Magnetoresistance of a two-dimensional electron gas in nearly parallel magnetic fields

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The transport properties of a two-dimensional electron gas in the presence of magnetic fields applied parallel to or slightly tilted from the plane of the confined electron gas are described. To analyze the experimental data the finite width of the electron layer has been taken into consideration. It is shown that the *Hall field* across the layer gives the significant contribution to the conductivity which might be dominant for the observed positive magnetoresistance.

The magnetoresistance of two-dimensional electron systems has attracted physicists for many years. The main attention has been paid to the configuration in which the magnetic field is applied perpendicularly to the system. Much less effort has been devoted to the study of the in-plane configuration, when the applied magnetic field is nearly parallel to the electron gas plane. In 1990 Leadley *et al.*[1] have reported positive magnetoresistance with the angle rising between the two-dimensional electron gas and the magnetic field direction in the range of a few degrees for a fixed magnetic field strength. It has been suggested that this effect is due to the pearlike shaped Fermi surface of electron gas, which arises due to the break of the time reversal symmetry in strong in-plane fields when the confining potential has an asymmetric, about triangular, shape.[2] However, neither quantitative nor qualitative analysis of the effect has been presented till now.

To understand the origin of the observed positive magnetoresistance, we have analyzed the electron conductivity for the particular case of a nearly in-plane configuration with a weak perpendicular component of the magnetic field, $B_\perp$. It allows us to include the effect of $B_\perp$ semiclassically, while the influence of the in-plane component, $B_\parallel$, to the electron energy spectrum will be fully taken into account. The conductivity will be interpreted in terms of the Chamber's solution of the Boltzmann's equation with a uniform relaxation time.[3] It will be shown that the *Hall electric field* across the electron gas layer has the significant effect on magnetotransport.

Let us first briefly summarize the properties of a free electron gas confined in the $\hat{z}$ direction by a potential $V_{\rm conf}(z)$ and subjected to the influence of magnetic fields,[4,5] which are controlled by the Hamiltonian,

$$H_0 = \frac{1}{2m}\left(\vec{p}+e\vec{A}\right)^2 + V_{\rm conf}(z) \quad , \tag{1}$$

where m and e are an effective electron mass and absolute value of the electron charge, respectively. In the case of strictly in-plane magnetic fields applied along $\hat{y}$ direction ($B_\parallel \equiv B_y$), it is convenient to choose the Landau gauge for the vector potential, $\vec{A} \equiv (B_\parallel z, 0, 0)$. The eigenfunctions of the Hamiltonian H_0 may be written as the product of a plane wave, $\exp(ik_x x + ik_y y)$ and the eigenfunction $\chi_0(z, k_x)$ of the k_x dependent Hamiltonian,

$$H_0(k_x) = \frac{1}{2}m\left(\frac{\hbar k_x}{m} + \omega_\parallel z\right)^2 + \frac{p_z^2}{2m} + V_{\rm conf}(z) \quad , \tag{2}$$

where $\omega_\parallel$ is the cyclotron frequency $eB_\parallel/m$. The energy spectrum $E(\vec{k})$ is given as the sum of kinetic energy of free electron motion in the $\hat{y}$ direction and eigenvalues $E_0(k_x)$ of the Hamiltonian $H_0(k_x)$,

$$E(\vec{k}) = \frac{\hbar^2 k_y^2}{2m} + E_0(k_x) \quad , \tag{3}$$

where $\vec{k} \equiv (k_x, k_y)$ is the two-dimensional wave vector. The index 0 points out the fact we are limiting our consideration to the two-dimensional electron gas with only the lowest energy subband occupied. Due to violation of the time reversal symmetry, caused by in-plane magnetic fields, the Fermi line has an asymmetric shape if $V_{\rm conf}(z) \neq V_{\rm conf}(-z)$, while in the zero field limit it is always a circle. With rising magnetic field strength, the perimeter along k_x direction increases on account of the perpendicular perimeter. This effect may be viewed as an increase of the electron effective mass $m_x^*(k_x)$ in $\hat{x}$ direction,

$$\frac{1}{m_x^*(k_x)} \equiv \frac{1}{\hbar^2}\frac{\partial^2 E_0(k_x)}{\partial k_x^2} \quad . \tag{4}$$

