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# Linear ac magnetic response near the vortex-glass transition in single-crystalline $YBa_2Cu_3O_7$

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By using a miniature two-coil mutual-inductance method and an exact inversion scheme, the ac penetration depth $\lambda_{ac}$ was measured in a twinned $YBa_2Cu_3O_7$ crystal in dc fields parallel to the $c$ axis. The Ohmic dc resistance was also measured on the same crystal. The behavior of these quantities was altogether consistent with the vortex-glass-transition scaling. However, if we calculate the Labusch parameter and the pinning relaxation time from $\lambda_{ac}$, they show unexpectedly large frequency dependencies, which can hardly be explained by the standard theory of the linear ac response of pinned vortices.

The phase diagram of the mixed state in high-$T_c$ superconductors has been a subject of intense study in the past few years. It has become rather clear that the second-order vortex-glass (VG) transition[1] occurs in disordered $YBa_2Cu_3O_7$ samples without columnar defects.[2–6] Although dc transport properties become highly nonlinear in the VG phase[2–4], the linear ac response persists to low temperatures.[1,5,6] The linear ac magnetic response is known to be characterized by the complex ac penetration depth $\lambda_{ac}$.[7–10] Theories of the linear ac magnetic response[7–10] have incorporated the effects of pinning and creep, and have shown how $\lambda_{ac}$ is related to pinning properties such as the Labusch parameter $\alpha_L$ and pinning relaxation time $\tau$. Recently, it was reported that the linear ac magnetic response measured through ac susceptibility obeys the critical scaling law near the VG transition temperature.[5,6] Such experiments naturally lead to the question: How should the description of the linear ac response through $\alpha_L$ and $\tau$ be understood in relation to the critical scaling of the VG transition? It is particularly interesting to see how the specific parameters characterizing vortex pinning, such as $\alpha_L$ and $\tau$, behave near the VG transition, since the VG transition is a continuous (second-order) transition where pinning becomes progressively effective.

In this work, we used a miniature two-coil mutual-inductance method and an exact inversion scheme to obtain a quantitatively reliable value of $\lambda_{ac}$ in a high-quality $YBa_2Cu_3O_7$ single crystal in magnetic fields applied along the $c$ axis. It is shown that $\lambda_{ac}(T)$ measured at different frequencies obeys the critical scaling[1] of the VG transition, where the VG transition temperature was determined from the Ohmic dc resistance measured on the same crystal. What is important in our result is that $\alpha_L$ and $\tau$ calculated from such $\lambda_{ac}(T)$ that obeys the critical scaling were dependent on the frequency. Since all the linear ac response theories of pinned vortices now at hand[7–10] consider frequency-independent pinning parameters, our result implies that such theories are incompatible with the critical phenomenon near the VG transition.

The two-coil mutual-inductance technique adopted in this work has been used by other groups for the study of superconducting *films*.[11] The pickup coil and the cancel coil were wound with a 25-$\mu$m-diam wire around a 0.5-mm-diam glass tube. On top of these coils the drive coil was wound with a 80-$\mu$m-diam wire; therefore, the typical dimension of the coil system was 0.6 mm. The sample was glued on one side of the coil system. The sample reported here was a 55-$\mu$m-thick heavily twinned $YBa_2Cu_3O_7$ single crystal, which has a trapezoidal $ab$ face with a base of 1.4 mm, top of 0.5 mm, and a height of 2.1 mm. The coils were placed at about 0.6 mm from the base of the trapezoid, so the typical sample dimension seen by the coil was about 1.2 mm. Note that it is essential to maximize the ratio of the sample size to the coil size to reduce the finite-size effect.[11] The dc magnetic field was applied parallel to the $c$ axis; therefore, the axis of the coils was parallel to the dc field. The dc resistance measured on the same sample showed $T_{c0}=92.1$ K and $\Delta T_c \approx 0.4$ K in zero field. All the data presented here were taken in the field-cooled procedure.

