Decrease in critical temperature due to disorder and magnetic correlations in two-dimensional superconductors

I. Grosu, T. Veres, and M. Crisan
Department of Physics, University of Cluj, 3400 Cluj, Romania
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We calculated the critical temperature of a two-dimensional superconductor taking into consideration the effect of disorder on the Coulomb pseudopotential and on the magnetic correlations. The results are in agreement with the experimental data for high-temperature superconductors.

I. INTRODUCTION

The discovery of high-critical-temperature superconductivity (HTS) has brought about many interesting problems to solve. One of the most important effects which makes the HTS's very different from the standard BCS superconductors is the reduction of the critical temperature T_c due to nonmagnetic impurity atoms. It is well known that in an isotropic BCS superconductor, a dilute concentration of nonmagnetic impurities does not change the superconducting properties. On the other hand, it is well known that the normal state of HTS's is very different from the normal state of BCS superconductors, and the doping of HTS's with magnetic or nonmagnetic impurities[1–11] has an important effect on the transport properties, suggesting the occurrence of localization effects. The importance of magnetic correlations has been generally accepted due to the successful explanation of nuclear magnetic resonance experiments in the Millis, Monien, and Pines[12] model. The connection between the magnetic correlations and disorder has been analyzed recently by Moshchalkov[13] in order to explain the quasilinear temperature dependence of resistivity, taking the magnetic correlation length to be of order of the inelastic scattering length. The same experimental data have been explained by Levine[14] by departure from marginal-Fermi-liquid (MFL) behavior.[15] The MFL behavior may cause a pair-breaking effect. At the present time, there is not a clear explanation of the decrease in T_c due to the doping of HTS's with nonmagnetic impurities. In this paper we consider a simple two-dimensional (2D) model for HTS's with disorder and magnetic correlations. Using the Eliashberg equations we can, in the McMillan limit, give an analytical expression for T_c which contains the influence of disorder. In Sec. II we consider a model in which the pairing is given by a bosonic attraction, and only the Coulomb interaction is affected by the disorder. The influence of the magnetic correlations, which can appear in a 2D disordered Fermi liquid, on T_c is treated in Sec. III using the t-matrix approximation. In Sec. IV we calculate T_c as a function of the inverse scattering time $1/\tau$ and the ratio $2\Delta(0)/T_c$ for a 2D disordered HTS with magnetic correlations. The scattering time is taken from the resistivity measurements[1–10] and we obtain a good agreement between the theoretical result and the experimental data using realistic parameters in the calculated expression of T_c. In Sec. V we discuss the results obtained.

II. COULOMB INTERACTION

The Coulomb interaction is very strongly affected by the disorder in the case of 2D superconductors. In order to evaluate the Coulomb pseudopotential, we will follow the method given by Belitz[16] and Entin-Wohlman, Gutfreund, and Weger[17] and used by the present authors to describe the disorder and the magnetic correlation effects in 2D HTS's. In this section we will consider this effect but using a diffusion coefficient which is frequency dependent, obtained by Gor'kov, Larkin, and Khmel'nitskii,[18]

$$D(\omega)=D_0\left[1-\lambda\ln\left|\frac{1}{\omega\tau}\right|\right], \tag{1}$$

where $D_0=v_F^2\tau/2$, $\lambda=1/2\pi E_F\tau$, E_F is the Fermi energy, and τ is the scattering time. The Coulomb pseudopotential is modified by the disorder as

$$\mu^*\simeq\frac{\mu}{1+\mu\int_{\omega_D}^{E_F}(d\omega/\omega)P(\omega)}, \tag{2}$$

ω_D being the bosonic characteristic energy, and

$$P(\omega)=\frac{1}{N(0)}\sum_{\mathbf{q}}P_{\mathbf{q}}(\omega) \tag{3}$$

with

$$P_{\mathbf{q}}(\omega)=-\frac{1}{\pi N(0)\omega}\mathrm{Im}\{\Pi(\mathbf{q},\omega)\}, \tag{4}$$

where $N(0)=m/2\pi$. The polarization function $\Pi(\mathbf{q},\omega)$ has the diffusive form

$$\Pi(\mathbf{q},\omega)=-N(0)\frac{D(\omega)q^2}{D(\omega)q^2-i\omega} \tag{5}$$

and using the cutoff wave vector $2q_c=2/\sqrt{D\tau}$ we obtain from Eqs. (3) and (4)

$$P(\omega)=\frac{1}{2}\lambda\frac{1}{(1+\lambda\ln|\omega\tau|)}\ln\left[1+\left[\frac{4}{\omega\tau}\right]^2\right], \tag{6}$$

which gives for the Coulomb pseudoptential μ^* the expression

$$\mu^* \simeq \mu \left\{ 1 + \frac{\mu}{2} \lambda \int_{\omega_D}^{E_F} \frac{d\omega}{\omega} (1 + \lambda \ln|\omega\tau|)^{-1} \times \ln \left[1 + \left[\frac{4}{\omega\tau} \right]^2 \right] \right\}^{-1} . \quad (7)$$

