Magnetostatic modes in semi-infinite magnetic-nonmagnetic superlattices for an arbitrary-angle magnetization geometry

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(Received 12 April 1994)

We discuss magnetostatic modes in semi-infinite magnetic-nonmagnetic superlattices in an arbitrary-angle magnetization geometry. In this case, some very interesting nonlinearity features appear. We also discuss the consistency condition for the existence of surface waves.

I. INTRODUCTION

In the past decade, the rapid progress in such techniques as molecular-beam epitaxy or metal-organic chemical vapor deposition has made it possible to prepare and investigate artificial layered systems, especially superlattices. Various kinds of collective excitations in superlattices attract a lot of studies, among which the magnetic collective excitations in magnetic superlattices have been an interesting subject.[1-11]

Camley and co-workers[1,2,6] and Barnas[3,4] discussed magnetic collective excitations in the cases where the saturation magnetization lies either parallel or perpendicular to the layers. According to them, there are two types of collective excitations in a magnetic-nonmagnetic superlattice. One is composed of surface waves in each magnetic layer and the other is composed of bulk waves in each magnetic layer. In a semi-infinite magnetic-nonmagnetic superlattice, these collective excitations are defined as bulk modes and surface modes, respectively, according to whether they are located inside the structure or near the surface. The features of the surface modes are sensitively dependent on the magnetization geometry. When the saturation magnetization lies parallel to the surface, the surface mode composed of surface waves in each magnetic layer has been found to be nonreciprocal with respect to the propagation direction. It propagates only along a restricted direction and the frequency is identical to the frequency of the Demon-Eshbach mode in a semi-infinite ferromagnet. But no surface modes can exist on the structure when the saturation magnetization lies along the normal to the surface, except that a deviation is introduced in the outermost elementary unit of the superlattice. This is due to the restriction from the consistency condition indicated by Camley and Cottam,[2] about which, however, there have been some differences.[1,6,7]

So far, to our knowledge, there is no discussion on the case where the magnetization lies neither parallel nor perpendicular to the layers. In this paper, we discuss magnetostatic modes in a semi-infinite magnetic-nonmagnetic superlattice in an arbitrary-angle magnetization geometry. In this case some very interesting nonlinearity features appear. Using algebraic theory, we also discuss the consistency condition for the existence of surface modes and confirm it mathematically in a more general magnetization geometry.

In Sec. II we develop a general relationship between the frequency of the surface mode and the direction of the applied field by solving the magnetostatic equations and employing the condition for surface modes. In Sec. III, we show some numerical results for general magnetization geometry.

II. THEORY

We consider the case where the wavelengths of spin waves are so long that the influence of short-range exchange interactions can be neglected but they are still short enough that[6]

$$(2\pi/\lambda)c \gg \omega ,$$

where λ is the length of the spin wave, ω the frequency, and c the speed of light in vacuum. Under such conditions, called the magnetostatic limit, the Maxwell's equations become the magnetostatic form

$$\nabla\times\mathbf{H}=0 , \tag{1}$$

$$\nabla\cdot(\mathbf{H}+4\pi\mathbf{M})=0 , \tag{2}$$

where the field $\mathbf{H}$ and the magnetization $\mathbf{M}$ can be written as sums of time-independent and time-dependent components in the forms

$$\mathbf{H}=\mathbf{H}_i+\mathbf{h}e^{-i\omega t} , \tag{3}$$

$$\mathbf{M}=\mathbf{M}_0+\mathbf{m}e^{-i\omega t} . \tag{4}$$

Here $\mathbf{h}$ and $\mathbf{b}=\mathbf{h}+4\pi\mathbf{m}$ also obey the magnetostatic equations. For sufficiently long wavelengths, the dynamic magnetic properties of the system can be described by the constitutive relation

$$\mathbf{m}=\chi\mathbf{h} , \tag{5}$$

where χ is the susceptibility tensor, the expression of which will be obtained by solving the Bloch equation of motion given by

$$\frac{d\mathbf{M}}{dt}=\gamma\mathbf{M}\times\mathbf{H} . \tag{6}$$

Here γ is the gyromagnetic ratio.