Figure 1(a) shows the in-phase and out-of-phase signals taken at 0.78 T at frequencies from 4.24 to 100 kHz. Similar data were taken in 0, 0.39, 1.18, and 1.57 T. Since the results were essentially the same, we will concentrate on the 0.78-T data below. The amplitude of the drive current $I_d$ was 5 mA, which produced an ac magnetic field of about $10^{-5}$ T at the sample. The response was linear with this much of the drive current, which was confirmed by taking the temperature-sweep data for three different drive-current amplitudes, 2.5, 5, and 7.5 mA, at 100 kHz; namely, the plots of the temperature vs the voltage divided by $I_d$ essentially collapsed on top of each other for the three drive currents within the experimental resolution. Also shown in the inset of Fig.1(a) is the resistive transitions measured with 1.8 A/cm$^2$.

For our geometry, the solution given by Clem and Coffey[10] can be used to calculate the voltage induced in the pickup coil as a function of $\lambda_{ac}^{-2}$. Note that the calculation by Clem and Coffey takes the nonlocality of vortex interaction into account. Once a table relating $\lambda_{ac}^{-2}$ to the induced

 

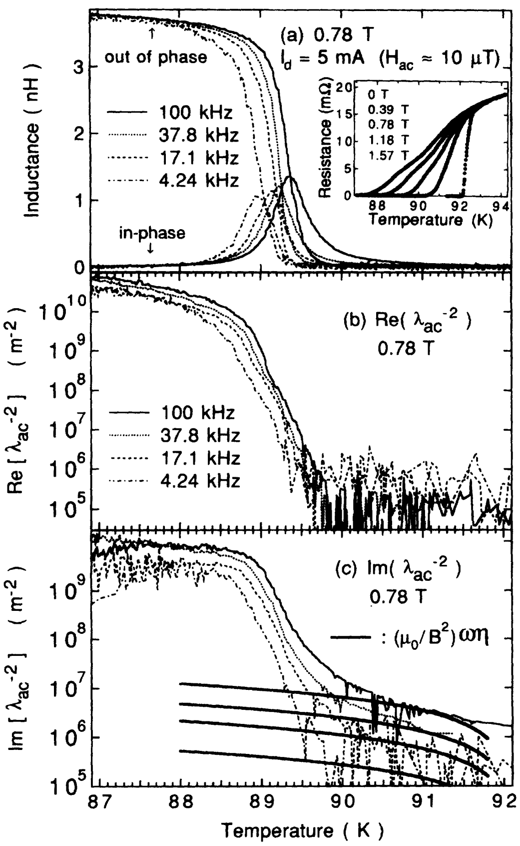


FIG. 1. (a) The in-phase and the out-of-phase signals at 0.78 T shown as $V/\omega I$. The inset shows the resistive transitions. (b) Real and (c) imaginary parts of $\lambda_{ac}^{-2}$. The thick solid lines in (c) are the estimation of the viscosity term.

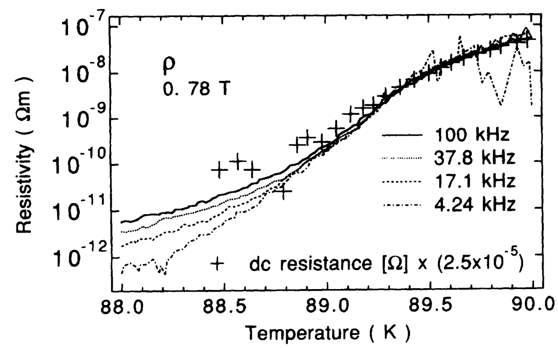


FIG. 2. The resistivity obtained from $\lambda_{ac}^{-2}$, together with the dc resistance times $2.5\times10^{-5}$.

voltage is constructed, the measured signals are converted to $\lambda_{ac}^{-2}$ with a numerical inversion method.[11] In the present experiment, the finite-size effect was not negligible. To take the the finite-size correction into the calculation, we introduced a lower wave-number cutoff in the numerical integration, which essentially corresponds to excluding the contribution from the current flowing outside the actual sample size. The cutoff value $q_{max}$ was determined by the condition that the low-temperature result in zero field gives a value that is consistent with the reported value [$\lambda(0)=0.14$ $\mu$m (Ref. 12)]. For the present sample, $q_{max}$ thus determined was $3.9\times10^3$ m$^{-1}$, which corresponds to the spatial extent of 1.6 mm, consistent with the actual sample dimension. The result of the inversion from the voltage data to $\lambda_{ac}^{-2}$ is shown in Figs. 1(b) and 1(c). The real (imaginary) part of $\lambda_{ac}^{-2}$ was indeterminate above 90 K (below 88 K) because the out-of-phase (in-phase) signal was almost zero there.