This generalizes the results obtained in Ref. 17 where the ω dependence of D was neglected. For $\mu=1.5$, $E_F=2$ eV, $\omega_D=5\times10^{-2}$ eV, and $E_F\tau=4$, we obtain $\mu^* \simeq 1$. In the absence of disorder, $\mu_0^* \simeq 0.23$.

III. INFLUENCE OF THE DISORDERED MAGNETIC CORRELATIONS

Magnetic correlations have a destructive effect on standard superconductors and the decrease in T_c due to magnetic correlations is known as the Berk-Schrieffer effect.[19] The effect of the disordered magnetic correlations can be expressed, as was shown in Ref. 20, by the parameter

$$\lambda_m = 2 \int_0^{z_c} \frac{dz}{z} g_m(z) , \quad (8)$$

where $g_m(z)$ is given by the t matrix as

$$g_m(z) = N(0) \int_0^{2q_c} \frac{dq}{2\pi p_F} \left[-\mathrm{Im} \left\{ \frac{1}{\pi} t(q,z) \right\} \right] . \quad (9)$$

Here

$$t(q,z) = -U^2 \chi_{\mathrm{RPA}}(q,z) , \quad (10)$$

where U is the Coulomb interaction and

$$\chi_{\mathrm{RPA}}(q,z) = \frac{\chi_0(q,z)}{1 - U\chi_0(q,z)} , \quad (11)$$

where

$$\chi_0(q,z) = \frac{\bar{N} D q^2}{D q^2 - iz} \quad (12)$$

with $\bar{N} = \mu_B^2 g^2 N(0)$. We approximate

$$\chi_0(z,q) = \begin{cases} \bar{N} , & 0 < z < z_m , \\ \bar{N} \dfrac{Dq^2}{Dq^2 - iz} , & z > z_m . \end{cases} \quad (13)$$

From Eqs. (10)–(13), we get

$$\mathrm{Im}\{t(q,z)\} = \begin{cases} 0 , & 0 < z < z_m , \\ -\dfrac{U^2 \bar{N} D q^2 z}{[(1 - U\bar{N}) D q^2]^2 + z^2} , & z > z_m , \end{cases} \quad (14)$$

where $D = D(z)$ and is given by Eq. (1). We introduced the approximation (13) for the diffusive susceptibility of the disordered Fermi liquid to avoid divergences from Eq. (8). Using (14) we calculate λ_m from (8) as

$$\lambda_m \simeq \frac{\sqrt{1 - U\bar{N}}}{8\pi^2 \sqrt{2}} \left[\frac{U\bar{N}}{1 - U\bar{N}} \right]^2 \frac{1}{E_F \tau} \times \int_{z_m}^{z_c} \frac{dz}{z} f(z) \left[\frac{2z\tau}{1 + \lambda \ln(z\tau)} \right]^{1/2} , \quad (15)$$

where

$$f(z) = \ln \frac{z\tau + 4(1 - U\bar{N}) - \sqrt{8z\tau(1 - U\bar{N})}}{z\tau + 4(1 - U\bar{N}) + \sqrt{8z\tau(1 - U\bar{N})}} + 2 \arctan \left[\left[\frac{8(1 - U\bar{N})}{z\tau} \right]^{1/2} - 1 \right] + 2 \arctan \left[\left[\frac{8(1 - U\bar{N})}{z\tau} \right]^{1/2} + 1 \right] . \quad (16)$$

IV. CRITICAL TEMPERATURE AND THE GEILIKMAN-KRESIN FORMULA

In order to calculate T_c and the expression for $2\Delta(0)/T_c$ we will use the Eliashberg equations written as

$$[1 - Z(\omega)]\omega = \frac{1}{2} \int_0^\infty dz' \int_0^{z_c} dz [\tilde{\alpha}^2(z) \tilde{F}(z) + g_m(z)] \left[\left[\tanh \frac{z'}{2T} + \coth \frac{z}{2T} \right] \left[\frac{1}{z' + z + \omega} - \frac{1}{z' + z - \omega} \right] - \left[\tanh \frac{z'}{2T} - \coth \frac{z}{2T} \right] \left[\frac{1}{z' - z + \omega} - \frac{1}{z' - z - \omega} \right] \right] \times \mathrm{Re} \left\{ \frac{z'}{\sqrt{z'^2 - \Delta^2(z')}} \right\} , \quad (17)$$

