Persistent currents in the presence of a transport current

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We have considered a system of a metallic ring coupled to two electron reservoirs. We show that in the presence of a transport current, the persistent current can flow in a ring, even in the absence of a magnetic field. This is purely a quantum effect and is related to the current magnification in the loop. These persistent currents can be observed if one tunes the Fermi energy near the antiresonances of the total transmission coefficient or the two-port conductance.

Experimental and theoretical research in mesoscopic systems have provided an opportunity of exploring truly quantum-mechanical effects beyond the atomic realm.[1] Persistent currents in small metal rings threaded by magnetic flux are a manifestation of quantum effects in submicrometer systems, and are prominent among the mesoscopic effects. Prior to the experimental observations,[2-4] Büttiker, Imry, and Landauer suggested the existence of persistent currents in an ordered one-dimensional ring threaded by a magnetic flux.[5] The coherent wave functions extending over the whole circumference of the loop lead to a periodic persistent current. General quantum-mechanical principles require that the wave functions, eigenvalues, and hence all observables be periodic in a flux ϕ threaded by the loop with a period ϕ_0, $\phi_0=hc/e$ being the elementary flux quantum. The magnetic field destroys the time-reversal symmetry and, as a consequence, the degeneracy of the states carrying current clockwise and anticlockwise is lifted. Depending on the position of the Fermi level, uncompensated current flows in either of the directions (diamagnetic or paramagnetic). For an ideal isolated ring without impurities and at zero temperature, the nature of the persistent current depends on the total number N of the electrons and the persistent current exhibits a sawtooth-type behavior as a function of magnetic flux. For even N, the jump discontinuities occur from the values $-(2ev_f/L)$ to $(2ev_f/L)$ at $\phi=0$, $\pm\phi_0$, and $\pm 2\phi_0$, and at $\phi=\pm\phi_0/2, \pm 3\phi_0/2$, etc., for odd N. Here v_f is the Fermi velocity, and L is the circumference of the ring. Studies have been extended to include multichannel rings, disorder, spin-orbit coupling, and electron-electron interaction effects.[6-12] The persistent current which flows without dissipation is an equilibrium property of the ring, and is given by a flux derivative of the total energy of the ring. These currents can also be thought to arise from the competing requirements of minimizing the free energy in the presence of flux at the same time maintaining the single valuedness of the wave function. Persistent currents are truly mesoscopic effects in the sense that they are strongly suppressed when the ring size exceeds the characteristic dephasing length of the electrons L_ϕ (i.e., the length scale over which the electron can be considered to be in a pure state).

Theoretical treatments to date have mostly been concentrated on isolated rings. Persistent currents occur not only in the isolated rings but also in rings connected via leads to electron reservoirs, namely in open systems.[13-19] In a recent experiment, Mailly, Chapelier, and Benoit measured the persistent current in both closed and open rings.[4] Büttiker gave a conceptually simple approach of a small metal loop connected to an electron reservoir (open system).[13] The reservoir acts as a source and a sink for electrons, and is characterized by a well-defined chemical potential μ, and by definition there is no phase relationship between the absorbed and emitted electrons by the reservoir. The reservoir acts as an inelastic scatterer and as a source of energy dissipation or irreversibility. All scattering processes in the leads are assumed to be elastic. Inelastic processes occur only in the reservoir, and hence there is a complete spatial separation between elastic and inelastic processes. Due to the presence of inelastic scattering (by definition) in open systems, the amplitude of the persistent current is smaller as compared to the closed systems. Weak inelastic scattering does not destroy the effect leading to persistent currents. We have extended Büttiker's discussions to a case wherein electrons from the reservoir enter and leave the ring in a subbarrier regime characterized by evanescent modes throughout the circumference of the loop.[17] In this situation the persistent current arises simultaneously due to two nonclassical effects, namely the Aharanov-Bohm effect and quantum tunneling. The dependence of the current on the length of the ring is similar to that arising due to states localized by static disorder. In our recent work we have calculated the persistent currents in a normal metal loop connected to two electron reservoirs in the presence of magnetic flux.[18] We have shown that in general the magnitude of persistent current in a loop depends on the direction of current flow from one reservoir to the other. Persistent currents in open systems are sensitive to the direction of the current, unlike physical quantities such as conductance. We hope that this effect is useful for separating persistent current from other parasital currents (noise) associated with experimental measurements.

