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Interconnection between the period of geometric oscillations in a longitudinal field and the Fermi surface

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We have shown that the period of geometric magnetoacoustic oscillations in a longitudinal magnetic field is uniquely defined by the configuration of the Fermi surface in the vicinity of the effective orbit. Hence, these oscillations can be used in the experimental characterization of the Fermi surfaces of metals.

I. INTRODUCTION

The geometric oscillations (GO's) in ultrasound attenuation and velocity have played an important role in investigations of Fermi surfaces (FS's). It must be mentioned, however, that the overwhelming majority of these works deal with the traditional geometry, where the magnetic field **H** is perpendicular to the sound wave vector **q** (the Bömmel-Pippard geometry).

The history of GO studies in the nontraditional geometry $\mathbf{q}\|\mathbf{H}$ is relatively short. Started in 1960 by the observation of unintelligible oscillations of ultrasound absorption,[1] it was interrupted by a paper by Miller[2] in 1966. The loss of interest in these themes was connected, in particular, with the opinion that this phenomenon is uninformative about the FS. We hope to demonstrate that this opinion is not fully justified.

The interest in GO's in $\mathbf{q}\|\mathbf{H}$ geometry was regenerated in 1992 following the discovery of an unusual polarization phenomenon: a quasiperiodic (against the reciprocal magnetic field H^{-1}) variation of ultrasound ellipticity.[3] Subsequent experiments have proved the effect to be the appearance of GO's in sound absorption for the two circularly polarized eigenwaves. Some time later, GO's in the sound velocity were observed with $\mathbf{q}\|\mathbf{H}$.[4] In Ref. 5, a mathematical description of GO's in a longitudinal field was partially provided. This work revealed a number of specific features of these nontraditional GO's compared to oscillations in the Bömmel-Pippard geometry.

In the following note we wish to complete this cycle of publications and to show that, for $\mathbf{q}\|\mathbf{H}$, the oscillation period t is *uniquely* connected with the FS geometry in the vicinity of the effective orbit.

II. PRINCIPAL RELATION

In early papers (by Mackinnon, Taylor, and Daniel[6] and by Mackintosh[7]), it was clearly demonstrated that the oscillation period versus reciprocal magnetic field in the case $\mathbf{q}\|\mathbf{H}$ is defined by the condition

$$q\Delta Z-\psi_0=2\pi N\ , \tag{1}$$

where ΔZ is the dimension of the effective electron trajectory in real space, measured along **H** (it is inversely proportional to H), and N is the serial number of an oscillation. (The value of the phase shift ψ_0 is not essential in this context.) Thus the *physical meaning* of the period was revealed right from the start. But the following questions arise: is the Fermi surface of the metal characterized by this period—and, if yes, is this connection of the period with the FS geometry unique? Those are the questions about the *geometric meaning* of the GO period.

Up to now, only Daniel and Mackinnon[8] to our knowledge have tried to answer these questions, but later experiments[9–11] with Sb demonstrated the inapplicability of their formula. As a matter of fact, a condition for the appearance of the Doppler-shifted cyclotron resonance (see Ref. 11, for example)—not for GO's—was derived in Ref. 8. The paper by Eckstein, Ketterson, and Eckstein[9] finished the discussion of the questions mentioned: "... we measure the linear dimension of the orbit parallel to the magnetic field. However, this gives no direct information about the orbit in p space ...". The following is an attempt to dispute this opinion.

It is known (see, for example, Ref. 5) that the effective orbit in our case is one having an average velocity (over the cyclotron period) in the direction $\mathbf{H}\|0_z\|\mathbf{q}$ that is equal to the sound velocity: $\bar{v}_z=s$ (as a rule, s is three orders of magnitude smaller than the characteristic value of the Fermi velocity v_F).

An expression for ΔZ may be derived easily in the framework of the quasiclassical description of electron dynamics in a magnetic field (for example, Ref. 5):

$$\Delta Z=\Omega^{-1}\int_{\theta_1}^{\theta_2}(v_z(\theta)-s)d\theta\ . \tag{2}$$

Here $\Omega = eH/(cm_c)$ is the cyclotron frequency, e is the absolute value of the electron charge, m_c is the cyclotron mass, c is the light velocity, and θ is the dimensionless time of the electron motion over the cyclotron orbit. θ_1 and θ_2 are the value of θ corresponding to the so-called turning points, where $v_z(\theta_1) = v_z(\theta_2) = s$ (we consider the simplest case with a minimal number of turning points R, i.e., $R=2$).

