ln\det[K_{ij}^{\alpha\beta}],$$

where $\mu_0 = \mu/(2q)$ and, setting $U_0 = U/(2q)$, $h = \mu_0 - U_0/2$,

$$K_{ij}^{11} = \left[\partial_\tau + 4q^2\left(\frac{U_0}{2} + h\,\Omega_{i,z}\right)\right]\delta_{ij} - 4q^2 t\hat{\Lambda}_{ij}\mathcal{P}_{ij}^{(+)},$$

$$K_{ij}^{12} = -4q^2 t\hat{\Lambda}_{ij}\mathcal{P}_{ij}^{(-)}, \qquad K_{ij}^{21} = -4q^2 t\hat{\Lambda}_{ij}\mathcal{P}_{ij}^{(-)}, \qquad (19)$$

$$K_{ij}^{22} = \left[\partial_\tau - 4q^2\left(\frac{U_0}{2} + h\,\Omega_{i,z}\right)\right]\delta_{ij} - 4q^2 t\hat{\Lambda}_{ij}\mathcal{P}_{ij}^{(+)}.$$

The square root of the determinant of the block matrix $K_{ij}^{\alpha\beta}$ enters into the effective action because,[14] due to the presence of $\mathcal{C}_{ij} = \frac{1}{2}(\eta_i\eta_j - \eta_j^*\eta_i^*)$ in the classical counterpart of $\hat{H}_H$, we have performed the integration with respect to the variables η_i^α, with $\eta_i^1 = \eta_i$, $\eta_i^2 = \eta_i^*$, satisfying to the condition $(\eta_i^\alpha)^* = \eta_i^\beta \sigma_{x,\beta\alpha}$. The Pfaffian structure of S_{eff}^H is quite natural: The off-diagonal blocks vanish only for a ferromagnetic background, and setting $\Omega_{i,\alpha} = \delta_{\alpha z}$ one can easily see that in this case both ef-

fective actions S_{eff} and S_{eff}^H correctly reduce to the logarithm of a free spinless fermion determinant. At half-filling one has $h = 0$, so that S_{eff}^H is rotationally invariant in the isospin. The symmetry is instead explicitly broken away from half-filling, as expected because of the relation $q\Omega_{i,z} = \langle\vec{\Omega}_i|S_{i,z} + q(1 - N_{\mathrm{el},i})|\vec{\Omega}_i\rangle$, where $N_{\mathrm{el},i}$ is the local electron number operator.

In general temporal fluctuations of the classical fields $\vec{\Omega}_i(\tau)$ (i.e., quantum fluctuations of the $\vec{Q}_i$'s) are suppressed by letting $q \to \infty$. Assuming the generalization (14), in Eq. (6) one has $c_{i,\uparrow}^\dagger \propto q \pm Q_{i,z}$ and $c_{i,\downarrow}^\dagger \propto Q_{i,-}$, and at large isospin we consistently[15] obtain $c_{i,\uparrow}^\dagger = O(q)$ and $c_{i,\downarrow}^\dagger = O(\sqrt{q})$, so that the propagation of the spin-$\downarrow$ electrons becomes smaller and eventually vanishes at $q = \infty$. Hence, at the mean-field level S_{eff}^H reduces to the effective action $S_{\mathrm{eff}}^{\mathrm{mf}}$ that one would have obtained by considering the Falikov-Kimball Hamiltonian.[1,16] This is a useful result, because one can reasonably obtain sensible information about the Hubbard model by systematically including corrections starting from a model, the Falikov-Kimball model, for which many exact results are known.[16]

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