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Josephson phenomena and quantum ferromagnetism in double-layer quantum Hall systems

Z. F. Ezawa
Department of Physics, Tohoku University, Sendai 980, Japan
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We analyze an effective theory for double-layer quantum Hall systems whose effective Hamiltonian consists of a kinetic energy term for the Goldstone mode and a tunneling term for electrons. This system corresponds to the quantum ferromagnet theory recently proposed. It is shown that Josephson phenomena occur except for the Meissner effect. A detailed physical picture of the commensurate-incommensurate phase transition in the magnetic order is described on the analogy of superconductor Josephson junction. Plasmon excitations are analyzed in each phase.

I. INTRODUCTION

It is intriguing that Josephson phenomena may occur in certain double-layer quantum Hall (DLQH) systems as a result of a spontaneous development of interlayer phase coherence.[1–5] The coherent phase θ is the Goldstone boson associated with a spontaneously broken $U(1)$ symmetry. This possibility has been pointed out based on the Chern-Simons (CS) theory of the planar electrons. A recent experiment[6] has revealed an anomalous behavior in the activation energy versus the parallel magnetic field $B_{\|}$ in systems where the Josephson effect had been predicted. It may well present, to our knowledge, the first indication of the Josephson phenomena.[3]

However, the problem seems to be still controversial. Recently, Yang *et al.*[7] proposed a quantum ferromagnet (QF) theory of DLQH systems, in which they argue that the Meissner effect does not exist. They suggest to relate the activation energy anomaly[6] to a commensurate-incommensurate (C-IC) phase transition in the magnetic order. It is an urgent problem to examine the fate of the Josephson phenomena in the QF theory. It is also important to clarify the relation between the QF theory and the CS theory.

There is one peculiar property in the CS scheme developed in Refs. 1–3, where the kinetic energy of the Goldstone mode θ is absent when the lowest Landau level projection is made in the mean-field approximation. On the contrary, the term exists in other approaches.[8] At least at the effective Hamiltonian level, the CS scheme[1–3] and the QF theory[7] are identical except for this term. We shall see that the term plays an important role quantitatively: The naive CS scheme without it turns out to be physically unacceptable. We wish to examine how various predictions[1,3] of Josephson phenomena are modified by the existence of the kinetic energy term.

Our results on the QF theory are as follows. Though the Meissner effect does not exist, the Josephson effect does exist. We account for the activation energy anomaly[6] by plasmon excitations intrinsic to the Josephson junction, where the critical point is determined in association with the C-IC phase transition. We should mention that all the naive CS results (including the Meissner effect[2]) are reproduced as the kinetic energy of the Goldstone mode is gradually suppressed. In this paper we use units such that $c = 1$ and $\hbar = 1$.

II. EFFECTIVE THEORY

The microscopic Lagrangian density is given by

$$\mathcal{L}_m = \sum_\alpha \Big(\psi_\alpha^\dagger (i\partial_0 + eA_0^\alpha)\psi_\alpha + \frac{1}{2M}(D_k^\alpha\psi_\alpha)^\dagger (D_k^\alpha\psi_\alpha) \Big) + \lambda(\psi_1^\dagger e^{-iedA_z}\psi_2 + \psi_2^\dagger e^{iedA_z}\psi_1), \qquad (1)$$

together with $\mathcal{L}_{EM} = (1/8\pi)(\varepsilon\mathbf{E}^2 - \mathbf{B}^2)$, where $iD_k^\alpha = i\partial_k - eA_k^\perp - eA_k^\alpha$; $k = x, y$. We have taken the layers parallel to the xy plane. Here, d is the interlayer distance; ε the dielectric constant; $-e$ the charge of the electron; M the effective mass of the electron; ψ_α the electron field and A_μ^α the electromagnetic potential at the layer α; $\mu = 0, x, y$. External magnetic field $B_\perp$ is applied perpendicular to the layers with $A_k^\perp = -\frac{1}{2}\varepsilon_{kj}x_j B_\perp$. Electrons tunnel across the layers with strength λ, which is assumed to be much smaller than the Coulomb interaction ($\lambda \ll e^2/\varepsilon\ell_B$) with $\ell_B = 1/\sqrt{eB_\perp}$. The tunneling gap is $\Delta_{\rm SAS} = 2\lambda$. This Lagrangian is manifestly invariant under the electromagnetic gauge transformation.