The effect of a weak perpendicular component of the magnetic field ($B_\perp \equiv B_z$) may be included through the dynamics of the electronic motion in $\vec{k}$ space, which is governed by the semiclassical equation,

$$\hbar\frac{\partial\vec{k}}{\partial t} = -eB_\perp\vec{v}(\vec{k}) \times \hat{z} - e\vec{\mathcal{E}} \quad , \tag{5}$$

where $\vec{\mathcal{E}}$ is the external electric field and $\vec{v}(\vec{k})$ is the velocity expectation value,

$$\vec{v}(\vec{k}) = \frac{1}{\hbar}\nabla_{\vec{k}}E(\vec{k}) \quad . \tag{6}$$

As a first approximation when studying electronic transport, all quantum phase coherence effects can be neglected. At zero electric field, electrons are considered to move along $\vec{k}$-space trajectories defined by constant energy contours and controlled by the semiclassical equation of motion, Eq. (5). Transport phenomena can be interpreted in terms of the electron distribution function $f(\vec{r}, \vec{k})$, which satisfies Boltzmann's equation. If we also assume that scattering can be treated in the uniform relaxation time approximation, linearizing the Boltzmann equation with respect to an external electric field leads to the following expression, due to Chambers,[3] for the conductivity tensor,

$$\sigma_{\alpha\beta} = \frac{e^2}{2\pi^2}\int \frac{df_0}{d\mu}\, v_\alpha(\vec{k}(t_0))\, \langle v_\beta(\vec{k}(t))\rangle_{t_0}\, d\vec{k} \quad , \qquad (7)$$

where f_0 is the equilibrium Fermi-Dirac distribution function and μ is the chemical potential. The term represented by angular brackets $\langle\rangle$ stands for the time average along the electron trajectory calculated using Eq. (5) in the absence of an electric field,

$$\langle v(t)\rangle_{t_0} = \int_{-\infty}^{t_0} v(t)\, e^{\frac{t-t_0}{\tau}}\, dt \quad , \qquad (8)$$

with τ being the relaxation time. The described concept has been successfully applied to explain low field magnetoresistance anomalies in periodically modulated two-dimensional systems.[6,7] In the considered case, however, it has to be modified due to the allowed electronic transport along $\hat{z}$ direction.

The in-plane magnetic field $B_\parallel$ induces a space separation of electron mass centra along $\hat{z}$ direction. Their location, often called a guiding center, is given by expectation values of the z coordinate,

$$Z(k_x) = -l_\parallel^2\, k_x + \frac{v_x(\vec{k})}{\omega_\parallel} = -\left(1 - \frac{m}{m_x^*(k_x)}\right) l_\parallel^2 k_x \ , \qquad (9)$$

where $l_\parallel$ stands for the magnetic length ($l_\parallel^2 = \hbar/m\omega_\parallel$). The effect of electron transitions in $\hat{z}$ direction of the real space could be taken into account by adding a corresponding diffusion term to the Boltzmann equation as has been done for example by Beenakker[8] for the case of periodically modulated two-dimensional electron gas. We will, however, use another procedure which is more convenient for the considered case. To stay within the uniform relaxation time approach, which models the energy dissipation, the collision term entering the Boltzmann's equation will be modified in the spirit of the Mermin's approach.[9] We will show that it leads to a correction term to the conductivity defined by Eq. (7).

The applied electric field $\vec{\mathcal{E}} \equiv (\mathcal{E}_x, \mathcal{E}_y)$ accelerates electrons in $\hat{x}$ and $\hat{y}$ directions. In the presence of in-plane magnetic field ($B_\parallel \equiv B_y$) the acceleration of an electron in the $\hat{x}$ direction is accompanied by a shift of its guiding center in $\hat{z}$ direction. This can be easily shown including $\vec{\mathcal{E}}$, which is adiabatically switched on, into the Hamiltonian H_0 by the time dependent vector potential,

$$\vec{A}(t) = -\tau\vec{\mathcal{E}}\, e^{\frac{t-t_0}{\tau}} \quad , \quad \vec{\mathcal{E}} = -\frac{\partial\vec{A}(t)}{\partial t} \quad . \qquad (10)$$

Treating the time t as a parameter, an electron which stays at the time $-\infty$ in a given state $\vec{k}$ will appear to change its properties in terms of the states classified by the wave vector $\vec{k} + e\vec{A}(t)/\hbar$. It results in the guiding center shift in $\hat{z}$ direction. All electrons are shifted in the same direction and the original equilibrium charge distribution, determined by the fixed positive background charge, is destroyed. The induced z-dependent electrostatic potential forces electrons to return back to their original positions $Z(k_x)$. The force can be characterized by a local electric field $\mathcal{E}_z$ acting on an electron in the state $\vec{k}$ and giving the additional contribution $e\mathcal{E}_z z$ to the Hamiltonian $H_0(k_x)$ defined by Eq. (2). The resulting eigenenergies can be expressed with the help of eigenenergies of the unperturbed Hamiltonian $H_0(k_x)$, as follows:

