Theory of electromagnetic modes of a magnetic superlattice in a transverse magnetic field: An effective-medium approach

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We derive effective-medium expressions, suitable for calculation of the properties of long-wavelength electromagnetic modes, for a magnetic superlattice in a magnetic field perpendicular to the interfaces. These are applied to a calculation of the dispersion relations for bulk and surface magnon polaritons of magnetic/nonmagnetic and magnetic/magnetic superlattices for both ferromagnetic and antiferromagnetic structures. Although we find no magnetostatic surface waves in such structures, in agreement with previous calculations, we predict retarded surface modes which are virtual. Such virtual modes have no magnetostatic limit.

I. INTRODUCTION

In recent years the investigation of collective excitations in magnetic superlattices has been a subject of increasing interest. Early theoretical work on magnetostatic modes in ferromagnetic superlattices was carried out by Camley, Rahman, and Mills[1] and by Grunberg and Mika.[2] They considered a ferromagnetic/nonmagnetic superlattice with magnetization and applied field $\mathbf{H}_0$ parallel to the interfaces. One of their most important results was that there is no surface magnetostatic mode (Damon-Eshbach mode) on a semi-infinite superlattice if the volume fraction occupied by the ferromagnet is less than 50%. Experimental confirmation by Brillouin light scattering on Ni/Mo superlattices was provided by Grimsditch *et al.*[3] A full account of magnetostatic modes with the field applied parallel to the surface was given by Camley[4] for both ferromagnetic and antiferromagnetic superlattices. Barnas[5] studied the retarded limit for an infinite layered structure consisting of alternating ferromagnetic and nonmagnetic films in the same geometry. He applied transfer-matrix methods to derive the general form of the dispersion relation. For wave-vector values away from the zone center, Albuquerque *et al.*[6] derived the dispersion equation for ferromagnetic superlattices for spin waves in the pure exchange region by means of the transfer-matrix method. Their numerical results are analogous to those found for phonons in diatomic superlattices.

The question of magnetostatic modes in a superlattice with magnetic field and magnetization normal to the interfaces was addressed by Camley and Cottam.[7] They gave a full account for bulk and surface waves on a ferromagnet/nonmagnetic (EuS/vacuum, Fe/vacuum), and an antiferromagnet/nonmagnetic (MnF_2/vacuum) superlattice. In particular, they showed that there are no magnetostatic surface modes in this geometry in the perfect semi-infinite structure.

It was realized later by Raj and Tilley[8] and Almedia and Mills[9] that when the excitation wavelength λ is much longer than the superlattice period D ($\lambda \gg D$), the magnetic superlattice can be described as an effective medium with permeability tensor elements given by averages over the unit cell of the superlattice. This followed similar earlier work[10–12] on dielectric superlattices; the expressions for dielectric media have been shown to give a good account of far-infrared spectra of long-period semiconductor superlattices.[13]

The earlier experimental work in magnetism was concerned with ferromagnet-based superlattices and most of the theoretical work concentrated on these. More recently, some attention has been given to antiferromagnetic-based superlattices. Neutron diffraction was used by Takano *et al.*[14] to study the CoO-NiO system. Ramos *et al.*[15] investigated the magnetic structure of FeF_2/CoF_2 superlattices by measuring the thermal-expansion coefficient.

A number of theoretical papers concerning antiferromagnetic superlattices have appeared. The effective-medium approximation was applied to uniaxial antiferromagnetic superlattices without an external field and with in-plane ordering by Almedia and Tilley.[16] This was extended to the case where ordering is perpendicular to the interfaces by Camley, Cottam, and Tilley.[17] For in-plane ordering, the effects of an applied magnetic field in plane and along the uniaxis were discussed by Oliveros *et al.*[18] They first derived the effective-medium permeability tensor and then applied it to a calculation of both magnetos-

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tatic (nonretarded) and polariton (retarded) modes. It is noteworthy that a magnetostatic surface mode on a semi-infinite superlattice occurs only if the magnetic volume fraction exceeds 50%, recalling the earlier theoretical and experimental results for ferromagnetic superlattices.[1–3] When the fraction is less than 50% surface polaritons are still found, but they are "virtual" with no magnetostatic limit.