An intuitive picture of the system is made by using the language of quantum ferromagnetism[8] with pseudospin $\mathbf{S} = (S_X, S_Y, S_Z)$. It has been argued[7] that the system resembles the XY model, where the ground state is a ferromagnet with a spontaneous magnetization: $\langle S_X\rangle = \sqrt{\rho_1\rho_2}\cos\theta$, $\langle S_Y\rangle = -\sqrt{\rho_1\rho_2}\sin\theta$, $\langle S_Z\rangle = \frac{1}{2}(\rho_1 - \rho_2) \equiv \Delta\rho$. Here, ρ_α are the electron densities in each layers. The Goldstone mode θ designates the orientation of the magnetization. The pseudospin stiffness is described by the Hamiltonian $\mathcal{H} = \frac{1}{2}\rho_s(\partial_k\theta)^2$. The tunneling interaction ($\propto S_X$) is equivalent to applying the pseudomagnetic field in the X direction. Its effect is to align the axis of the magnetization with the X axis ($\theta = 0$). This axis modulates locally in the presence of the external parallel magnetic field $B_{\|}$: as we shall see, $\theta \propto xB_{\|}$ in the commensurate (C) phase and $\theta = 0$ in the incommensurate (IC) phase. The oscillation of the pseudospin $\mathbf{S}$ in the SU(2) pseudospace is observed as a plasma oscillation since $\Delta\rho = S_Z \propto \sin(\omega_P t)$ with ω_P the plasma frequency.

We construct an effective theory describing the low energy dynamics of the collective modes θ and $\Delta\rho$. Within the subspace spanned by the quantum Hall states, we have $\langle\psi_\alpha^\dagger\psi_\alpha\rangle = \rho_\alpha$, $\langle\psi_1^\dagger\psi_2\rangle = \sqrt{\rho_1\rho_2}e^{-i\theta}$, corresponding to the spontaneous magnetization of the pseudospin density. Note that $\psi_1^\dagger\psi_2$ is a bosonic operator and that $\langle\psi_1^\dagger\psi_2\rangle$ states explicitly the existence of the interlayer phase coherence. Then, the effective Lagrangian density for θ and $\Delta\rho$ reads

$$\mathcal{L}_m = -(\partial_t\theta - e\Delta A_0)\Delta\rho + 2\lambda\sqrt{\rho_0^2 - (\Delta\rho)^2}\cos(\theta + edA_z),$$

with $\Delta A_\mu \equiv A_\mu^1 - A_\mu^2$, where the kinetic energy has been quenched by the lowest Landau level projection.

For simplicity we consider the case in which there is no parallel electric field ($E_k = 0$) and hence no Hall current. The Coulomb energy $\mathcal{H}_C$ can be evaluated in $\mathcal{L}_{EM} + \mathcal{L}_m$. The main terms are the electric capacitive energy and the pseudospin stiffness energy.[7] Keeping only these two terms, the effective Hamiltonian is given by

$$\mathcal{H} = \frac{d}{8\pi}B_k^2 + \frac{2\pi e^2 d}{\varepsilon}(\Delta\rho)^2 + \tfrac{1}{2}\rho_s(\partial_k\theta + e\Delta A_k)^2 - 2\lambda\sqrt{\rho_0^2 - (\Delta\rho)^2}\cos(\theta + edA_z). \quad (2)$$

Comments are in order. When the screening effect is neglected and when $(\Delta\rho)^2$ is neglected with respect to ρ_0^2, this Hamiltonian is the same one proposed in the QF theory[7] where $\rho_s = e^2/(16\sqrt{2\pi}\varepsilon\ell_B)$. On the other hand, in the mean-field approximation of the CS theory,[1] the same Hamiltonian has been derived together with the lowest Landau level constraint, $\partial_k\theta + e\Delta A_k = 0$, which is equivalent to the infinite pseudospin stiffness ($\rho_s \to \infty$). Actually, the constraint is a weak condition, and we expect to obtain a finite pseudospin stiffness by improving an approximation also in the CS theory.[9]

We apply the constant parallel magnetic field $B_\parallel$ in the y direction and regard the system to be uniform in the y and z directions. This allows us to set $B_x = 0$ and $\partial_y\theta = 0$. Choosing the gauge such that $B_y = -\partial_x A_z$, we solve the Maxwell equation $\partial_z B_y = -4\pi J_x$, where the drift current takes value $J_x = \mp e\rho_s\partial_x\theta$ on the layer located at $z = z_\alpha$ ($z_1 > z_2$). Hence, we obtain

$$B_y = -\partial_x A_z = -4\pi e\rho_s\partial_x\theta + B_\parallel, \quad (3)$$

or $A_z = 4\pi e\rho_s\theta - xB_\parallel + (1/ed)\vartheta_0$, where $B_\parallel$ is the external parallel magnetic field outside the junction ($z > z_1$ or $z < z_2$), and ϑ_0 is an integration constant. The term $4\pi e\rho_s\partial_x\theta$ represents the screening effect.