$$Z(\omega)\Delta(\omega) = \frac{1}{2} \int_0^\infty dz' \int_0^{z_c} dz [\tilde{\alpha}^2(z) \tilde{F}(z) - g_m(z)] \left[\left[\tanh \frac{z'}{2T} + \coth \frac{z}{2T} \right] \left[\frac{1}{z' + z + \omega} + \frac{1}{z' + z - \omega} \right] - \left[\tanh \frac{z'}{2T} - \coth \frac{z}{2T} \right] \left[\frac{1}{z' - z + \omega} + \frac{1}{z' - z - \omega} \right] \right] \times \mathrm{Re} \left\{ \frac{\Delta(z')}{\sqrt{z'^2 - \Delta^2(z')}} \right\} - \mu^* \int_0^{\omega_c} dz' \tanh \frac{z'}{2T} \mathrm{Re} \left\{ \frac{\Delta(z')}{\sqrt{z'^2 - \Delta^2(z')}} \right\} , \quad (18)$$

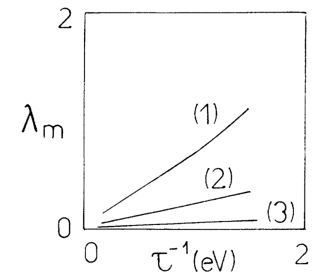

FIG. 1. Dependence of the parameter λ_m on τ^{-1} for curve (1), $U\bar{N}=0.9$, curve (2), $U\bar{N}=0.8$, and curve (3), $U\bar{N}=0.6$, with $z_m=0.0005$ eV, $z_c=0.2$ eV, and $E_F=2$ eV.

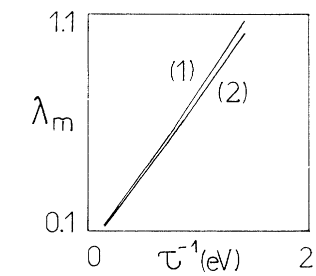

FIG. 2. Dependence of the parameter λ_m on τ^{-1} for $U\bar{N}=0.9$ and $z_c=0.2$ eV. Curve (1), $z_m=0.0005$ eV, curve (2), $z_m=0.001$ eV.

where

$$\tilde{\alpha}^2(z)\tilde{F}(z)\simeq\alpha^2(z)F(z)+\frac{\mu z}{\pi E_F} . \tag{19}$$

The critical temperature T_c can be obtained from the linearization of Eqs. (17) and (18) and we get, if the magnetic correlations are neglected, the equation for T_c:

$$T_c\simeq 1.134\omega_D \times\exp\left[-\frac{1+\tilde{\lambda}}{\tilde{\lambda}-[\mu^*/(1+\mu^*\ln\omega_c/\omega_D)](I_1+1)}\right] , \tag{20}$$

where

$$I_1=2\int_{\omega_D}^{\infty}\frac{dz'}{z'}\int_0^{z_c}\frac{dz}{z'+z}\left[\alpha^2(z)F(z)+\frac{\mu P_1 z}{\pi}\right] , \tag{21}$$

$\tilde{\lambda}=\lambda_{ph}+2\mu z_c/\pi E_F$, $P_1\simeq 1/E_F$, and μ^* is given by Eq. (7). Using $\omega_D=5\times 10^{-2}$ eV, $\lambda_{ph}=3.5$, $\mu=1.5$, $E_F=2$ eV, and $z_c\simeq 4\omega_D$ we obtain an increasing of T_c with the disorder. For $E_F\tau=4$ we get $T_c\simeq 73$ K. (When disorder is absent the critical temperature, using the same parameters, has a larger value.) If we consider now the magnetic correlations in Eqs. (17) and (18), we obtain

$$T_c\simeq 1.134\omega_D\exp\left[-\frac{1+\lambda_m+\tilde{\lambda}}{\tilde{\lambda}-\lambda_m-[\mu^*/(1+\mu^*\ln\omega_c/\omega_D)](I_1+1)}\right] . \tag{22}$$

In Figs. 1 and 2 we calculate λ_m as function of the inverse scattering time $1/\tau$ for different values of $\bar{N}U$ and z_m.

The critical temperature T_c as a function of $1/\tau$ for different values of the Coulomb interaction is given in Fig. 3. We see that T_c decreases with $1/\tau$ and in Fig. 4 we compare our theoretical prediction with the experimental results.[3] The values of $1/\tau$ has been taken from the resistivity measurements (Ref. 3) and from Fig. 4 we see that there is good agreement between the proposed mechanism of decrease of T_c and experimental data.