In this paper we derive effective-medium expressions suitable for the calculation of properties of long-wavelength modes for a magnetic superlattice with perpendicular ordering and applied field. These expressions apply for both ferromagnetic and antiferromagnetic superlattices. They are applied to a calculation of dispersion equations for bulk and surface polariton modes. We present numerical examples for the ferromagnetic system YIG/YAG (yttrium iron garnet/yttrium aluminium garnet), and the antiferromagnetic systems FeF_2/ZnF_2 and FeF_2/MnF_2.

The organization of the paper is as follows. In Sec. II we derive the general forms of the effective-medium tensors. In Sec. III the results are used to derive the general dispersion relation for bulk and surface polariton modes. Numerical results are presented in Sec. IV and Sec. V presents the conclusions.

II. PERMEABILITY TENSOR AND DIELECTRIC TENSOR

We consider a magnetic superlattice with interfaces perpendicular to the y axis and with a magnetic field in the y-axis direction as shown in Fig. 1. Each layer $\beta(=a$ or $b)$ is characterized by a dielectric constant ε^{β} and permeability tensor

$$\overleftrightarrow{\mu}^{\beta}(\omega)=\begin{bmatrix}\mu_1^{\beta} & 0 & -i\mu_2^{\beta}\\ 0 & \mu_3^{\beta} & 0\\ i\mu_2^{\beta} & 0 & \mu_1^{\beta}\end{bmatrix} . \tag{1}$$

Here μ_1^{β} and μ_2^{β} have resonances at the corresponding ferromagnetic or antiferromagnetic resonance frequencies. The explicit forms of μ_1^{β} and μ_2^{β} for a ferromagnet are[19,20]

$$\mu_1^{\beta}=1+\frac{4\pi\gamma^2 H_i M}{\gamma^2 H_i^2-\omega^2} , \tag{2}$$

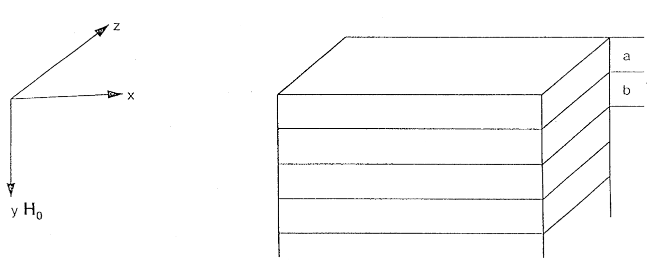

FIG. 1. Magnetic superlattice. Layers of thickness d_a and d_b alternate and a magnetic field is applied perpendicular to the interface along the y axis.

$$\mu_2^{\beta}=\frac{4\pi\gamma^2 \omega M}{\gamma^2 H_i^2-\omega^2} , \tag{3}$$

where γ, H_i, and M depend on the layer index β. Here H_i is the internal magnetic field in the y direction, which is given by

$$H_i=H_0-4\pi M , \tag{4}$$

where H_0 is the external applied field, M is the saturation magnetization perpendicular to the surface, and γ is the gyromagnetic ratio. The reduction of the internal magnetic field is caused by the demagnetization effect. For an antiferromagnet, the elements of $\overleftrightarrow{\mu}^{\beta}(\omega)$ are[19,20]

$$\mu_1^{\beta}=1+\frac{4\pi\gamma^2 H_{an} M}{\omega_0^2-(\omega+\gamma H_0)^2}+\frac{4\pi\gamma^2 H_{an} M}{\omega_0^2-(\omega-\gamma H_0)^2} , \tag{5}$$

$$\mu_2^{\beta}=\frac{4\pi\gamma^2 H_{an} M}{\omega_0^2-(\omega+\gamma H_0)^2}-\frac{4\pi\gamma^2 H_{an} M}{\omega_0^2-(\omega-\gamma H_0)^2} , \tag{6}$$

where

$$\omega_0=|\gamma|[H_{an}(2H_e+H_{an})]^{1/2} \tag{7}$$

is the antiferromagnetic resonance frequency. In the above equations H_{an} is the anisotropy field, H_e is the exchange field, and μ_3^{β} is nonresonant for both ferromagnets and antiferromagnets, and is a positive constant. Again the material parameters in (5) to (7) depend on the layer index β.