The Gibbs free energy density is $\mathcal{G} = \mathcal{H} - (d/4\pi)B_y B_\parallel$, which is derived as

$$\mathcal{G} = \frac{2\pi e^2 d}{\varepsilon}(\Delta\rho)^2 + \frac{\kappa}{8\pi e^2 d}(\partial_x\vartheta + edB_\parallel)^2 - 2\lambda\sqrt{\rho_0^2 - (\Delta\rho)^2}\cos\vartheta - \frac{d}{8\pi}B_\parallel^2, \quad (4)$$

where $\kappa = 4\pi e^2\rho_s d/(1 + 4\pi e^2\rho_s d)$, with $1 \geq \kappa \geq 0$, and

$$\vartheta \equiv \theta + edA_z = (1 + 4\pi e^2\rho_s d)\theta - edxB_\parallel + \vartheta_0. \quad (5)$$

The screening effect is maximum (minimum) when $\kappa = 1$ ($\kappa = 0$). The field equations read

$$\partial_t\vartheta = -\frac{4\pi e^2 d}{\varepsilon}\Delta\rho - \frac{2\lambda\Delta\rho}{\sqrt{\rho_0^2 - \Delta\rho^2}}\cos\vartheta, \quad (6)$$

$$\partial_t\Delta\rho = -\frac{\kappa}{4\pi e^2 d}\partial_x^2\vartheta + 2\lambda\sqrt{\rho_0^2 - \Delta\rho^2}\sin\vartheta. \quad (7)$$

The static equations are $\Delta\rho = 0$ and the sine-Gordon equation $\lambda_J^2\partial_x^2\vartheta - \sin\vartheta = 0$, with $\lambda_J = \sqrt{\kappa/8\pi e^2 d\lambda\rho_0}$.

III. C AND IC PHASES

Recall that the magnetostatic property of the superconductor Josephson junction is also governed by the sine-Gordon system. Thus, making a well-known analysis,[10] we obtain the following physical picture of the ground state.

When $B_\parallel$ is sufficiently small, B_y is equal to $B_\parallel$ at the edge and gradually decreases to the screened value B_0 deep inside the junction with the penetration depth λ_J. Far from the edges we find that $\vartheta = 0$, and hence that $\theta = (\kappa/4\pi e\rho_s)[xB_\parallel - (\vartheta_0/ed)]$ and $B_0 = B_\parallel/(1 + 4\pi e^2\rho_s d)$. The free energy density is estimated as

$$\mathcal{G}_C = \mathcal{H} - \frac{d}{4\pi}B_0 B_\parallel = -\frac{d}{8\pi}\frac{B_\parallel^2}{1 + 4\pi e^2\rho_s d} - 2\lambda\rho_0. \quad (8)$$

This is the C phase.[11] It should be mentioned that there is no screening ($B_0 = B_\parallel$) if $\kappa = 0$ (or $\rho_s \to 0$) while there is the complete screening ($B_0 = 0$) if $\kappa = 1$ (or $\rho_s \to \infty$). The latter is the Meissner effect obtained in the naive CS scheme:[2] The Meissner effect is realized when the pseudospin stiffness is infinite.

