Thermoelectric power above the Kosterlitz-Thouless transition

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Thermoelectric transport due to the motion of free unbound vortices above the Kosterlitz-Thouless transition temperature is considered. It is shown that the Magnus force, which appears under the influence of the temperature gradient, leads to a nonzero thermopower at a temperature above the Kosterlitz-Thouless transition temperature $T_{\rm KT}$, but below the mean-field (Ginzburg-Landau) critical temperature $T_{\rm GL}$. The exponential inverse-square-root reduced-temperature dependence of the thermopower above $T_{\rm KT}$ is predicted with a consequent broadening of the superconducting transition detected by means of thermopower measurements. The case of the clean superconductor is analyzed.

Transport properties in the vicinity of the Kosterlitz-Thouless (KT) transition have attracted the close attention of both theorists and experimentalists since the discovery of this phenomenon.[1] Kosterlitz-Thouless behavior associated with the dissociation of vortex-antivortex pairs due to thermal fluctuations above the critical temperature $T_{\rm KT}$ is a characteristic feature of two-dimensional superconductors. In view of the fact that the main structural feature of high-T_c superconductors is the large value of spatial anisotropy and the essentially two-dimensional character of their physical properties, in the recent past there has been considerable effort to investigate experimentally the possibility of the KT transition in these materials. Kosterlitz-Thouless properties have been examined in bulk single crystals of $YBa_2Cu_3O_7$ (Ref. 2) and $Bi_2Sr_2CaCu_2O_8$,[3] oriented $Tl_2Ba_2CaCu_2O_8$, $YBa_2Cu_3O_{7-x}$, and $Bi_2Sr_2CaCu_2O_8$ thin films,[4–6] and $YBa_2Cu_3O_{7-x}$ monolayers[7]. In these papers the resistivity data just below the mean-field critical temperature were quantitatively compared with the predictions of KT theory and a rather good agreement between theory and experiment was demonstrated.

In the present paper we study the thermoelectric transport in high-temperature superconductors above $T_{\rm KT}$. Taking into account that high-T_c materials are in the clean limit ($l \gg \xi_0$, where l is the electron mean free path and ξ_0 is the zero-temperature coherence length),[6,8] we consider below the model of a clean two-dimensional superconductor. We show that the thermoelectric effect in zero magnetic field appears due to the Magnus force, which causes the motion of free unbound vortices above $T_{\rm KT}$. Vortex motion under the influence of a temperature gradient is widely discussed now in connection with studies of the Seebeck and Nernst effects in the mixed state of high-T_c oxides.[9,10] In the present work, we follow the assumption that the Magnus force which leads to the vortex motion has its origin in the fulfillment of boundary conditions at the vortex core boundary, as was suggested by Samoilov *et al.*[10]

We consider now a layered clean superconductor with in-plane temperature gradient applied near the KT transition point. We neglect below all effects of interlayer interaction, so the superconductivity is assumed to be strictly two-dimensional. As was shown,[1] the KT transition in two-dimensional superconductors involves vortex-antivortex pairs bound at low temperatures which dissociate into free vortices at the characteristic temperature $T_{\rm KT}$. The energy of the vortex-antivortex pair is a logarithmic function of the separation and in the absence of electrical current has the form

$$U(r) = 2E_c + 2\pi KT \ln(r/\xi) \qquad \text{for } \xi \ll r \ll \lambda, \qquad (1)$$

and for $r \gg \lambda$ the interaction energy falls off as $1/r$. Here E_c is the vortex core energy, T is the temperature, K is the renormalized stiffness constant, r is the separation of the vortex-antivortex pair, ξ is the Ginzburg-Landau coherence length, which for high-T_c materials would be the a-b plane coherence length, and λ is the two-dimensional magnetic penetration depth. The temperature range of our interest is $T_{\rm KT} < T < T_{\rm GL}$. As was discovered by Nelson and Kosterlitz,[11] there exists a universal relation between $T_{\rm KT}$ and the two-dimensional superfluid density n_s:

$$T_{\rm KT} = \frac{\pi n_s \hbar^2}{8m} = \frac{c^2\hbar^2}{16e^2\lambda} \qquad (2)$$