As in the usual ac-susceptibility measurement, the peak in the in-phase signal should occur when $|\lambda_{ac}|$ becomes comparable to a characteristic sample dimension,[13] the thickness in this case. From Figs. 1(a)–1(c), one sees that, at each $\omega$, the peak occurs when $|\lambda_{ac}|\approx 67$ $\mu$m, a value consistent with the sample thickness. This justifies our numerical procedure. Furthermore, it is useful to compare the dc-transport result and the resistivity calculated from $\lambda_{ac}^{-2}$. The (real) resistivity $\rho$ can be calculated[7] with the equation $\rho=\mathrm{Re}(i\omega\mu_0\lambda_{ac}^2)=\omega\mu_0\mathrm{Im}(\lambda_{ac}^{-2})/[\mathrm{Re}(\lambda_{ac}^{-2})^2+\mathrm{Im}(\lambda_{ac}^{-2})^2]$. The result for $\rho$ is shown in Fig. 2, together with the dc resistance times $2.5\times10^{-5}$. Considering the sample dimensions and the fact that the current leads were placed on top of the $ab$ plane, the factor $2.5\times10^{-5}$ is consistent with the conversion factor from resistance ($\Omega$) to resistivity ($\Omega$m). Note that the dc-transport result and $\rho$ calculated from $\lambda_{ac}^{-2}$ at the four different frequencies all coincide with each other above about 89.3 K. This coincidence gives further justification to our numerical procedure.

Now let us see if our $\lambda_{ac}$ thus obtained obeys the critical scaling of the VG transition. We do not determine the critical exponents from our data, but we use the values reported in the literature.[2–5] First, we obtain the VG transition temperature from the Ohmic dc resistance data, as shown in the inset of Fig. 3. Here $\partial T/\partial \ln R$ is plotted vs $T$, and the tail of the data is fitted with a line of slope $1/\nu(z-1)=0.154$,[2–5] where $\nu$ and $z$ are the static and dynamical critical exponents, respectively. $T_g$ is given by the temperature where this line extrapolates to zero;[3] for example, $T_g\approx 88.2$ K at 0.78 T. Of course, these fits alone are not convincing enough to believe

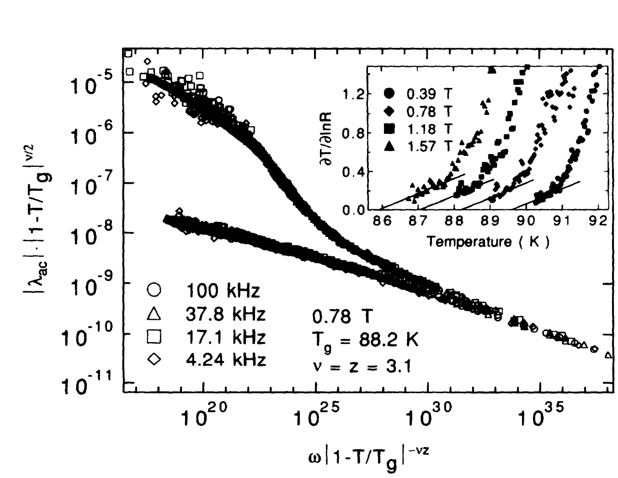


FIG. 3. Scaling plot of $|\lambda_{ac}|$ in 0.78 T from 85 to 92 K. The inset shows the plot of $\partial T/\partial \ln R$ vs $T$ for the dc resistance data.

that the VG transition indeed occurs; it is due to our lack of picovolt sensitivity[3] in measuring the resistance of a single crystal. However, as long as the value of $1/\nu(z-1)$ is fixed, the data give a certain range of temperature (about $\pm 0.2$ K) where $T_g$ should lie, if it ever exists.