In order to calculate the ratio $2\Delta(0)/T_c$ we use the McMillan approximation in Eqs. (17) and (18); and $\Delta(0)$ is obtained, following the standard method, as

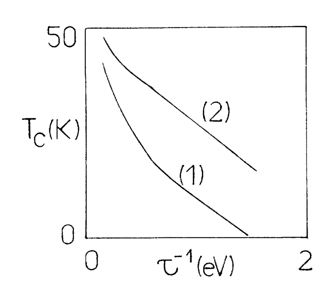

FIG. 3. T_c dependence on τ^{-1} given by Eq. (22) for $z_m=0.0005$ eV, $z_c=0.2$ eV and $\lambda_{ph}=3.5$. Curve (1), $U\bar{N}=0.93$, curve (2), $U\bar{N}=0.9$.

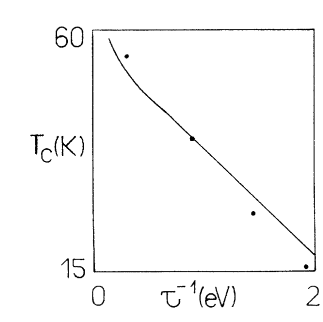

FIG. 4. T_c dependence on τ^{-1} given by Eq. (22) with realistic parameters and $z_m=0.0005$ eV, $z_c=0.2$ eV, $U\bar{N}=0.9$, and $\lambda_{ph}=4$; the experimental data are taken from Ref. 3.

$$\Delta(0) \simeq 2\omega_D \exp\left[-\frac{1+\lambda_m+\tilde{\lambda}}{\tilde{\lambda}-\lambda_m-[\mu^*/(1+\mu^*\ln\omega_c/\omega_D)](I_2+1)}\right], \tag{23}$$

where

$$I_2 = 2\int_0^{z_c} \frac{dz}{z}\left[\alpha^2(z)F(z)+\frac{\mu P_1 z}{\pi}-g_m(z)\right] \times \ln\left[1+\frac{z}{\omega_D}\right]. \tag{24}$$

From Eqs. (23) and (22) we calculate

$$\frac{2\Delta(0)}{T_c} \simeq 3.53 \exp\left\{\frac{AC(I_1-I_2)}{[B-C(I_2+1)][B-C(I_1+1)]}\right\}, \tag{25}$$

where

$$A = 1+\tilde{\lambda}+\lambda_m, \quad B = \tilde{\lambda}-\lambda_m, \quad C = \mu^* \Big/ \left[1+\mu^*\ln\frac{\omega_c}{\omega_D}\right]. \tag{26}$$

For $U\bar{N}=0.9$, $\lambda_{\rm ph}=4$, and $\tau=0.7$ eV^{-1}, we obtain $2\Delta(0)/T_c \simeq 7$. The deviations of the Geilikman-Kresin[21] ratio $2\Delta(0)/T_c$ for HTS's which contain a strong retardation effect due to disorder are large, and can be caused by different mechanisms such as the 2D character of the electronic excitations.[22,23]

V. DISCUSSIONS

We showed that the disorder introduced by nonmagnetic impurities give rise to drastic effects on the critical temperature T_c of a 2D HTS. Taking a frequency-dependent diffusion coefficient, we calculated the Coulomb pseudopotential and the effect of the magnetic correlations on the critical temperature T_c. If the magnetic correlations are neglected T_c can increase with disorder. This can be explained by the dependence of the Coulomb pseudopotential of the disorder, but for the HTS's it is well known that nonmagnetic impurities give a strong depression in T_c. We showed that it can be explained by the contribution of disorder in the 2D magnetic correlation factor λ_m, which maintains its singularity of the form $[\bar{N}U/(1-\bar{N}U)]^2$ but also contains a factor which depends on $1/\tau$, and gives a decrease in T_c. However, we have to mention that the magnetic correlations in a 2D disordered Fermi system are a very difficult problem, because at low temperatures the susceptibility varies as $\chi(T)\sim T^{-\gamma}$ ($\gamma=\frac{4}{3}$) but in the high-temperature domain the susceptibility is constant. The Moriya[24] self-consistent spin-fluctuation theory has been applied recently to HTS's but the effect of disorder was not considered. The agreement with the experimental data can also be explained if we use the predictions made by Sachdev[25] and Sing[26] concerning the possible occurrence of magnetic moments in the disordered Fermi system. A qualitative explanation of the decrease of the critical temperature has been given by one of us,[23] but neglecting the frequency dependence in the diffusion coefficient and taking into consideration the renormalization in the diffusive dynamic susceptibility due to the scattering on the magnetic moments which may appear in the disordered interacting Fermi liquid. The problem is open but it is one of the keys for the selection of the appropriate pairing mechanism for HTS's.

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