Proceeding from Eq. (1), the oscillation period can be written in the form

$$t = \frac{2\pi}{q}(\Delta G)^{-1} , \tag{3}$$

where a definition for the field-independent quantity $\Delta G \equiv \Delta Z \times H$ has been introduced. One must bear in mind the following: in cases of practical interest, a typical value of the velocity $v_z(\theta) \gg s$, otherwise the oscillation period would be extremely small at frequencies up to 1 GHz, and the GO's would be inaccessible to observation. We can assume, therefore, that $s=0$ in Eq. (2). The calculation of the integral in Eq. (2) is reduced to an integration over the orbital arc dk_l in k space ($\mathbf{k}$ is the electron wave vector). As a result,

$$(\Delta G) \simeq \frac{H}{\Omega}\int_{\theta_1}^{\theta_2}[v_z(\theta)-0]d\theta \equiv \frac{hc}{2\pi e}\int_{k_1}^{k_2} v_z \frac{dk_l}{v_\perp} . \tag{4}$$

$v_\perp$ is the transverse (relative to $\mathbf{H}$) component of the electron velocity at a given point on the orbit. Here we define the orbit as a closed curve in k space, resulting from the intersection of the FS with the plane $k_z = \text{const}$. The turning points k_1 and k_2 are defined by the condition $v_z(k_l) \simeq 0$. Figure 1 shows a plane that is perpendicular to an orbit element dk_l, a velocity vector $\mathbf{v}$, the effective orbit $k_z = k_{\text{eff}}$, and also an orbit $k_z = k_{\text{eff}} + \Delta k_z$ (with its projection). The electron velocity is perpendicular to the FS:

$$\frac{v_z}{v_\perp} \equiv -\lim_{\Delta k_z \to 0}\left[\frac{\Delta k_\perp}{\Delta k_z}\right] . \tag{5}$$

It follows from Eq. (5), in particular, that the integration limits for the second integral in Eq. (4) are the intersection points of the effective orbit ($k_z = k_{\text{eff}}$) with the projection of the orbit ($k_z = k_{\text{eff}} + \Delta k_z$). Taking into account that $\Delta k_\perp dk_l$ is the area element [see Fig. 1(b)], one obtains

$$\Delta G_{kl} \simeq \frac{hc}{2\pi e}\lim_{\Delta k_z \to 0}\left[\frac{\Delta\Sigma^C}{\Delta k_z}\right] \equiv \frac{hc}{2\pi e}\frac{\partial \Sigma^C}{\partial k_z} . \tag{6}$$

In Eq. (6), Σ^C means the area of the dashed crescentlike figure in Fig. 1, or the area of the orbit region intercepted by the line k_1-k_2. Finally, we have

$$t = 4\pi^2 e(hcq)^{-1}\left[\frac{\partial \Sigma^C}{\partial k_z}\right]^{-1} . \tag{7}$$

This formula defines a direct relation between the GO period t and the FS geometry, thus giving a solution of the problem discussed.

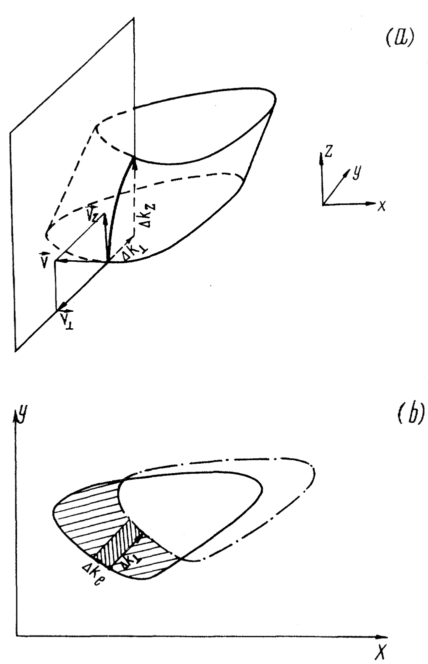

FIG. 1. (a) The Fermi surface fragment in the vicinity of the effective orbit. (b) The orbits ($k_z = k_{\text{eff}}$) and ($k_z = k_{\text{eff}} + \Delta k_z$), projected onto the plane (k_x, k_y).

III. VERIFICATION

The validity of the formula derived can be checked through the experimental data of Beckmann, Eriksson, and Hörnfeldt[10] on Sb. As shown in Ref. 10, the GO's in Sb are connected with "ellipsoids" inclined to the threefold axis (the magnetic field vector is directed along this axis). The model for such a sheet of the Sb FS is known, and it gives us the possibility for explicit calculation of $\partial\Sigma^C/\partial k_z$. On the other hand, there are data for the period t; hence the experimental value of the geometric parameter discussed can be deduced on the basis of Eq. (7).