Using the value of $T_g$ thus determined and the critical exponents in the literature, we can analyze $\lambda_{ac}$ according to the critical scaling. The scaling theory[1] implies that $\lambda_{ac}$ should obey the scaling relation

$$|\lambda_{ac}(\omega)| = \xi_g^{1/2}\tilde{\lambda}_{\pm}(\omega \xi_g^z), \tag{1}$$

where $\tilde{\lambda}_{+}$ and $\tilde{\lambda}_{-}$ are the universal scaling functions for $T>T_g$ and $T<T_g$, respectively, and $\xi_g$ is the VG correlation length. Since $\xi_g \sim |1-T/T_g|^{-\nu}$, Eq. (1) means that a plot of $|\lambda_{ac}||1-T/T_g|^{\nu/2}$ vs $\omega|1-T/T_g|^{-\nu z}$ for different frequencies should collapse onto two universal curves. The result of such a scaling plot is shown in Fig. 3, where the critical exponents were taken to be $\nu = z = 3.1$,[14] the values that Kötzler *et al.* have argued[5] to be universal for twinned crystals in magnetic fields parallel to the $c$ axis. The collapse of the data onto universal curves is remarkable, which gives evidence that our data from dc transport and linear ac response are altogether consistent with the VG critical scaling.

The linear ac response theory for pinned vortices shows[7–10] that $\lambda_{ac}$ at an angular frequency $\omega$ is expressed as

$$\lambda_{ac}^2 = \lambda^2 + \frac{B^2}{\mu_0}\left(\frac{\alpha_L}{1-i/\omega\tau} + i\omega\eta\right)^{-1}, \tag{2}$$

where $\lambda$ is the London penetration depth, $B$ is the magnetic induction, and $\eta$ is the volume viscosity. Here, $\alpha_L$ is defined to be the elastic restoring force per unit volume, and $\tau$ is written as $\tau \approx (\eta/\alpha_L)\exp(U/k_BT)$, where $U$ is the energy barrier for thermal activation. When we analyze $\lambda_{ac}^{-2}$ in magnetic fields, the $\lambda^2$ term in Eq. (2) can be neglected because $|\lambda_{ac}|$ in magnetic fields were much longer than $\lambda$ obtained in zero field. Then, the relevant parameters in Eq. (2) are $\alpha_L$, $\tau$, and $\eta$. Measurements in microwave frequencies show[15–17] that $\eta$ can be described by the Bardeen-Stephen expression $\eta = BB_{c2}/\rho_n$, where $B_{c2}$ is the upper critical field and $\rho_n$ is the normal-state resistivity. Although $\rho_n$ of $YBa_2Cu_3O_7$ has been reported to become anomalously small at low temperatures,[17,18] $\rho_n$ is not so much different from the value just above $T_c$ at the temperatures of our interest ($T>88$ K).[15,18] Therefore, the contribution from $\omega\eta$ to $\mathrm{Im}(\lambda_{ac}^{-2})$ can be well estimated by $\omega BB_{c2}/\rho_n$. Such an estimation, with $dB_{c2}/dT = -1.9$ T/K (Ref. 19) and $\rho_n = 0.8$ $\mu\Omega$ m, is plotted in Fig. 1(c). It is understood that, in our frequency range, the contribution from $\omega\eta$ to $\mathrm{Im}(\lambda_{ac}^{-2})$ becomes less than 3% below 89.3 K, where we can safely neglect $\omega\eta$ in Eq. (2). When $\omega\eta$ and $\lambda$ are neglected, $\tau$ and $\alpha_L$ can be calculated at each $\omega$ by $\tau = \mathrm{Re}(\lambda_{ac}^{-2})/\omega\,\mathrm{Im}(\lambda_{ac}^{-2})$ and $\alpha_L = \mathrm{Re}(\lambda_{ac}^{-2})B^2[1+1/(\omega\tau)^2]/\mu_0$. The results for $\tau$ and $\alpha_L$ thus calculated are shown in Figs. 4(a) and 4(b). It is evident that $\tau$ and $\alpha_L$ are $\omega$ dependent. Note that there is an order of magnitude variance in $\tau$ for frequencies from 4.24 to 100 kHz at all temperatures in Fig. 4(a). $\alpha_L$ also shows an order of magnitude variance at higher temperatures, though its variance becomes smaller for lower temperatures.