$$\tilde{E}_0(\tilde{k}_x) = E_0(\tilde{k}_x) - v_d\hbar\tilde{k}_x + \frac{1}{2}mv_d^2 \quad , \qquad (11)$$

where $\tilde{k}_x = k_x + mv_d/\hbar$. The expectation velocity value in $\hat{x}$ direction is changed by the drift velocity $v_d \equiv \mathcal{E}_z/B_\parallel$ and the guiding center is shifted by $\Delta Z = -v_d/\omega_\parallel$ due to the effect of $\mathcal{E}_z$. To restore the equilibrium charge distribution, it is suggested to compensate for the guiding center shift, due to the electric field $\mathcal{E}_x$, by the shift ΔZ induced by $\mathcal{E}_z$. It gives the following value of the drift velocity:

$$v_d(t) = \frac{\mathcal{E}_z}{B_\parallel} = -\frac{1}{l_\parallel^2}\frac{\partial Z(k_x)}{\partial k_x}\frac{e\mathcal{E}_x\tau}{m}\, e^{\frac{t-t_0}{\tau}} \quad . \qquad (12)$$

The Hall field builtup in real systems is, however, modified by the screening effect.[10] In the considered case of noninteracting electrons the introduced electric field $\mathcal{E}_z$, Eq. (12), may be viewed as a *slave* field defining the transformation $\tilde{k}_x = k_x + mv_d/\hbar$, which leads to the equilibrium charge distribution and, consequently, to a local equilibrium across the electron gas layer. This procedure excludes z dependence of the transport problem with the persisting meaning of τ as an energy relaxation time.

At the time $t = t_0$ the final value of $\vec{\mathcal{E}}$ is reached and up to linear terms in applied electric field the corresponding contribution to the conductivity is given as

$$\Delta\sigma_{xx}(B_\perp = 0) = -\frac{1}{2\pi^2}\frac{e^2\tau}{m}\int f_0\, \frac{1}{l_\parallel^2}\frac{\partial Z(k_x)}{\partial k_x}\, d\vec{k} \quad . \quad (13)$$

Integrating by parts and introducing the time average integral, we get

$$\Delta\sigma_{xx} = -\frac{e^2}{2\pi^2}\int \frac{\partial f_0}{\partial\mu} v_x(\vec{k}(t_0))\, \langle\omega_\parallel Z(k_x(t))\rangle_{t_0}\, d\vec{k} \ . \quad (14)$$

Here, we have included electron dynamics due to the nonzero perpendicular magnetic field $B_\perp$ inserting the time dependence of k_x into the time average in the spirit of Chambers' arguments. This conductivity contribution $\Delta\sigma_{xx}(B_\perp)$ to the xx component of the conductivity tensor, given by Eq. (7), is analogous to the guiding-center-drift term for the periodically modulated two-dimensional electron gas.[8]

For the simplest form of the confining potential,

$$V_{\text{conf}} = \frac{1}{2}\, m\, \Omega^2\, z^2 \quad , \qquad (15)$$

analytic expressions for the conductivity components can be obtained. The effective mass given by Eq. (4) becomes independent[4] of k_x, $m_x^*(k_x) = (1 + \omega_\parallel^2/\Omega^2)m$, the Fermi surface has the elliptic shape and integrals entering Eqs. (7) and (14) can be easily evaluated. The resulting expressions for the resistivity are given as follows:

$$\rho_{xx} = \rho_{yy} = \frac{m}{e^2\tau n_s}\frac{1+\omega_c^2\tau^2}{1+\frac{m}{m_x^*}\omega_c^2\tau^2} \quad , \tag{16}$$

$$\rho_{xy} = -\rho_{yx} = -\frac{B_\perp}{en_s}\frac{1+\omega_c^2\tau^2}{\frac{m_x^*}{m}+\omega_c^2\tau^2} \quad , \tag{17}$$

where $\omega_c = eB_\perp/\sqrt{mm_x^*}$ is the cyclotron frequency and n_s is the areal electron concentration.