As $B_\parallel$ is increased, the magnetic flux begins to penetrate into the junction as sine-Gordon vortices on top of B_0. Each vortex separates two C domains. When $B_\parallel$ is sufficiently large, all the flux penetrates into the junction freely. Then, also inside the junction we have $\theta = 0$, $\vartheta = \vartheta_0 - xedB_\parallel$, and $B_y = B_\parallel$. A naive estimation of the Gibbs free energy is

$$\mathcal{G}_{\rm IC} = \mathcal{H} - \frac{d}{4\pi}B_\parallel^2 \approx -\frac{d}{8\pi}B_\parallel^2. \quad (9)$$

This is the IC phase.[11]

The phase transition point $B_\parallel^*$ is determined by equating $\mathcal{G}_C = \mathcal{G}_{\rm IC}$. When we use (8) and (9) we find that $B_\parallel^* \approx \sqrt{2}/ed\lambda_J$. Actually the transition point is slightly below than this, because the true free energy is less than the value given in (9). A better estimation is made by taking into account the free energy of sine-Gordon vortices penetrated into the junction,[10] which gives $B_\parallel^* = 4/\pi ed\lambda_J$. It is easy to see that $B_\parallel^*$ agrees precisely with the C-IC transition point obtained in Ref. 7 for $0 < \kappa \ll 1$.

We examine the results numerically. In mks units it follows that $\lambda_J = \hbar\sqrt{\kappa/2\mu_0 e^2 d\lambda\rho_0}$, $B_\parallel^* = 4\hbar/\pi ed\lambda_J$, $\kappa =$

$\mu_0 e^2\rho_s d/(\hbar^2+\mu_0 e^2\rho_s d)$. Typical sample parameters[6] are $2\rho_0 \sim 1.26\times 10^{15}$ m^{-2}, $d\sim 2.1\times 10^{-8}$ m, and $\Delta_{\rm SAS} = 2\lambda \sim 0.8$ K $\sim 1.1\times 10^{-23}$. Using these values we estimate that $\lambda_J \sim 4.9\times 10^{-5}\sqrt{\kappa}$ m and $B_\parallel^* \sim 8.2\times 10^{-4}/\sqrt{\kappa}$ T. It is notable that $\kappa = 1$ in the naive CS scheme ($\rho_s \to \infty$) but that $\kappa \sim 2.7\times 10^{-7}$ in the QF theory ($\rho_s \sim 0.32$ K). Hence, the phase transition point is $B_\parallel^* \sim 8.2\times 10^{-4}$ T in the naive CS scheme, while it is $B_\parallel^* \sim 1.6$ T in the QF theory.

Here we argue that the naive CS scheme with $\kappa = 1$ would be physically unacceptable. The quantum Hall effect is known[12] to break down above the critical current density ~ 1 A/m. In the C phase the screening current $J_x^{\rm sc} \equiv e\rho_s\partial_x\theta$ reads $J_x^{\rm sc} = (\kappa/\mu_0)B_\parallel$ in mks units. We estimate that $J_x^{\rm sc} \sim 650$ A/m in the naive CS scheme and $J_x^{\rm sc} \sim 0.37$ A/m in the QF theory at each C-IC phase transition point, respectively. Obviously, the screening current is too large to be physical in the naive CS scheme. Hereafter we only consider the QF theory.

IV. PLASMONS EXCITATIONS

We go on to analyze the low lying excitation modes of θ and $\Delta\rho$, which we call plasmon excitations.[13] They are very different in the C and IC phases since the ground states are very different. For simplicity we only consider the plasmon excitations at zero momentum.

In the C phase we analyze the small uniform fluctuation of ϑ around $\vartheta = 0$ in (6) and (7), finding that $\partial_t^2\vartheta + \omega_C^2\vartheta = 0$, where

$$\omega_C^2 = \omega_J^2 + \Delta_{\rm SAS}^2, \tag{10}$$

with $\omega_J^2 = 4\pi e^2 d\rho_0\Delta_{\rm SAS}/\varepsilon$. The plasmon energy ω_C is independent of the external parallel field $B_\parallel$.

In the IC phase we analyze the small uniform fluctuation of θ around $\theta = 0$, finding[3] that $\partial_t^2\theta + \omega_{IC}^2\theta = 0$ with

$$\omega_{\rm IC}^2 = \omega_J^2 \left| \frac{\sin(ed\ell B_\parallel/2)}{ed\ell B_\parallel/2} \right| + \Delta_{\rm SAS}^2. \tag{11}$$

Here, since we are considering a uniform fluctuation, we have taken a spatial average over the size ℓ of the region of the IC phase.[13] The plasmon energy $\omega_{\rm IC}$ depends on the parallel field $B_\parallel$ in an interesting way: It is equal to ω_C at $B_\parallel = 0$, and decreases rapidly as $B_\parallel$ increases, and takes the minimum value $\Delta_{\rm SAS}$ at $B_\parallel = B_\parallel^c(\ell)$ given by $B_\parallel^c(\ell) = 2\pi/ed\ell$. It displays an interferencelike pattern for $B_\parallel > B_\parallel^c(\ell)$.