As described in Fig. 1, we take the z axis of the Cartesian coordinate system along the normal to the surface of

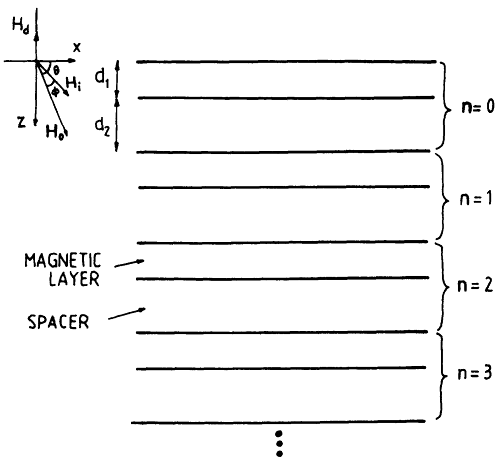

FIG. 1. The geometry of the semi-infinite magnetic-nonmagnetic superlattice. The z axis is taken along the normal to the surface. The thickness of the magnetic film is d_1 and the thickness of the nonmagnetic spacer is d_2. The length of an elementary unit is $L=d_1+d_2$. The elementary units are indexed by n. θ and ϕ show the directions of the applied field $\mathbf{H}_0$ and the internal field H_i, respectively.

the layered structure which occupies the half-space $z \geq 0$. The structure under consideration consists of magnetic layers and non-magnetic spacers which are labeled by the indices 1 and 2, respectively. Here d_1 is the thickness of magnetic layers and d_2 the thickness of spacers. The elementary units, with the length $L=d_1+d_2$, are indexed by the integer n and the first elementary unit corresponds to $n=0$. The applied field H_0 is restricted to the x-z plane. Following Camley and Cottam[2] we neglect the anisotropic field and thus the saturation magnetization lies parallel to the internal field $\mathbf{H}_i$. In our case, $\mathbf{H}_i=\mathbf{H}_d+\mathbf{H}_0$. Here $\mathbf{H}_d$ is the demagnetization field which is antiparallel to the z axis and has the amplitude $|\mathbf{H}_d|=4\pi M_0\sin(\theta+\phi)$, where θ and ϕ show the directions of $\mathbf{H}_0$ and $\mathbf{H}_i$, respectively. Solving Eq. (6) and keeping only linear time-varying terms, we obtain

$$\chi=\begin{bmatrix} \chi_1\sin^2\theta & i\chi_2\sin\theta & -\chi_1\sin\theta\cos\theta \\ -i\chi_2\sin\theta & \chi_1 & i\chi_2\cos\theta \\ -\chi_1\sin\theta\cos\theta & -i\chi_2\cos\theta & \chi_1\cos^2\theta \end{bmatrix} . \tag{7}$$

For a ferromagnet

$$\chi_1=\frac{H_iM}{H_i^2-(\omega/\gamma)^2} , \tag{8}$$

$$\chi_2=\frac{M(\omega/\gamma)}{H_i^2-(\omega/\gamma)^2} . \tag{9}$$

In terms of the magnetic scalar potential Φ, defined by $\mathbf{h}=-\nabla\Phi$, the magnetostatic equations (1) and (2) reduce to

$$\sum_i \mu_{ii}\frac{\partial^2\Phi}{\partial x_i^2}+2\mu_{zx}\frac{\partial^2\Phi}{\partial x\partial z}=0 . \tag{10}$$

Here the magnetic permeability tensor μ is defined by $\mu=I+4\pi\chi$. We search for a solution in the form of a plane wave propagating parallel to the surface as follows:

$$\Phi=\phi(z)e^{i(q_xx+q_yy-\omega t)} . \tag{11}$$

Following Camley and Cottam,[2] we assume the form of $\phi(z)$ as

$$\phi(z)=\begin{cases} Ce^{qz} , & z\leq 0 , \\ e^{-\beta nL}(A_+e^{\alpha_+(z-nL)}+A_-e^{\alpha_-(z-nL)}) , & nL\leq z\leq nL+d_1 , \\ e^{-\beta nL}(B_+e^{q(z-nL-d_1)}+B_-e^{-q(z-nL-d_1)}) , & nL+d_1\leq z\leq(n+1)L , \end{cases} \tag{12}$$

where

$$q^2=q_x^2+q_y^2$$

and

$$\alpha_\pm=\pm\left[\frac{\mu_{xx}q_x^2+\mu_{yy}q_y^2}{\mu_{zz}}-\frac{\mu_{zx}^2q_x^2}{\mu_{zz}^2}\right]^{1/2}-i\frac{\mu_{zx}q_x}{\mu_{zz}} .$$