In the case of $B_\perp = 0$, the diagonal resistivity components have the Drude form, $m/(e^2\tau n_s)$, with the zero field electron mass m. Neglecting the correction term $\Delta\sigma_{xx}$ would lead to the same form for the resistivity component ρ_{xx}, but with electron mass m_x^*. The correction term, Eq. (14), just eliminates the effect of increasing electron effective mass with in-plane magnetic field. On the other side, the relaxation time may generally depend on $B_\parallel$. In the simplest approach of isotropic scattering within electron gas plane, it is inversely proportional to the density of states and we get

$$\tau = \sqrt{\frac{m}{m_x^*}}\tau_0 \quad , \quad \frac{m_x^*}{m} = 1 + \frac{\omega_\parallel^2}{\Omega^2} \ , \tag{18}$$

where τ_0 is the zero field relaxation time. This gives rise to the positive magnetoresistance with respect to $B_\parallel$.

The perpendicular component $B_\perp$ of the magnetic field induces the resistance increase till a saturation value for high enough $B_\perp$ ($\omega_c\tau \gg 1$) is reached. The equality of both diagonal components is the result of the considered harmonic confinement Eq. (15). Any asymmetry of the Fermi line in k_x direction leads to the increase of ρ_{xx} on account of ρ_{yy}. Let us point out that while such an asymmetry itself may cause similar behavior of the ρ_{xx} component, the magnetoresistance of ρ_{yy} component becomes independent on $B_\perp$ if the conductivity contribution $\Delta\sigma_{xx}$, Eq. (14), is not taken into account.

The harmonic confinement, Eq. (15), and the uniform relaxation time we used are not too realistic assumptions. Nevertheless the resistivity given by Eq. (16), with the confinement frequency Ω being the single fitting parameter, qualitatively describes the main features of the experimental magnetoresistance traces as shown in Fig. 1 and Fig. 2. The data have been taken at the temperature 4.2 K on standard $Al_xGa_{1-x}As$-GaAs heterostructures grown by the molecular beam epitaxy. The typical layer structure consists of a 2 μm GaAs buffer, 100 Å spacer, 1000 Å of Si-doped $Al_xGa_{1-x}As$ ($x \sim 0.32$), and 200 Å cap layer of GaAs. The Hall bar samples of 400 μm conducting channel width have been used for the measurement. At helium temperature 4.2 K, they have typically the carrier density and the mobility in the range $n_s = 3.0$–5.5×10^{15} m^{-2} and 20–41 m^2/Vs, respectively. The relatively small carrier density ensures the existence of two-dimensional electron gas with only the lowest subband occupied in the whole range of the used magnetic field strength, up to 5 T. All samples show negative magnetoresistance applying magnetic field perpendicularly to the electron gas, as shown in the inset of Fig. 1. This often observed effect, more pronounced at lower temperatures, is supposed to be caused by the localization effects which are excluded from our model calculation. There is also a slight instability of our samples to the thermal cycling. Note, that for a technical reason the configuration can be changed at room temperatures only. This effect is responsible for the difference in magnetoresistance traces as functions of $B_\perp$.

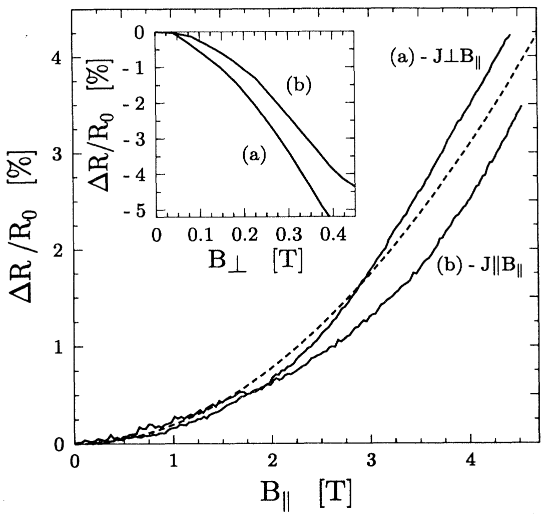

FIG. 1. Magnetoresistances $\Delta R_{xx}/R_0$ [(a) $J \perp B_\parallel$] and $\Delta R_{yy}/R_0$ [(b) $J\|B_\parallel$] as functions of the in-plane magnetic field $B_\parallel$ for $B_\perp = 0$; R_0 denotes the zero field resistance. The dashed line represents the model calculation for harmonic confinement frequency $\Omega = eB_{\rm conf}/m$ with $B_{\rm conf} = 16$ T. The inset shows negative magnetoresistance for the case of zero in-plane field.

The effects of in-plane magnetic fields and those slightly tilted by an angle θ from the direction parallel to the two-dimensional electron gas plane have been studied in two different configurations with respect to the applied current direction. The resistivity component ρ_{xx} corresponds to the current applied perpendicularly to the magnetic field direction ($J \perp B_\parallel$), while ρ_{yy} has been measured with the current applied along the in-plane component of the magnetic field ($J\|B_\parallel$).

