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Nonadiabatic analysis of the Josephson critical current influenced by quantum phase fluctuations

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The influence of the electromagnetic environment on the dc Josephson current is studied allowing for nonadiabatic motion of the phase difference across a junction. The critical current is evaluated nonperturbatively within mean-field theory, fully taking into account the nonlocal kernels in the Ambegaokar-Eckern-Schön effective action. It turns out that the adiabatic approximation, which can be justified when the superconducting energy gap exceeds the charging energy, yields *qualitatively* correct results even for the opposite case, although *quantitative* deviations from the adiabatic approximation are found to be substantial in some experimentally accessible regions.

I. INTRODUCTION

The dynamics of a submicrometer tunnel junction with capacitance C of the order of 10^{-15} F or less can be described in terms of the charge on the capacitor and the phase defined as the time integral of the voltage across it. These variables are canonically conjugate,[1] and their quantum fluctuations cannot be suppressed simultaneously. Which variable undergoes smaller quantum fluctuations depends on the impedance of the electromagnetic (EM) environment surrounding the junction. For a low-impedance environment, quantum fluctuations of the phase are suppressed at the expense of enhanced charge fluctuations, and vice versa for a high-impedance environment.[2–4] The crossover impedance is set by the resistance quantum $R_Q \equiv h/4e^2$ constructed from Planck's constant h and the elementary charge e.

For normal junctions, quantum charge fluctuations obscure the Coulomb blockade of tunneling — the suppression of the low-bias tunneling conductance due to the elementary charging energy.[5] For superconducting junctions, quantum fluctuations bring about an even more drastic effect: If the dc impedance of the EM environment exceeds the critical value R_Q, quantum phase fluctuations completely suppress the supercurrent through the junction. The quenching of the supercurrent at this critical value is called the dissipative phase transition[6] (also referred to as the Schmid transition), which has been studied theoretically for a quantum particle moving in a cosine potential and coupled to a linear dissipative environment (for a review, see Ref. 7). In fact, however, this model can describe a Josephson junction only when the phase changes little on the time scale $\hbar/\Delta$ set by the superconducting energy gap Δ. The junction capacitance C plays the role of a mass for the phase degree of freedom, and a large capacitance hinders rapid phase motion. Therefore, only for junctions with capacitance large enough to make the charging energy $E_C \equiv e^2/2C$ much smaller than Δ (i.e., in the *adiabatic limit*) are results derived from the cosine potential expected to be valid.

By way of illustration, let us consider aluminum which is often used for submicrometer junctions[8–10] owing to its favorable processing properties. It has an energy gap of about 2 K which equals the charging energy corresponding to capacitance about 0.5×10^{-15} F. With the rapid advance in microfabrication technique, it will soon be possible to further miniaturize junctions, reducing their capacitance to one-tenth of this value or less. Then the restriction of the theory to the adiabatic limit will clearly not be justified.

It is therefore interesting to study how quantum phase fluctuations alter the dc Josephson current without recourse to the adiabatic approximation. For resistive Cooper-pair and quasiparticle current[11] at nonzero voltage, Zaikin has recently examined nonadiabatic effects perturbatively.[12] The dc supercurrent that we address in this paper, however, cannot be studied perturbatively because any finite-order expansion in the tunneling amplitude leads to a complete suppression of the supercurrent if the dc impedance of the EM environment is nonzero. We shall therefore work with a variational method, the self-consistent harmonic approximation (SCHA), which in the adiabatic limit was used by Fisher and Zwerger.[13]

This paper is organized as follows. Section II provides a brief summary of the path-integral description of a tunnel junction to establish notation and to make this paper self-contained. Section III uses a variational method to obtain a set of integral equations for a nonadiabatic expression of the Josephson current. Section IV numerically solves these equations and discusses the consequences. Section V summarizes the main results of this paper.

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II. PATH-INTEGRAL FORMULATION OF THE PROBLEM

A. Effective action

While the operator description of a tunnel junction is convenient when tunneling can be treated perturbatively, the path-integral formulation has advantages if nonperturbative effects need to be taken into account. In this formulation the key quantity is the effective action for the degrees of freedom of our concern.