In our present work, we have considered a metallic loop coupled to two electron reservoirs (characterized by chemical potentials μ_1 and μ_2) via ideal wires as shown in Fig. 1. For the sake of simplicity we have restricted ourselves to the case of one-dimensional structure. The length of the upper arm of the loop is l_1, and that of the

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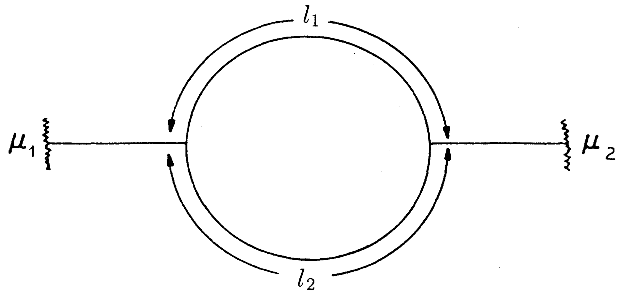

FIG. 1. An open metallic loop connected to two electron reservoirs.

lower arm is l_2, such that the circumference of the ring is $L=l_1+l_2$. When the chemical potential μ_1 is greater than μ_2, the net current flows from left to right, and vice versa when μ_2 is greater than μ_1. We show that in the presence of a current flow through the sample (the nonequilibrium situation), a net circulating current flows in a loop in the absence of magnetic field in certain range of Fermi energies. In a sense the persistent current is induced by incident carriers. The existence of such currents was first discussed by Büttiker;[14] however, our analysis is qualitatively different from that of the earlier study. The current injected by the reservoir into the lead around the small energy interval dE is given by $dI_{\text{in}}=ev(dn/dE)f(E)dE$. Here $v=\hbar k/m$ is the velocity of the carriers at the energy E, $(dn/dE)=1/(2\pi\hbar v)$ is the density of states in the perfect wire, and $f(E)$ is the Fermi distribution. The total current flow I in a small energy interval dE through the system is given by the current injected into the leads by reservoirs multiplied by the transmission coefficient T. This current splits into I_1 and I_2 in the upper and lower arms, respectively, at the junction, such that $I=I_1+I_2$ (the conservation of current or Kirchoff's law). Since the upper and lower arm lengths are unequal, in general these two currents differ in magnitude. Büttiker[14] suggests that this difference arises due to a circulating current I_0, such that the current in the upper branch is then given by $I_1=I/2+I_0$, and the current in the lower branch is given by $I_2=I/2-I_0$. Such a construction always results in a persistent current. However, if this definition is taken seriously, then even in a classical loop with different resistances in different arms one obtains different currents in the presence of a dc current and hence persistent current. It is clear then that with this definition one can obtain persistent currents even classically without invoking quantum mechanics at all. In our present quantum problem, when one calculates the currents (I_1,I_2) in two loops, there exists two distinct possibilities. In the first possibility, for a certain range of incident Fermi wave vectors (or energies), the current in the two arms I_1 and I_2 are individually less than the total current I, such that $I=I_1+I_2$. In such a situation both currents in two arms flow in the direction of the applied field. In such a situation we do not assign any persistent current flowing in the ring. However, in a certain energy interval, it turns out that current in one arm is larger than the total current I (magnification property). This implies that, to conserve the total current at the junctions, the current in the other arm must be negative, or should flow against the applied external field induced by difference in the chemical potentials. In such a situation one can interpret that the negative current flow in one arm of the loop continues to flow in a loop as a circulating (or persistent) current. Thus the magnitude of the persistent current is the same as that of the negative current. The direction of the persistent current can be inferred as follows. Consider a case when the net current flows in the right direction (i.e., $\mu_1>\mu_2$). If for this case negative current flows in the lower arm, then persistent current flows in a clockwise (or positive) direction. If, on the other hand, the negative current flows in the upper arm, then the persistent current flows in an anticlockwise (or negative) direction. The negative current in one arm of the loop is purely a quantum-mechanical effect. Our procedure of assigning persistent current only when negative current flows in one of the arms is the same procedure that is well known in classical ac network analysis.[20] It is well known that, when a parallel resonant circuit (capacitance C connected parallel with a combination of inductance L and resistance R) is driven by external electromotive force (generator), the circulating current arises in an LCR circuit at a resonance frequency. This effect is sometimes referred to as a current magnification. In this classical network, when the external driving frequency is around a resonance frequency, circulating currents are possible. Moreover, at the resonance the total net current amplitude in the circuit is at its minimum value. It turns out that even in our quantum problem the circulating current arises near the antiresonances (or transmission zeros) of the loop structure coupled to leads.