Both positive magnetoresistances do not differ substantially in the case of in-plane magnetic fields. A little stronger magnetoresistance of ρ_{xx} in contrast to ρ_{yy}, shown in Fig. 1 for the sample with $n_s = 5.2\times10^{15}$ m^{-2} and the mobility 35 m^2/Vs, is not an universal feature. Very often just the opposite behavior has been observed. The result of the model calculation presented in Fig. 1 corresponds to the case of harmonic confinement represented by the frequency Ω equal to $eB_{\rm conf}/m$ with $B_{\rm conf} = 16$ T. The value of τ_0 has been determined from the zero field mobility of the sample.

The result of the model calculation for the same parameter values and experimental magnetoresistance traces obtained by tilting the direction of the magnetic field from the inplane orientation (with the fixed magnetic field strength) are shown in Fig. 2 as the function of resulting perpendicular magnetic field component $B_\perp$. To exclude partly localization effects, we have slightly modified the definition of the magnetoresistance in this

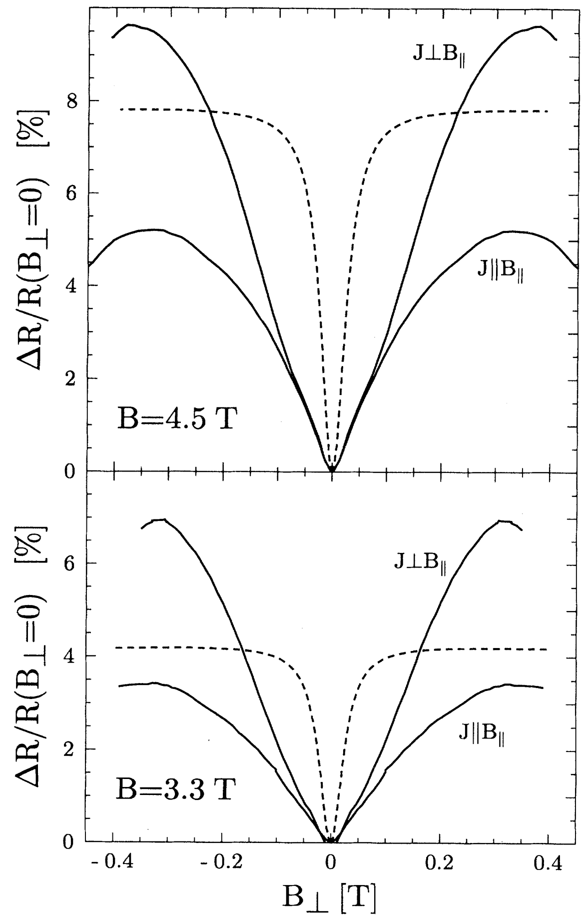

FIG. 2. Angle dependencies of the magnetoresistance shown as functions of $B_\perp = B \sin\theta$ for two fixed values of the magnetic field strength B, $R(B_\perp = 0)$ denotes the resistance for the in-plane field orientation. Dashed lines represent the model calculation with $B_{\rm conf} = 16$ T.

case. The value ΔR is the difference between measured resistance at $B_\perp \neq 0$ and that at $B_\perp = 0$ for fixed value of the total magnetic field strength enlarged by the same difference taken under the condition of the zero in-plane field, $B_{\|} = 0$. In the temperature range 2.1–4.2 K, the results are practically temperature independent. The observed higher saturation value for ρ_{xx} is the universal feature of all samples indicating an asymmetric confinement within the studied structures. The accuracy of the measurement of $B_\perp$ did not allow us to check small deviations of the Hall resistivity, Eq. (17), from the free electron value $B_\perp/(en_s)$ predicted for a weak perpendicular component of magnetic fields.

The used model is too simplified to expect the quantitative agreement with the experimental data. While the electron energy spectrum can easily be improved to correspond to a real electron systems,[5] the essential limitation is the used uniform relaxation time approach applicable for the case of an isotropic scattering within two-dimensional $\vec{k}$ space. The electron scattering on ionized impurities should be considered and the electron transition probability between states k_x and $-k_x$ is expected to decrease at high in-plane fields when the magnetic length becomes comparable to the width of the potential confinement. Also, the localization effect may influence the magnetotransport substantially and we consider this effect responsible for the negative magnetoresistance observed at $B_\perp > 0.3$ T. Nevertheless, despite of the used simplified model, the presented analysis allows us to conclude that any more realistic treatment has to take into account the *Hall field* across the electron gas layer which, under particular conditions, might have the dominant effect on the magnetoresistance.

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