As was argued previously[8,9] the field components H_x, H_z, and B_y are constant over many layers since they are continuous at the interfaces, whereas the other components are given by spatial averages over the values in each layer. Thus

$$\langle H_{x,z}\rangle=H_{x,z}^{a}=H_{x,z}^{b} , \tag{8a}$$

$$\langle H_y\rangle=f_a H_y^a+f_b H_y^b , \tag{8b}$$

$$\langle B_{x,z}\rangle=f_a B_{x,z}^a+f_b B_{x,z}^b , \tag{8c}$$

and

$$\langle B_y\rangle=B_y^a=B_y^b , \tag{8d}$$

where $f_a(=d_a/D)$ and $f_b(=d_b/D)$ are the volume fractions occupied by layers a and b. With use of

$$\mathbf{B}=\overleftrightarrow{\mu}\cdot\mathbf{H} , \tag{9}$$

we derive from (8) the following expression for the effective permeability tensor:

$$\overleftrightarrow{\mu}(\omega)=\begin{bmatrix}\mu_1 & 0 & -i\mu_2\\ 0 & \mu_3 & 0\\ i\mu_2 & 0 & \mu_1\end{bmatrix} , \tag{10a}$$

where

$$\mu_1=\mu_1^a f_a+\mu_1^b f_b , \tag{10b}$$

$$\mu_2=\mu_2^a f_a+\mu_2^b f_b , \tag{10c}$$

and

$$\mu_3^{-1}=(\mu_3^a)^{-1}f_a+(\mu_3^b)^{-1}f_b \ . \tag{10d}$$

The effective dielectric tensor can be derived by applying the same procedures to the averaged fields $\langle \mathbf{D} \rangle$ and $\langle \mathbf{E} \rangle$. Straightforward algebra gives the effective dielectric tensor in the well-known form[10–12]

$$\overleftrightarrow{\varepsilon}=\begin{bmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\parallel & 0 \\ 0 & 0 & \varepsilon_\perp \end{bmatrix} , \tag{11a}$$

where

$$\varepsilon_\perp=f_a\varepsilon^a+f_b\varepsilon^b \tag{11b}$$

and

$$\varepsilon_\parallel=\frac{\varepsilon^a\varepsilon^b}{f_a\varepsilon^b+f_b\varepsilon^a} \ . \tag{11c}$$

The effective-medium tensors $\overleftrightarrow{\mu}$ and $\overleftrightarrow{\varepsilon}$ may now be utilized to obtain the polariton excitations in various systems.

III. BULK AND SURFACE POLARITONS

Now we apply the results of Sec. II to determine bulk and surface magnon polariton modes. The dispersion relation for bulk polaritons propagating in an infinite effective medium follows from Maxwell's equations. After eliminating $\mathbf{E}$ in the curl equations, one obtains the following wave equation for $\mathbf{H}$:

$$\mathbf{k}\times\overleftrightarrow{\varepsilon}^{-1}\cdot(\mathbf{k}\times\mathbf{H})+q_0^2\overleftrightarrow{\mu}\cdot\mathbf{H}=0 \ , \tag{12}$$

where $q_0=\omega/c$ is the vacuum wave vector, ω the angular frequency and $\overleftrightarrow{\varepsilon}^{-1}$ is the inverse of (11a). Since the x and z axis are equivalent we take $\mathbf{k}=(k_x,k_y,0)$ and (12) is rewritten

$$\begin{bmatrix} q_0^2\varepsilon_\perp\mu_1-k_y & k_xk_y & -iq_0^2\varepsilon_\perp\mu_2 \\ k_xk_y & q_0^2\varepsilon_\perp\mu_3-k_x & 0 \\ iq_0^2\varepsilon_\parallel\varepsilon_\perp\mu_2 & 0 & q_0^2\varepsilon_\parallel\varepsilon_\perp\mu_1-k_x-k_y \end{bmatrix}\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}=0. \tag{13}$$