We now consider a case where the current is injected from the left reservoir (i.e., the current flow is in the right direction). The total current flow around a small energy interval is given by $I=(e/2\pi\hbar)T$, where T is the total transmission coefficient. It is straightforward exercise to set up a scattering problem for this case and to calculate the transmission coefficient and the currents in the upper (I_1) and lower (I_2) arms. We closely follow our earlier method of quantum waveguide transport on networks[17,18,21,22] to calculate these quantities. We have imposed Griffths boundary conditions (conservation of current) and single-valuedness of the wave functions at the junctions. For details see Refs. 17, 18, and 21–23. The expressions for I, T, I_1, and I_2 are given by

$$I=(e/2\pi\hbar)T\ , \tag{1}$$

$$T=\{8(2-\cos[2kl_1]-\cos[2kl_2]+4\sin[kl_1]\sin[kl_2])\}/\Omega\ , \tag{2}$$

$$I_1=(e/2\pi\hbar)8(1-\cos[2kl_2]+2\sin[kl_1]\sin[kl_2])/\Omega\ , \tag{3}$$

$$I_2=(e/2\pi\hbar)8(1-\cos[2kl_1]+2\sin[kl_1]\sin[kl_2])/\Omega\ , \tag{4}$$

where

$$\Omega=(37-5\cos[2kl_1]-32\cos[kl_1]\cos[kl_2]$$
$$-5\cos[2kl_2]$$
$$+5\cos[2kl_1]\cos[2kl_2]+48\sin[kl_1]\sin[kl_2]$$
$$-4\sin[2kl_1]\sin[2kl_2])\ . \tag{5}$$

Here k is the incident wave vector. Our expression for the transmission coefficient agrees with the earlier known expression[23] for the case of $l_1=l_2$. The transmission coefficient across a metallic loop connected to two reservoirs and in the presence of magnetic flux has been investigated by several authors[24,25] in connection with the Aharonov-Bohm effect. We have studied the behavior of the currents I_1 and I_2 as a function of the Fermi wave vectors. We then identified the wave-vector intervals, wherein either I_1 or I_2 flows in the negative direction, and by their magnitudes we calculated the persistent currents as described in earlier paragraphs. In Fig. 2 we plotted the circulating currents (solid curves) in the dimensionless units ($I_c \equiv 2\pi\hbar I_c/e$) in the small energy interval dE around the Fermi energy as a function of dimensionless wave vector kL. We have taken $l_1/l_2=5.0/3.0$. In Fig. 2 we have also plotted the transmission coefficient T for the same parameter values. We notice that the persistent current changes sign as we cross the energy or the wave vector at the first antiresonance (transmission zero or minimum) in the transmission coefficient. It does not change sign as we cross the second antiresonance. The first antiresonance is characterized by a asymmetric zero pole in the transmission amplitude [zero occurs at a value of $kL=(2\pi)$ and poles are given by $kL=(6.25495\text{-}i\ 0.299976)$ and $(6.46865\text{-}i\ 1.90045)$]. The proximity of the zero and the pole lead to the sharp variations in the transmission coefficient around the magnitude zero as a function of energy and lead to a asymmetrical behavior in the transmission coefficient (around antiresonance), sometimes termed as a Fano resonance.[26] The second antiresonance is characterized by a zero along with two symmetrically placed poles and the transmission coefficient is symmetric around the antiresonance. The zero is at a value $kL=(4\pi)$, and the poles are given by $kL=(12.4105\text{-}i\ 1.07584)$ and $(12.7222\text{-}i\ 1.07584)$. We have thus shown that the persistent current arises near the vicinity of the antiresonances, and that the nature of the persistent current as we cross the antiresonance depends on the zero-pole structure in the transmission amplitude around the antiresonance.