V. ACTIVATION ENERGY ANOMALY

How can we observe these plasmon excitations experimentally? The plasmon is neutral but affects the transport property since it induces the oscillation of the charging $e\Delta\rho$ in each layer together with the electric field associated with it.[14] Hence, the plasmon energy would be detected as an activation energy, which is determined by measuring the resistivity ρ_{xx} of the system. The activation energy must be given by the smallest plasmon energy, i.e., by the plasmon energy at zero momentum.

In the IC phase the plasmon energy depends on the size parameter ℓ. When ℓ is the size of the sample for which $ed\ell B_\parallel \gg 1$, it follows that $\omega_{\rm IC} = \Delta_{\rm SAS}$. Then, the activation energy is given by two constants, ω_C in the C phase ($B_\parallel < B_\parallel^*$) and $\Delta_{\rm SAS}$ in the IC phase ($B_\parallel > B_\parallel^*$), as in Fig. 1. This behavior accounts for the observed data[6] for $B_\parallel > B_\parallel^*$. Here, in a typical sample the theoretical prediction is $B_\parallel^* \sim 1.6$ T, which is compatible with the observed critical value $B_\parallel^* \sim 0.8$ T.

In order to account for the data also for $B_\parallel < B_\parallel^*$, we question the rigidity of the C phase against the penetration of the external parallel magnetic field. Note that the screening effect, being given by $\kappa B_\parallel$, is negligibly small since $\kappa \sim 2.7\times 10^{-7}$. We may evaluate the difference between the Gibbs energies (8) and (9) in the two phases, finding that $|\Delta\mathcal{G}| < \kappa(d/8\pi)B_\parallel^2$, which is also negligibly small. This implies that the parallel magnetic field may penetrate into the junction rather freely as sine-Gordon vortices in the C phase. Now, each vortex separates two C domains, and vortices themselves act as IC domains. Thus, when $B_\parallel < B_\parallel^*$, the sample contains many C domains separated by IC domains (vortices).

The contribution to the activation energy from the C domains is ω_C, while the one from the IC domains is $\omega_{\rm IC}(\ell_M)$ where ℓ_M is the size of the vortex. We may estimate it as $\ell_M \approx \pi\lambda_J$, for which $B_\parallel^c(\ell_M) \approx B_\parallel^*$. Now, the activation energy is given by $\omega_{\rm IC}(\ell_M)$ for $B_\parallel < B_\parallel^*$, which decreases rapidly from ω_C to $\Delta_{\rm SAS}$ as $B_\parallel$ increases from $B_\parallel = 0$ to $B_\parallel^*$, as in Fig. 1. This rapid decrease is precisely the observed feature of the data.[6]

The activation energy formula contains three parameters; the maximum energy ω_C, the minimum energy $\Delta_{\rm SAS}$, and the critical point $B_\parallel^*$; see Fig. 1. If we may adjust them phenomenologically, our formula explains all the reported data[6] quite well, as is seen in Figs. 3(a)–(e) of Ref. 3. Note that the fitting by the plasmon formula

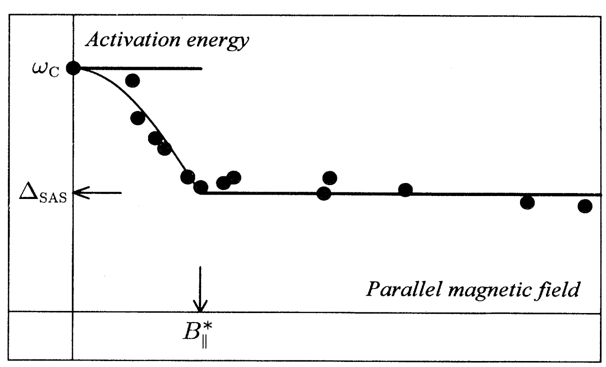

FIG. 1. The activation energy is given theoretically by two constants (thick lines) in the C and IC phases, which is actually modified (thin curve) in the C phase by vortices penetrated into the junction. Solid circles represent data points in a typical sample.

is precisely the same in the naive CS scheme[3] and in the present QF theory, although the mechanisms of obtaining the size parameter ℓ_M are very different. In the naive CS scheme, without any justification we made a working hypothesis of decomposition of the sample to domains, where ℓ_M was the maximum size of such domains. This hypothesis is redundant in the QF theory. In particular, we now identify ℓ_M with the size of the sine-Gordon vortex penetrated into the C phase, which is found to be $\ell_M \sim 2.5 \times 10^{-7}$ m from $B^*_{\|} \sim 0.8$ T in a typical sample.