Here q_x and q_y are the components of the wave vector along the x and y directions, respectively. The coefficients $A_\pm$, $B_\pm$, and C will be determined by employing boundary conditions. In order to guarantee that the solution we find is a true surface wave β must satisfy the inequality given by

$$\mathrm{Re}(\beta)>0 . \tag{13}$$

Here we confine ourselves to the Voight geometry where the in-plane propagation wave vector of the spin wave is restricted to be perpendicular to the saturation magnetization. In our case, this condition becomes $q_x=0$. Thus $\alpha_+=-\alpha_-=\alpha$, with $\alpha=\sqrt{\mu_{yy}/\mu_{zz}}q$. The usual electromagnetic boundary conditions are that the tangential $\mathbf{h}$ and the normal $\mathbf{b}$ components are continuous across all boundaries. In the case described in Fig. 1, the boundary conditions become that Φ and $b_z=-\sum_i\mu_{zi}\partial\Phi/\partial x_i$ are continuous at $z=nL$ and $z=nL+d_1$. The application of the boundary conditions results in

$$\begin{aligned}
&C=A_{+}+A_{-} ,\\
&qC=\lambda_{+}A_{+}+\lambda_{-}A_{-} ,\\
&e^{\alpha d_1}A_{+}+e^{-\alpha d_1}A_{-}=B_{+}+B_{-} ,\\
&\lambda_{+}e^{\alpha d_1}A_{+}+\lambda_{-}e^{-\alpha d_1}A_{-}=q(B_{+}-B_{-}) ,\\
&e^{-\beta L}(A_{+}+A_{-})=e^{qd_2}B_{+}+e^{-qd_2}B_{-} ,\\
&e^{-\beta L}(\lambda_{+}A_{+}+\lambda_{-}A_{-})=q(e^{qd_2}B_{+}-e^{-qd_2}B_{-}) ,
\end{aligned} \tag{14}$$

with

$$\lambda_{\pm}=\pm\alpha\mu_{zz}+i\mu_{zy}q_y .$$

Eliminating B_+, B_-, and C from Eqs. (14) we can obtain a set of three linear homogeneous equations in two unknowns A_+ and A_- as follows:

$$\begin{aligned}
&(\lambda_{+}-q)(1-e^{\beta L+\alpha d_1-qd_2})A_{+}\\
&\qquad+(\lambda_{-}-q)(1-e^{\beta L-\alpha d_1-qd_2})A_{-}=0 ,\\
&(\lambda_{+}+q)(1-e^{\beta L+\alpha d_1+qd_2})A_{+}\\
&\qquad+(\lambda_{-}+q)(1-e^{\beta L-\alpha d_1+qd_2})A_{-}=0 ,\\
&(\lambda_{+}-q)A_{+}+(\lambda_{-}-q)A_{-}=0 ,
\end{aligned} \tag{15}$$

with the attenuation constant β being a parameter. According to algebraic theory, a system of linear homogeneous equations has a nontrivial solution only if the rank of the coefficient matrix is smaller than the number of unknowns. In the case under consideration, the number of unknowns is 2 and thus for a nontrivial solution the rank of the coefficient matrix must equal 1, which implies that the determinants of the three 2×2 submatrixes in the coefficient matrix all vanish, i.e.,

$$(\lambda_{+}-q)(\lambda_{-}+q)(1-e^{\beta L+\alpha d_1-qd_2})(1-e^{\beta L-\alpha d_1+qd_2})-(\lambda_{+}+q)(\lambda_{-}-q)(1-e^{\beta L-\alpha d_1-qd_2})(1-e^{\beta L+\alpha d_1+qd_2})=0 , \tag{15a}$$

$$(\lambda_{+}+q)(\lambda_{-}-q)(1-e^{\beta L+\alpha d_1+qd_2})-(\lambda_{-}+q)(\lambda_{+}-q)(1-e^{\beta L-\alpha d_1+qd_2})=0 , \tag{15b}$$

$$(\lambda_{+}-q)(\lambda_{-}-q)\sinh(\alpha d_1)=0 . \tag{15c}$$

Here β should satisfy all three equations at the same time. According to Camley and Stamps.[6] Eq. (15c) has three possible cases, $\lambda_{+}-q=0$, $\lambda_{-}-q=0$, and $\alpha d_1=im\pi$ (here $m=0,\pm1,\pm2,\cdots$). Combining Eqs. (15a) and (15c), we have