(e is the electron charge and c is the velocity of light), where n_s and λ are functions of temperature. To evaluate $T_{\rm KT}$ explicitly we note that in the clean case[12]

$$\lambda = 1.33\frac{\lambda_L^2(0)}{d}\left[\frac{\Delta(T)}{\Delta(0)}\tanh\left(\frac{\Delta(T)}{2T}\right)\right]^{-1}, \qquad (3)$$

where λ_L is the London penetration depth, $\Delta(T)$ is the BCS energy gap, and d is the effective correlation length along the c axis, which would be the actual interlayer distance in our consideration. By substitution of Eq. (3) into Eq. (2) one obtains the implicit equation for $T_{\rm KT}$:

$$\frac{T_{\rm KT}}{T_{\rm GL}}A^{-1}\left(\frac{T_{\rm KT}}{T_{\rm GL}}\right) = 3.93\frac{\xi}{l}\frac{\hbar d}{e^2\rho_n}, \qquad (4)$$

where $A(T/T_{\rm GL})$ is the temperature-dependent function

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contained within the brackets in Eq. (3) and ρ_n is the normal-state resistivity. For temperatures near $T_{\rm KT}$ Eq. (5) reduces to the simple form

$$\tau_c = \frac{T_{\rm GL} - T_{\rm KT}}{T_{\rm KT}} = 0.23 \frac{l}{\xi} \frac{e^2 \rho_n}{\hbar d}, \tag{5}$$

which is valid when $\tau_c \ll 1$. Compared to the dirty limit calculations,[13] the additional factor l/ξ arises in Eq. (5).

Under an external temperature gradient, free vortex motion above $T_{\rm KT}$ results in the thermoelectric effect due to the Magnus force acting on vortices. As was shown in Ref. 10, the Magnus force in this case has the form

$$\mathbf{F}_M = \Phi_0 \left[\left(\frac{S_n}{\rho_n} \nabla T - n_s e \mathbf{v}_L \right) \times \mathbf{z} \right], \tag{6}$$

where $\Phi_0 = \pi c\hbar/e$ is a flux quantum, S_n is the normal-state thermopower, $\mathbf{v}_L$ is the drift velocity of the vortex, and $\mathbf{z}$ is a vorticity vector. The vortex velocity can be found from the force balance equation $\mathbf{F}_M + \mathbf{F}_T + \mathbf{f} = \mathbf{0}$, where $\mathbf{F}_T = -S_\phi \nabla T$ is the so-called thermal force (S_ϕ is the entropy per unit length of the vortex line) and $\mathbf{f} = -\eta \mathbf{v}_L$ is the viscous drag force. We note that the thermal force $\mathbf{F}_T$ acts on the vortex independently of the direction of the vector $\mathbf{z}$. By this we mean that this force does not lead to movement of the vortex and antivortex in opposite directions and therefore to the appearance of a longitudinal voltage. For this reason we can ignore the thermal force in our further consideration. Another simplification is the neglect of the second term in square brackets in Eq. (6), which results in a transverse voltage only. Under these assumptions one finds the vortex velocity

$$\mathbf{v}_L = \frac{\Phi_0}{\eta} \frac{S_n}{\rho_n} \left[\nabla T \times \mathbf{z} \right]. \tag{7}$$

Using the Josephson relation[14] we obtain the electric field across the sample,

$$\mathbf{E} = \frac{\pi}{e} n_s [\mathbf{v}_L \times \mathbf{z}] = \frac{c n_s}{\eta} \frac{S_n}{\rho_n} \nabla T. \tag{8}$$

The last equation gives the general expression for the Seebeck coefficient due to free vortex motion. Now we have to evaluate the magnitudes of n_s and η above the KT transition temperature for the case of a clean superconductor.

At temperatures just above $T_{\rm KT}$, n_s is proportional to the inverse square of the average distance between thermally induced free vortices ξ_+. The correlation length $\xi_+(T)$ was calculated by Halperin and Nelson[15] and then modified by Minnhagen, taking into account the temperature dependence of n_s.[16] The result is

$$\xi_+ = a\xi(T) \exp \left[B \left(\frac{T_{\rm GL} - T}{T - T_{\rm KT}} \right)^{1/2} \right], \quad n_s = 2\pi C \xi_+^{-2}, \tag{9}$$

where a, B, and C are sample-dependent constants of order unity.

Let us turn now to evaluation of the viscous drag coefficient η. This value was calculated in a number of papers, but mainly for dirty systems. We need now the magnitude of η in the clean limit. Because KT theory provides the kinetic coefficients only with an accuracy of constants of order unity [see Eq. (9)] we can determine η also within this accuracy. In this case we do not need the microscopic calculations which alone allow to obtain the exact values of the numerical coefficients. Therefore we restrict ourselves to simple estimations. As was pointed out in Refs. 17 and 18, there are two main mechanisms of dissipation near the critical temperature. The first one is associated with the heating of normal excitations inside the vortex core. The corresponding value of the coefficient η has the form[17]