VI. JOSEPHSON EFFECT

The system shows a new aspect when external leads are attached to the layers. Let us make a thought experiment in which the external current is supplied uniformly to the layers. Then, our analysis is parallel to the well-known one in the superconductor Josephson junction though it is slightly complicated due to the existence of the tunneling term. In order to see what would happen, we study the physical meaning of the basic equations (6) and (7). (For simplicity we set $A_z = 0$ and $\partial_x\theta = 0$ by assuming $B_{\|} = 0$; then, $\theta = \vartheta$.) First, (6) is equivalent to the formula $d\theta/dt = -\partial\mathcal{H}/\partial\Delta\rho \equiv eV_{\rm ext}$. It implies that the time evolution of the phase θ is controlled by the energy change induced by the movement of electrons from one layer to the other layer, which is equal to the work done by the external voltage $V_{\rm ext}$. Second, the physical meaning of (7) is simply the charge conservation in the isolated system. In the presence of the external current J, the charge conservation leads to $J = e\partial_t\Delta\rho - e\Delta_{\rm SAS}\sqrt{\rho_0^2 - \Delta\rho^2}\sin\theta$. Here, $J_z \equiv (1/d)(\delta\mathcal{L}_m/\delta A_z) = -e\Delta_{\rm SAS}\sqrt{\rho_0^2 - \Delta\rho^2}\sin\theta$ is the tunneling current, and $e\partial_t\Delta\rho$ represents the displacement current. Equations (6) and (7) are replaced by

$$\partial_t\theta = -\frac{\omega_J^2}{\Delta_{\rm SAS}}u\left(1 + \frac{\Delta_{\rm SAS}^2}{\omega_J^2}\frac{\cos\theta}{\sqrt{1-u^2}}\right), \qquad (12)$$

$$\frac{J}{J_0} = \frac{1}{\Delta_{\rm SAS}}\partial_t u - \sqrt{1-u^2}\sin\theta, \qquad (13)$$

where $J_0 \equiv e\rho_0\Delta_{\rm SAS}$ and $u \equiv \Delta\rho/\rho_0$.

In the system with dc current feed, when $|J/J_0| < 1$, the solution is given by $u = 0$ ($\Delta\rho = 0$) and $\theta = \theta_0 =$ const with $\sin\theta_0 = J/J_0$. Therefore, we conclude that the Josephson current J flows between the two layers with $J = -e\rho_0\Delta_{\rm SAS}\sin\theta_0$. The tunneling current is a superconducting current ($V_{\rm ext} = 0$ for $\theta = \theta_0$). On the other hand, for $|J/J_0| > 1$ the current-voltage characteristic becomes complicated compared with the case of the superconductor Josephson junction[10] although the modification is quite small since $\Delta_{\rm SAS}^2/\omega_J^2 \ll 1$ in actual samples. The system with dc voltage feed may be similarly discussed, where $\partial_t\theta = eV_{\rm ext}$, or $\theta = \theta_0 + eV_{\rm ext}t$. Solving u as a function of t in (12) and substituting it into (13), we can determine the tunneling current J as a function of t.

It is very interesting that the Meissner effect does not exist although the Josephson effect does exist. The reason is that the junction is made of the layers. In the case of the superconductor Josephson junction, because the junction is made between two superconducting bulks into which the parallel magnetic field cannot penetrate, we need to set $B_{\|} = 0$ in (3) as the boundary condition imposed outside the junction (or inside the bulk): thus, the Meissner effect follows inevitably. This explains why we can have the Josephson current without being accompanied by the Meissner effect in the Josephson junction in the DLQH system.

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[14] The following physical picture will also be useful [A. Iwazaki) (private communication)]. The plasmon is neutral but it could be regarded as a bound state of a pair of quasiparticle and quasihole, just as the magnetoroton in the single-layer quantum Hall system. In the presence of the external electric field, plasmons would be dissociated into quasiparticles and quasiholes.