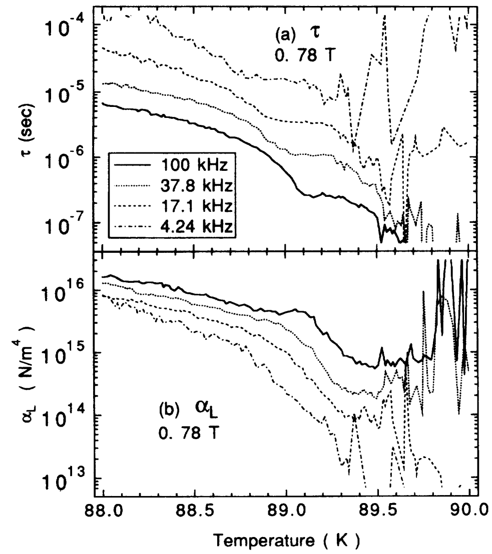


FIG. 4. (a) $\tau$ and (b) $\alpha_L$ obtained from $\lambda_{ac}^{-2}$ at each $\omega$.

As will be shown below, the trend in the $\omega$ dependence of $\alpha_L$ can be qualitatively explained by taking the distribution of the Labusch parameter into consideration; however, it is difficult to explain the order-of-magnitude variance. To see the essence of the effect of distribution, let us assume that, for simplicity, the Labusch parameter is uniformly distributed from 0 to $\alpha_1$. A careful consideration of the vortex dynamics reveals that the quantity to be averaged is $\lambda_{ac}^2$. The average of $\lambda_{ac}^2$ is given by $\bar{\lambda}_{ac}^2 = (1/\alpha_1)\int_0^{\alpha_1}\lambda_{ac}^2 d\alpha_L$. Then the apparent $\bar{\alpha}_L$ calculated from $\bar{\lambda}_{ac}^2$ becomes $\bar{\alpha}_L = 2\alpha_1/\ln(1+\alpha_1^2/\omega^2\eta^2)$, where an approximation $\exp(-U/k_BT)\to 0$ was made. This $\bar{\alpha}_L$ is larger for larger $\omega$, which agrees with the trend observed in our result. However, since the $\omega$-dependent factor appears only in the logarithm, the order-of-magnitude variance can hardly be reproduced. Therefore, it seems that an incorporation of the distribution of pinning parameters into the linear ac response theory is not enough to explain our result.

It should be mentioned that both $\tau$ and $\alpha_L$ in Fig. 4 show an anomalous change in their behavior at some frequency-dependent temperature which lies around 89 K. The reason for this change is not clear; however, a careful analysis of the data tells us that such change occurs when $|\lambda_{ac}(\omega)|$ becomes 33 $\mu$m at all frequencies. This fact suggests that the change in the behavior of $\tau$ and $\alpha_L$ is likely to be related to a crossover between some different length scales. One possibility is the crossover between the sample thickness and the length scale that characterize the correlation between vortex segments along the $c$ axis, which is expected to occur in the vortex liquid state.[20] Further study is necessary to clarify the origin of this anomaly.

In conclusion, we found that the description of the linear ac magnetic response of pinned vortices through the Labusch parameter $\alpha_L$ and the relaxation time $\tau$ (Refs. 7–10) is hardly compatible with the critical scaling of the vortex-glass transition.[1] This fact was manifested in unexpectedly large frequency dependencies in $\alpha_L$ and $\tau$, which was calculated from such $\lambda_{ac}$ that obeys the critical scaling.

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