Equation (13) is a set of three linear homogeneous equations satisfied by $\mathbf{H}$ in the effective medium. The condition for a nontrivial solution is

$$\varepsilon_\parallel\mu_3\alpha^4-(\varepsilon_\parallel\mu_1k_3^2+\varepsilon_\perp\mu_3k_1^2)\alpha^2 +k_3^2(\varepsilon_\perp\mu_1k_1^2+q_0^2\mu_2^2\varepsilon_\parallel\varepsilon_\perp)=0 \ , \tag{14}$$

where

$$k_1^2=k-q_0^2\varepsilon_\parallel\mu_1 \tag{15a}$$

and

$$k_3^2=k^2-q_0^2\varepsilon_\perp\mu_3 \ . \tag{15b}$$

Here with a view to later applications to surface modes, we define $\alpha^2=-k_y^2$, and we have put $k=k_x$. The solution of (14) is given by

$$\alpha^2=k^2\left[\frac{\varepsilon_\parallel\mu_1+\varepsilon_\perp\mu_3}{2\varepsilon_\parallel\mu_3}\right]-q_0^2\varepsilon_\perp\mu_1 \pm\left[k^4\left[\frac{\varepsilon_\parallel\mu_1-\varepsilon_\perp\mu_3}{2\varepsilon_\parallel\mu_3}\right]^2+\frac{q_0^2k_3^2\varepsilon_\perp\mu_2^2}{\varepsilon_\parallel\mu_3}\right]^{1/2} , \tag{16}$$

and henceforth α_1 is the root with the positive sign and α_2 the root with the negative sign.

Depending upon the position in the ω-k plane, the following possibilities may arise, (i) α_1 and α_2 are both real and positive, (ii) one is real and the other is pure imaginary, (iii) both are complex in which case they are conjugate, and (iv) both are pure imaginary. One can classify the surface modes corresponding to these possibilities[22] as (i) bonafide surface modes, (ii) pseudosurface modes, (iii) generalized surface modes and, (iv) bulk modes. In all these cases (16) shows that the medium is birefringent meaning that for a given frequency there are two values of the wave vector $\mathbf{k}$. As in all birefringent crystals[24] each of the two modes has a definite polarization determined by (13). In this paper we concentrate on the bonafide surface modes for which α_1 and α_2 are real and positive.

To derive the surface polariton dispersion relation, we consider an interface at $y=0$ (Fig. 1) between the superlattice and vacuum and seek solutions of Maxwell's equations of the form

$$\mathbf{H}=(H_{0x},H_{0y},H_{0z})\exp(\alpha_0y), \quad y<0 \tag{17}$$

$$\mathbf{H}=(H_{1x},H_{1y},H_{1z})\exp(-\alpha_1y) +(H_{2x},H_{2y},H_{2z})\exp(-\alpha_2y), \quad y>0 \ , \tag{18}$$

where $\exp[i(kx-\omega t)]$ is an implicit common factor in (17) and (18). Here $\alpha_0^2=k^2-q_0^2$ and α_1,α_2 are the two solutions of (16). As the effective medium is birefringent, the general solution of the wave equation is a linear superposition of the two terms, as written in (18). The polarization in each term is determined so the value of only one field component, say H_z, determines the others. Thus there are just two unknowns in (18), say H_{1z} and H_{2z}. On the other side of the interface, since (17) relates to an isotropic medium, the polarization is not definite. Therefore (17) also includes two unknowns, say H_{0x} and H_{0z}. The other component H_{0y} and the components of $\mathbf{E}$ are found in terms of these from Maxwell's equations.