In Fig. 3 we plotted persistent currents in dimensionless units (solid curves) and transmission coefficient (dashed curves) versus kL for a case when $l_1/l_2=(3.0)$. For this particular case the transmission coefficient is symmetric around the antiresonances, and the persistent current does not change sign as we cross the antiresonance. In general the zero-pole structure in the transmission coefficient is sensitive to the ratio l_1/l_2, whether commensurate or not . For an incommensurate ratio we mostly obtain Fano-type antiresonances. For the commensurate case, depending on the degree of commensuration we can have both Fano-type and symmetric antiresonances. The magnitude and width of the persistent current peak in the vicinity of antiresonances depend on the strength of the imaginary part of the pole. If the two poles have different imaginary parts, the peak value of the persistent current will be higher (along with a smaller width) for the persistent current behavior near the pole, with a smaller imaginary part as compared to the larger one.

We have shown above that persistent currents can arise in the absence of a magnetic field in an open loop connected to two reservoirs in the presence of a transport current. For a fixed value of the Fermi energy the persistent current changes sign as we change the direction of the current flow. In equilibrium (i.e., $\mu_1=\mu_2$) we do not obtain any persistent currents in the absence of a magnetic field. In the nonequilibrium situation (i.e., $\mu_1\neq\mu_2$) it is possible to observe persistent currents. If $\mu_1>\mu_2$, then at zero temperature the total magnitude of the persistent current is given by $I_T=\int_{\mu_1}^{\mu_2} I_c dE$. Experimentally it is possible to observe these currents if one tunes the Fermi

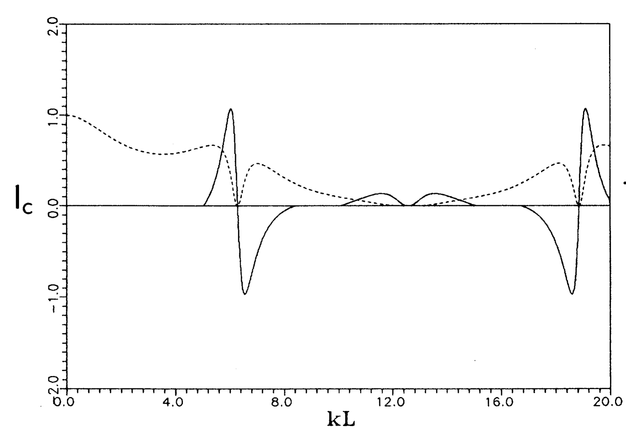

FIG. 2. Plot of persistent current I_c vs dimensionless wave vector kL (solid curve), and transmission coefficient T vs kL (dashed curve). We have taken $l_1/l_2=5.0/3.0$.

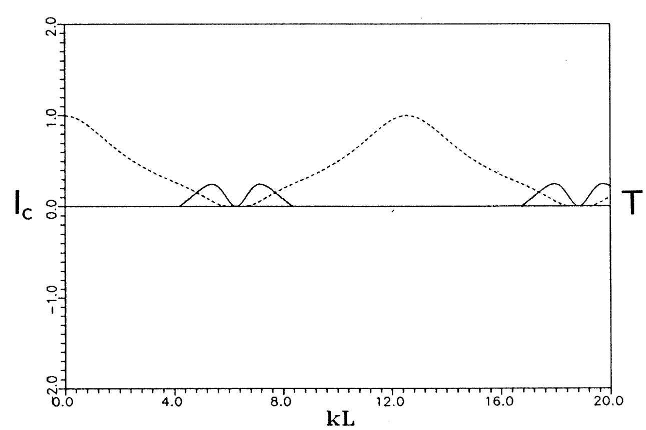

FIG. 3. Plot of persistent current I_c vs dimensionless wave vector kL (solid curve) and transmission coefficient T vs kL (dashed curve). We have taken $l_1/l_2=3.0$.

energy around the antiresonances in the two-port conductance (or transmission coefficient). Moreover, it is better to tune the Fermi energy around the symmetric antiresonance so that at finite temperature the effect survives, i.e., the current on both sides of this antiresonance has the same sign and hence the finite temperature does not lead to cancellations, as opposed to the case of Fermi energy around asymmetrical antiresonances.

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