$$\eta_1 = \frac{\pi \hbar^2}{4 \rho_n e^2 \xi^2}. \tag{10}$$

It is important that dissipation due to this mechanism is not sensitive to whether the system is in the clean or dirty limit. The second important mechanism is connected with the inhomogeneity of the order parameter in the vortex resulting in additional dissipation, as was proposed by Tinkham.[18] If the characteristic time of vortex motion is $t_0 \sim \xi/v_L$ and the relaxation time of the order parameter $\tau_0 \sim \Delta^{-1}(T)$ during slow movement of the vortex ($\tau_0 \ll t_0$), the energy dissipated per unit volume is $W \approx F\tau_0 \xi^2 n_s / t_0$, where F is the average free energy per unit volume. In this case the viscous drag coefficient can be found as[17]

$$\eta_2 = \frac{W}{v_L^2} \approx \frac{\hbar^2}{e^2 \xi l \rho_n}. \tag{11}$$

Compared with the dirty case the additional factor $\xi/l \ll 1$ appears in η_2. As a result, the second mechanism of dissipation dominates over the first one in the clean limit, giving the main contribution to the thermopower (as well as to the resistivity).

Combining Eqs. (8), (10), and (11), we find the Seebeck coefficient due to the free vortex motion just above the KT transition point in the form

$$\frac{S_s}{S_n} = A \frac{l}{\xi} \exp \left[-B \left(\frac{T_{\rm GL} - T}{T - T_{\rm KT}} \right)^{1/2} \right], \tag{12}$$

where A and B are nonuniversal constants of order unity.

Equation (12) is our final result. First of all it is worth mentioning that the temperature dependence of the thermopower is the same as the temperature dependence of the resistivity in the temperature interval between $T_{\rm KT}$ and $T_{\rm GL}$. As the thermopower becomes zero in the superconducting state, the effect discovered should be observed experimentally as a tail in the temperature dependence of the Seebeck coefficient at the edge of the superconducting transition. Experimentally, the shape of the $S(T)$ curve in the proximity of the transition temperature is still not clearly understood. Nevertheless, pronounced tails have been observed in a number of experimental papers (see, for example, Ref. 19). We emphasize that generally speaking the broadening of the transition in zero magnetic field can be attributed to the effect of sample inhomogeneities. On the other hand, even high-quality samples of high-temperature superconductors demonstrate a broadening of the resistive transition, which was found to be well consistent with KT theory, but the length and shape of the resistive tails were shown to be strongly dependent on the oxygen content of the sample.[6] The calculations above show that similar "intrinsic" tails should be observed in the Seebeck coefficient. More detailed measurements and quantitative analysis of the experimental data are required to establish the intrinsic nature of the broadening of the transition detected by means of thermopower measurements.

Another important point we have to discuss is the applicability of the theory proposed to high-temperature superconductors. The central point here is the dimensionality of the order parameter, which can easily be studied through investigations of its thermodynamic fluctuations above T_{GL}. Results reported in a large number of papers demonstrate that the fluctuation enhancement of conductivity in Bi- and Tl-based oxides is well consistent with the two-dimensional (2D) Aslamazov-Larkin theory and there is no crossover to the three-dimensional regime in the temperature range of Gaussian fluctuations.[20] This fact strongly supports the possibility of interpretation of transport properties in these materials at the edge of the superconducting transition within the context of KT theory. On the other hand, the treatment of data obtained on $YBa_2Cu_3O_{7-x}$, for which the fluctuation conductivity was shown to have a complex behavior with a 2D-3D crossover, requires a comprehensive theory taking into account the effects of interlayer interaction.

It is also interesting to note that application of the KT theory to clean systems leads to an essential increase of the difference between T_{KT} and T_{GL} and therefore to broadening of the transition. The resulting value of the reduced temperature interval $\tau_c = (T_{GL} - T_{KT})/T_{KT}$ turns out to be greater than τ_c for the dirty case by the factor $l/\xi \sim$ 5–8 in real high-temperature materials. This fact allows one to improve the agreement between theory and experiment.[3] Thus taking into account the typical values of $l \sim 80$ Å, $\xi \sim$ 10–15 Å, $\rho_n \sim$ 150–200 $\mu\Omega$ cm, and $d \sim 20$ Å for $Bi_2Sr_2CaCu_2O_x$ and $Tl_2Ba_2CaCu_2O_x$ monocrystal films,[20] we find from Eq. (5) an estimation for τ_c in the range 0.01–0.03. The last prediction is in close agreement with experimentally found values of this parameter.[3–6]

In summary, we have studied the thermoelectric effect due to the motion of free unbound vortices just above the Kosterlitz-Thouless transition temperature. For this purpose the model of a clean two-dimensional superconductor in the proximity of the superconducting transition was considered. We found that the motion of vortices and antivortices produces the thermoelectric effect due to the Magnus force which appears under the influence of a temperature gradient. This effect manifests itself as tails with characteristic exponential inverse-square-root temperature dependence on the Seebeck coefficient at the edge of the superconducting transition.

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