The determination of the surface polariton dispersion relation requires the imposition of electromagnetic boundary conditions at $y=0$, namely continuity of the tangential components of the magnetic and electric fields H_x, H_z, E_x, and E_z. Making use of the wave equation (13) with Maxwell's curl equations, we express the tangential components in terms of the unknowns H_{0x}, H_{0z}, H_{1x}, and H_{2z}. The boundary conditions then yields

$$H_{0z}=H_{1z}+H_{1z} \ , \tag{19a}$$

$$iq_0^2\varepsilon_\parallel\varepsilon_\perp\mu_2H_{0x}=\sigma_1H_{1z}+\sigma_2H_{2z} \ , \tag{19b}$$

$$\frac{iq_0^2k_3^2\mu_2\varepsilon_\parallel\varepsilon_\perp}{\alpha_0\mu_3}H_{0x}=\alpha_1H_{1z}+\alpha_2H_{1z} \ , \tag{19c}$$

$$\varepsilon_\perp \alpha_0 H_{0z} = \alpha_1 H_{1z} \alpha_2 H_{2z} \ , \tag{19d}$$

where

$$\sigma_{1,2} = \varepsilon_\perp k_1^2 - \varepsilon_\parallel \alpha_{1,2}^2 \ . \tag{20}$$

Equations (19) are four homogeneous equations in terms of four unknown amplitudes H_{0x}, H_{0z}, H_{1z}, and H_{2z}. Setting up these equations in matrix form and applying the condition for a nontrivial solution gives us the required dispersion equation in the form

$$\frac{k_3^2}{\alpha_0 \mu_3}[\varepsilon_\perp k_1^2 + \varepsilon_\parallel \alpha_1 \alpha_2 + \varepsilon_\parallel \varepsilon_\perp \alpha_0(\alpha_1+\alpha_2)] + \varepsilon_\parallel \alpha_1 \alpha_2(\alpha_1+\alpha_2) + \varepsilon_\parallel \varepsilon_\perp \alpha_0(\alpha_1^2+\alpha_2^2+\alpha_1\alpha_2) - \varepsilon_\perp^2 \alpha_0 k_1^2 = 0 \ . \tag{21}$$

It is noted that this dispersion relation has the same mathematical form as that obtained for a magnetoplasma medium.[22,23] We are now in a position to investigate numerically the magnon polaritons of various magnetic multilayers.

IV. NUMERICAL RESULTS

In (16) and (21) k only appears in even powers, so that the dispersion curves for positive and negative k are identical. That is, the surface mode propagation is reciprocal, as required on symmetry grounds for the present case where the applied magnetic field is perpendicular to the surface.[4]

It will be useful to obtain the expression for the nonretarded, or magnetostatic, limit $k \gg q_0$ which is mathematically equivalent to taking $c \to \infty$. In this case $k_1 = k_3 = \alpha_0 = k$, together with

$$\alpha_1 = \left[\frac{\mu_1}{\mu_3}\right]^{1/2} k \tag{22a}$$

and

$$\alpha_2 = \left[\frac{\varepsilon_\perp}{\varepsilon_\parallel}\right]^{1/2} k \tag{22b}$$

and (21) now reduces to

$$\mu_3 \left[\frac{\mu_1}{\mu_3}\right]^{1/2} = -1 \ . \tag{23}$$

Equations (22a) and (23) represent the dispersion relation for the magnon polaritons in the nonretarded limit for any effective-medium superlattice in a transverse field. One can derive (22a) and (23) directly by solving Maxwell's equations in the magnetostatic limit. It is immediately obvious from (22a) and (23) that μ_1 and μ_3 must both be negative in order for the modes to have a magnetostatic limit. The expression for μ_3 given by (10d), where μ_3^a and μ_3^b are positive constants implies that $\mu_3 > 0$. There are therefore, no magnetostatic surface modes for this geometry which is clear from our result (when $k \to \infty$). This is in accord with the conclusion of Camley and Cottam.[7]

The frequency windows where surface magnon polaritons may reside and the boundaries of the bulk continua can be determined from (16) by putting $\alpha = 0$. There are two solutions

$$k^2 = q_0^2 \varepsilon_\parallel \mu_v \tag{24}$$

for the surface windows, where $\mu_v = \mu_1 - \mu_2^2/\mu_1$. The boundaries of the bulk continua are given by

$$k^2 = q_0^2 \varepsilon_\perp \mu_3 \ , \tag{25}$$

which is analogous to a light line since $\varepsilon_\perp$ and μ_3 are constant. We henceforth refer to this line as the bulk line.