The mentioned sheet is described satisfactorily by the equation[10]

$$\frac{2m_0\varepsilon_F}{(h/2\pi)^2} = \alpha_{11}k_x^2 + \alpha_{22}k_y^2 + 2\alpha_{23}k_yk_z + \alpha_{33}k_z^2$$
$$= \alpha_{11}k_x^2 + \alpha_{22}\left[k_y + \frac{\alpha_{23}}{\alpha_{22}}k_z\right]^2 + \tilde{\alpha}_{33}k_z^2 . \tag{8}$$

Here m_0 is the electron rest mass, $\varepsilon_F = 18\times10^{-14}$ erg, $\alpha_{11}=15.8$, $\alpha_{23}=7.6$, $\alpha_{23}=5.9$, and $\tilde{\alpha}_{33} = \alpha_{33} - \alpha_{23}^2/\alpha_{22}$. The FS section intersected by the plane $k_z=0$ (for which, as a consequence of the inversion symmetry of the ellipsoid, $\bar{v}_z=0$) is an ellipse with the following semi-axes:

$$a_0 = \frac{2\pi}{h}\left[\frac{2m_0\varepsilon_F}{\alpha_{11}}\right]^{1/2}, \quad b_0 = \frac{2\pi}{h}\left[\frac{2m_0\varepsilon_F}{\alpha_{22}}\right]^{1/2} . \tag{9}$$

A movement onto another cross section ($k_z = \delta k$) leads to the following two effects:

(a) a displacement of the ellipse from $k_y=0$ to the point $k_y=-\delta k\alpha_{23}/\alpha_{22}$.

(b) a change of the line dimensions; for example, the semi-axis (a) becomes equal to

$$a\simeq a_0\left[1-\frac{\tilde{\alpha}_{33}(\delta k)^2}{4m_0\varepsilon_F}\right].$$

The last effect can be disregarded because it is quadratic in the infinitesimal value of (δk).

Next, in the limit of $(\delta k)\rightarrow 0$, the intersection points of the two almost equal elliptic trajectories $k_z=0$ and $k_z=\delta k_z$ approach the positions (turning points) $(a_0,0)$ and $(-a_0,0)$. This fact implies that Σ^C is the area of the part of the ellipse which is cut off by the line $k_y=0$. For the effective orbit $k_z=0$, this is shown in Fig. 2(a) and for the section $k_z=\delta k$ in Fig. 2(b). The difference of the two areas shown, $\delta\Sigma^C$, is equal to the strip area which is shown cross hatched in Fig. 2(b). Thus,

$$\frac{\partial\Sigma^C}{\partial k_z}\equiv 2a_0\frac{\alpha_{23}}{\alpha_{22}}\ . \tag{10}$$

Equation (10) can be rewritten in the form

$$\frac{\partial\Sigma^C}{\partial k_z}\equiv A\tan\phi\ , \tag{11}$$

where A is the effective orbit size measured between the turning points, and ϕ is the inclination angle of the given FS sheet relative to **H**. It is clear from the above that the form Eq. (11) is of greater generality than Eq. (10), which is valid for an ellipsoidal FS only. At the same time, Eq. (11) can be regarded as an interpretation of $\partial\Sigma^C/\partial k_z$ by means of simpler geometric quantities (a linear size and an inclination angle).

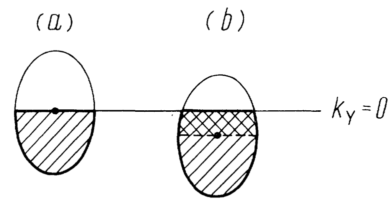

FIG. 2. Cross sections of an inclined elliptical FS intersected by the planes (a) $k_z=0$ and (b) $k_z=\delta k_z$.

The calculated value is obtained from Eqs. (9) and (10):

$$\left[\frac{\partial\Sigma^C}{\partial k_z}\right]_{\text{calc}}=0.11\ \text{Å}^{-1}\ .$$

On the other hand, starting from the experimental data[10] on the oscillation period ($t=28\times10^{-4}$ Oe^{-1} at frequency $f=146.5$ MHz with $s=2.71\times10^5$ cm/s), we obtain from Eq. (7)

$$\left[\frac{\partial\Sigma^C}{\partial k_z}\right]_{\text{expt}}=\frac{2\pi e}{hc}\frac{s}{f}\frac{1}{t}=0.10\ \text{Å}^{-1}\ .$$

As is seen, a good agreement is obtained between the experimental and calculated values, thus confirming the validity of Eq. (7).

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