$\lambda_{+}-q=0$ requires $\beta L=\pm(\alpha d_1+qd_2)$,

$\lambda_{-}-q=0$ requires $\beta L=\pm(\alpha d_1-qd_2)$,

$\alpha d_1=im\pi$ requires $\beta L=\pm qd_2+i(2m+1)\pi$.

Similarly, combining (15b) and (15c), we obtain

$\lambda_{+}-q=0$ requires $\beta L=-(\alpha d_1+qd_2)$,

$\lambda_{-}-q=0$ requires $\beta L=\alpha d_1-qd_2$,

$\alpha d_1=im\pi$ requires $\beta L=-qd_2+i(2m+1)\pi$.

Obviously, the three consistency solutions for β, which satisfy the three equations simultaneously, are

$$\beta L=-(\alpha d_1+qd_2) ,$$
$$\beta L=\alpha d_1-qd_2 ,$$
$$\beta L=-qd_2+i(2m+1)\pi .$$

After giving up the solution which does not represent a true surface mode due to $\mathrm{Re}(\beta)<0$, we only have the solution $\beta L=\alpha d_1-qd_2$ with $\lambda_{-}-q=0$. In the geometry under consideration, the wave vector is restricted to the y direction, i.e., $q=|q_y|$. After substituting the definition of λ_- into the equation, we have

$$(\sqrt{\mu_{yy}\mu_{zz}}+1)q-i\mu_{zy}q_y=0 . \tag{16}$$

When α is real and $\alpha d_1-qd_2>0$, this solution represents the type of surface modes which are composed of surface waves in each magnetic layer. Obviously, the other type of surface modes, which consist of bulk waves in each magnetic film, cannot exist due to the consistency requirement. Mathematically we confirm the conclusion made by Camley and Cottam[2] in the case of perpendicular magnetization and extend it to a more general magnetization geometry.

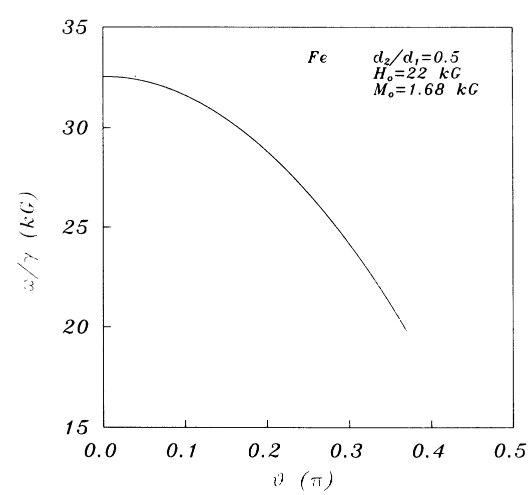

FIG. 2. Frequency of the surface wave versus the direction of the external field.

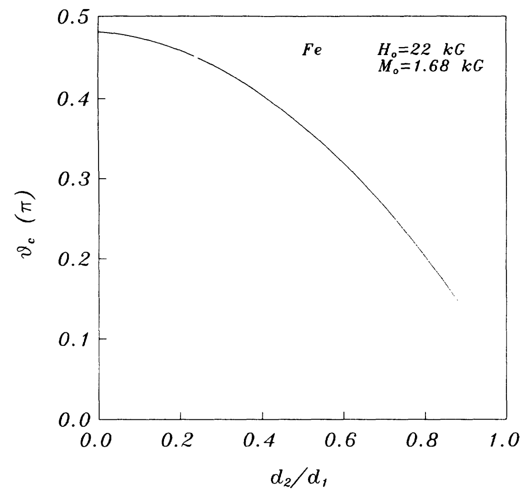

FIG. 3. Critical angle of the surface wave versus dimensionless geometry parameter d_2/d_1.