We have solved the dispersion relations numerically to determine the surface modes of ferromagnetic and antiferromagnetic superlattices. We start with the former. Typical results are shown in Fig. 2 for the case when layer a is ferromagnetic YIG and b is nonmagnetic with dielectric constant $\varepsilon^b > 0$. Figure 2(a) illustrates the case $f_a = 1$ corresponding to a semi-infinite YIG sample. There are three bulk regions (shaded area). The surface polariton curve is located in the reststrahl regions between the bulk continuum regions. We find only a single virtual surface mode (virtual modes have no magnetostatic limit, that is they do not persist to $k \to \infty$), in the lowest window, merging at the high-frequency end into a bulk region at finite k. These features of Fig. 2(a) are preserved in Figs. 2(b)–2(d), but with a change of scale. The reststrahl regions and the middle bulk region become narrower. For example, the widths of the two reststrahl bands in Fig. 2(a) are in the frequency ranges $\omega = (1.29-1.53 \text{ cm}^{-1})$ and $(1.74-2.04 \text{ cm}^{-1})$ whereas in Fig. 2(d) the ranges are $\omega = (1.29-1.35 \text{ cm}^{-1})$ and $(4.14-1.45 \text{ cm}^{-1})$.

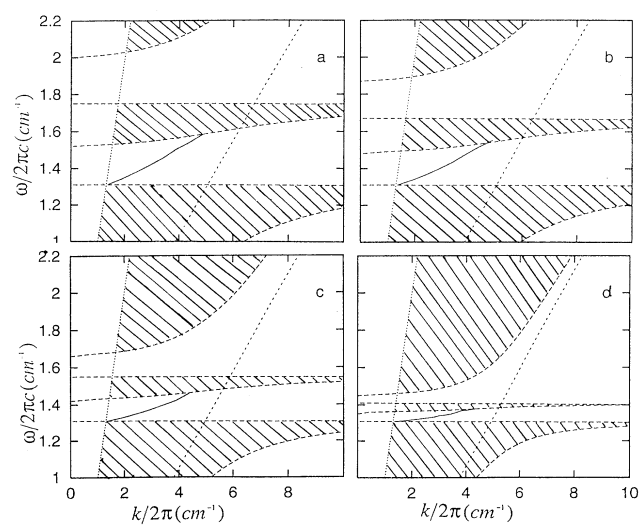

FIG. 2. Bulk continua and surface polariton dispersion curves ω versus k for a ferromagnetic superlattice, where layer a is YIG, with $H_0 = 5$ T, $\varepsilon^a = 15.0$, $M_a = 0.175$ T, and $\gamma_a = 0.467$ cm^{-1}/T [Raj and Tilley (Ref. 8)] and layer b is nonmagnetic with $\varepsilon^b = 14.8$ shaded area bulk continuum, —— surface wave, — — — bulk line, · · · · light line, (a) $f_a = 1.0$, (b) $f_a = 0.8$, (c) $f_a = 0.5$, and (d) $f_a = 0.2$.

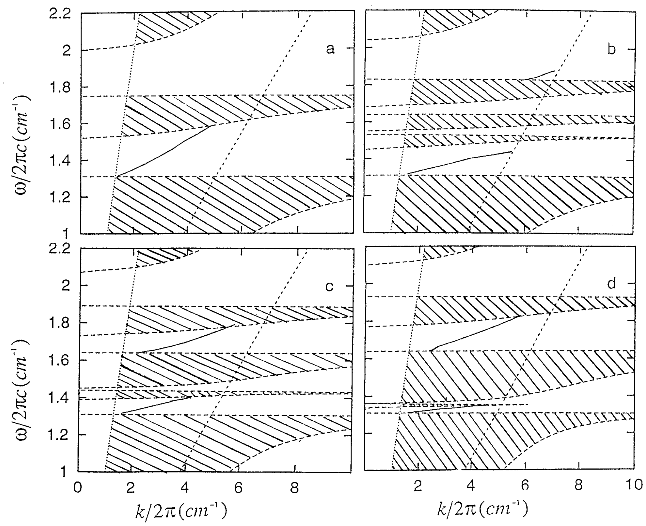

FIG. 3. Bulk continua and surface polariton dispersion curves ω versus k for YIG/YAG with same parameters as in Fig. 2 except $\varepsilon^b=14.8$, $M_b=0.12$ T, $\gamma_b=\gamma_a$ [Raj and Tilley (Ref. 8)]; shaded area bulk continuum —— surface waves, — — — bulk line, · · · · light line, (a) $f_a=1.0$, (b) $f_a=0.8$, (c) $f_a=0.5$, and (d) $f_a=0.2$.

We now consider the case where both layers are ferromagnetic. We give in Fig. 3 bulk and surface dispersion curves for a YIG/YAG superlattice (the material parameters may be found in Raj and Tilley[8]). Figure 3(a) shows $f_a=1.0$, like Fig. 2(a). The results including a YAG layer are illustrated in Figs. 3(b)–3(d), and one can see that two new bulk regions appear. In Fig. 3(b) there are two virtual surface modes, terminating on a bulk line. In Figs. 3(c) and 3(d) there are two surface modes which behave in the same manner as those of Fig. 3(a).

We now turn to an antiferromagnetic superlattice and describe the bulk and surface modes in the long-wavelength limit for FeF_2/ZnF_2, (antiferromagnet/nonmagnetic) and FeF_2/MnF_2, (antiferromagnet/antiferromagnet). The parameters used for FeF_2 are given by Brown *et al.*[21] and for MnF_2 by Camley and Cottam.[7] The results for the former are illustrated in Figs. 4. In Fig. 4(a), for $f_a=1$ (bulk FeF_2), there are three bulk continuum regions. The two surface modes are located between the bulk regions starting on the vacu-

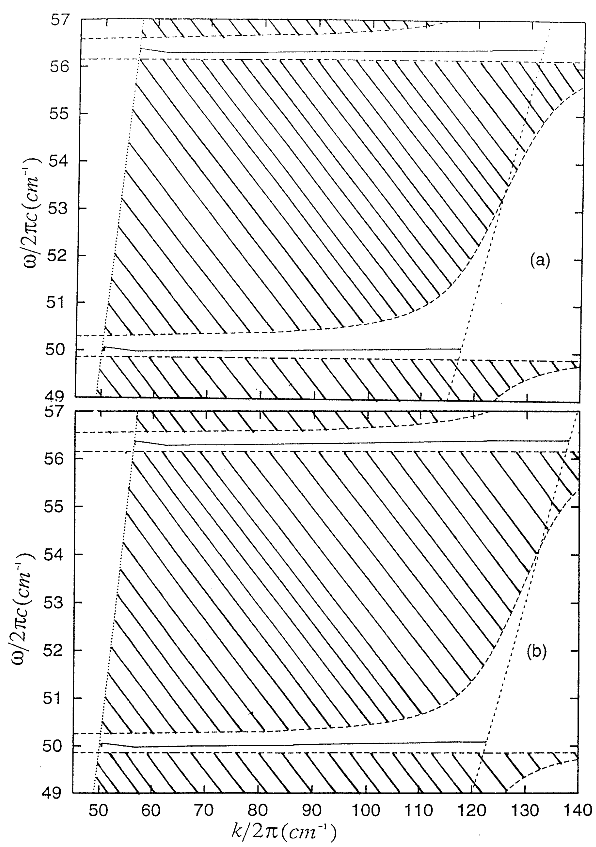

FIG. 4. Bulk continua and surface polariton dispersion curves ω versus k for FeF_2/ZnF_2, with [Brown *et al.* (Ref. 21)] $H_0=3$ T, $M_a=0.05T$, $H_{an}^a=20$ T, $H_e^a=54$ T, $\gamma_a=1.05$ cm^{-1}/T, and $\varepsilon_a=5.5$, and $\varepsilon_b=8$, shaded area bulk continuum, —— surface waves, — — — bulk line, · · · · light line, (a) $f_a=1.0$, (b) $f_a=0.8$.