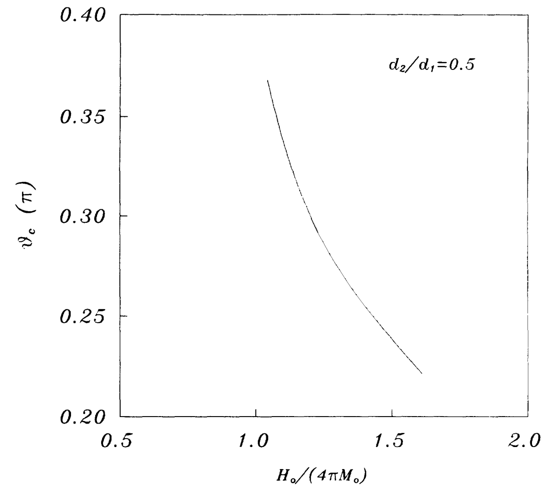

FIG. 4. Critical angle of the surface wave versus dimensionless magnetic parameter $H_0/(4\pi M_0)$.

It can be easily confirmed from Eq. (16) that the surface wave has the frequency $\omega=\gamma(H_0+2\pi M_0)$ and propagates only along the $+y$ direction when $\theta=0$, but cannot exist when $\theta=\pi/2$. This agrees with the earlier results given by Camley and co-workers.[1,2,6] In the next section, by solving Eq. (16) numerically we will show some interesting features of the surface mode in the case of arbitrary-angle magnetization.

III. RESULTS

In this section, we present a numerical solution of Eq. (16). It can be seen that under the case considered the nonlinearity plays an important role in the collective excitations in the layered structure. We use the parameters appropriate for Fe: $M=1.68$ kG, $H_0=22$ kG. For Eq. (16), there are two possible cases where either $q_y=-q$ or $q_y=q$, corresponding to the propagation of the surface wave in the $+y$ and $-y$ directions, respectively. But we do not find that Eq. (16) has physical solutions for $q_y=q$. This means that the surface wave cannot propagate along the $-y$ direction. This is the same as in the situation of parallel-magnetization geometry.

In Fig. 2 we explore the frequency as a function of the angle between the applied field and the surface, taking $d_2/d_1=0.5$. It can be observed that the frequency decreases as θ decreases. When $\theta=0$, the frequency $\omega/\gamma=32.6$ kG, which is identical to the frequency of the Damon-Eshbach surface wave on a semi-infinite ferromagnet. There exists a critical angle $\theta_c=0.37\pi$, above which the surface wave cannot exist.

We further investigate the influences of structure parameters on the critical angle. In Fig. 3 we plot the critical angle θ_c versus the dimensionless geometry parameter d_2/d_1. It can be seen that θ_c increases as d_2/d_1 decreases. This is reasonable in terms of the physics because the nonmagnetic spacer suppresses magnetic excitations. When $d_2/d_1=0$, i.e., $d_2=0$, θ_c presents the critical angle in the case of a semi-infinite ferromagnet under the same conditions.

In Fig. 4 we show the relationship between the critical angle θ_c and the dimensionless magnetic parameter $H_0/(4\pi M_0)$, assuming $d_2/d_1=0.5$. One can see a rapid decrease of θ_c with increase of $H_0/(4\pi M_0)$. Therefore we can conclude that for a given saturation magnetization a larger applied field will result in a smaller critical angle.

In summary, we discuss the consistency condition for the existence of surface modes and support the conclusion, made by Camley and Cottam,[2] that the surface wave composed of bulk waves in individual magnetic layers cannot exist on a semi-infinite magnetic-nonmagnetic superlattice due to the consistency requirement. We further extend the study of surface modes to a more general case where the saturation magnetization makes an angle with respect to the surface. Some interesting features are found as follows.

(1) Like the case of parallel magnetization, in any-angle magnetization geometry the surface mode is nonreciprocal with respect to propagation direction. It can only propagate along the $+y$ direction in our system. The frequency decreases with increase of the angle between the applied field and the surface.

(2) There exists a critical value of the angle, above which the surface wave cannot exist. The critical angle depends sensitively on the structure parameters. As other parameters are fixed, the increase of applied field or the increase of thickness of spacers will result in the decrease of the critical angle.

These features of the surface wave reflect a nonlinearity effect in the system. We hope our results can be confirmed experimentally by use of Brillouin light scattering.

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