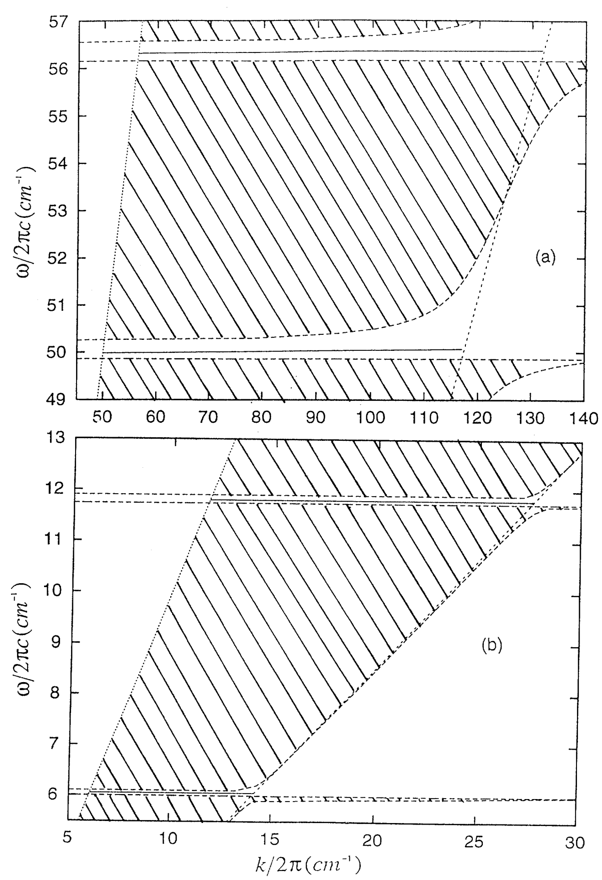

FIG. 5. Bulk continua and surface polariton dispersion curves ω versus k for FeF_2/MnF_2, with $f_a=0.8$ and [Camley and Cottam (Ref. 7)] $M_b=0.06T$, $H_{an}^b=0.785$ T, $H_e^b=55T$, $\gamma_b=4.5$ cm^{-1}/T, $H_0=3$ T and $\varepsilon_b=5.5$, shaded area bulk continuum, —— surface waves, — — — bulk line, · · · · light line. (a) FeF_2 bulk and surfacelike modes, (b) MnF_2 bulk and surfacelike modes.

um light line ($\omega=ck$) and ending at the bulk line. These features of Fig. 4(a) are preserved in Fig. 4(b) (where $f_a=0.8$). In fact, we find that the surface modes energies vary only marginally with f_a, although with decreasing f_a the windows become narrower.

Figure 5 illustrates the surface modes for an FeF_2/MnF_2 superlattice with $f_a=0.8$. There are four surface modes, two reside in the FeF_2 resonance region [Fig. 5(a)] and the other two in the MnF_2 resonance region [Fig. 5(b)]. Again, if f_a is decreased further, the energy of these surface modes is only marginally changed, whilst the windows in which FeF_2 surfacelike modes reside become narrower and the windows in which MnF_2 surfacelike modes appear become wider.

V. CONCLUSION

We have derived a general effective-medium theory of bulk and surface polaritons which applies to both ferromagnetic and antiferromagnetic superlattices in the limit of long wavelength, when the static magnetic field is applied parallel to the magnetization and perpendicular to the surface. Our numerical results illustrate modes with α_1 and α_2 both real and positive. In other words, they are for bonafide surface modes.[22]

The main new results are the expressions in (16) and (21) for the bulk and the surface polariton dispersion equations. The formal equations apply for both ferromagnetic and antiferromagnetic superlattices. The surface modes are virtual, that is they have no magnetostatic limit. The absence of magnetostatic surface modes in the perpendicular configuration is in agreement with previous calculations.[7]

The surface polaritons discussed here could be investigated by attenuated total reflection (ATR). Expressions for ATR reflectivity could be derived from the effective permeability tensor (10), in a similar way to the corresponding semiconductor calculation.[13] Although the surface polaritons are often found in very narrow frequency windows, recent experimental developments in far-infrared reflectivity spectroscopy[25] allow frequency scans with experimental resolution down to 0.02 cm^{-1} and thus the prospect of their detection. It is hoped that such experimental investigation of multilayer systems may now be attempted.

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