The system we study consists of a Josephson tunnel junction, its electromagnetic environment, and the external driving source. Correspondingly, the effective action has three parts S_J, S_0, and S_x, respectively. The relevant variable in the problem is the phase φ whose time derivative is related to the voltage V across the junction by

$$\frac{d\varphi}{dt} = 2eV. \qquad (2.1)$$

Here and henceforth, we shall use a system of units in which $\hbar$ and the Boltzmann constant k_B are equal to unity. Electronic degrees of freedom involved in tunneling and in the electrodynamics of the external circuit can be traced out, yielding an effective action which depends solely on φ. The part of the action describing Cooper-pair and quasiparticle tunneling is given in imaginary time by[14,7]

$$S_J[\varphi] = \int_0^{1/T} d\tau_1 \int_0^{1/T} d\tau_2 \Big(\beta(\tau_1-\tau_2)\cos\frac{\varphi(\tau_1)+\varphi(\tau_2)}{2} - \alpha(\tau_1-\tau_2)\cos\frac{\varphi(\tau_1)-\varphi(\tau_2)}{2}\Big), \qquad (2.2)$$

where the integrals extend from 0 to the reciprocal temperature $1/T$. This action is obtained by means of a cumulant expansion of the partition function to second order in the tunneling amplitude. The partition function calculated from S_J therefore includes contributions of arbitrarily high order in the tunneling amplitudes. The kernels α and β reflect properties of the normal and anomalous electron Green functions of the junction electrodes. The nonlocal nature in time of these kernels arises from the elimination of the electronic degrees of freedom that are coupled to the phase φ. According to BCS theory, the kernels can be expressed at zero temperature[14] in terms of the modified Bessel functions K_0 and K_1 (Ref. 15) as

$$\alpha(\tau) = \frac{\Delta^2}{2\pi e^2 R_T}\Big[K_1(\Delta|\tau|)\Big]^2,$$
$$\beta(\tau) = -\frac{\Delta^2}{2\pi e^2 R_T}\Big[K_0(\Delta|\tau|)\Big]^2, \qquad (2.3)$$

where the junction electrodes are assumed to have the same superconducting energy gap Δ, and R_T is the tunnel resistance of the junction in the normal state.

To the action (2.2) must be added a contribution S_0 describing the quantum dynamics of the phase in the absence of tunneling. This dynamics is governed by the coupling of the phase to the EM environment of the junction, and the strength of the coupling can be characterized by the frequency-dependent impedance $Z_0(\omega)$ of the entire system. Since we are interested in time and length scales that are large compared to the atomic ones, S_0 may be assumed to be quadratic in φ (cf. also Appendix A of Ref. 16),

$$S_0[\varphi] = -\frac{1}{2}\int_0^{1/T} d\tau_1 \int_0^{1/T} d\tau_2\, \varphi(\tau_1) G_0^{-1}(\tau_1-\tau_2)\varphi(\tau_2), \qquad (2.4)$$

where $G_0(\tau) = -\langle \mathcal{T}\{\varphi(\tau)\varphi(0)\}\rangle_0$ is the time-ordered (indicated by $\mathcal{T}$) Green function of the phase in imaginary time, evaluated with the action S_0. Taking a quadratic form for S_0 implies linear response of the EM environment in the relevant frequency region — a condition well satisfied if the leads attached to the junction are metallic. The fluctuation-dissipation theorem then lets us express the Fourier transform of the retarded Green function of the phase (in the absence of tunneling) in terms $Z_0(\omega)$. Analytic continuation of this relation to the upper half of the complex ω plane and use of the symmetry of G_0 lead to[17]

$$G_0(\omega_\nu) = -\frac{2\pi}{R_Q}\frac{Z_0(i|\omega_\nu|)}{|\omega_\nu|}, \qquad (2.5)$$

where $\omega_\nu = 2\pi T\nu$ ($\nu = 0, \pm1, \pm2, \ldots$) are the Matsubara frequencies for bosons. For a simple circuit with an Ohmic resistance R shunting the junction with capacitance C, one has $Z_0(\omega) = R/(1 - i\omega RC)$.

We also need to include the action

$$S_x[\varphi] = -\frac{1}{2e}\int_0^{1/T} I_x \varphi(\tau) d\tau, \qquad (2.6)$$

which describes an external source generating the current I_x.[7] The total action $S = S_J + S_0 + S_x$ describes a current-biased, shunted junction.

We are interested in the expectation value of the supercurrent through the junction, which in the path-integral formalism is given by[7]

$$\langle I_J\rangle = \frac{\int \mathcal{D}\varphi\, I_J[\varphi]\exp(-S[\varphi])}{\int \mathcal{D}\varphi\, \exp(-S[\varphi])}, \qquad (2.7)$$

with

$$I_J(\tau) = -2e\int_0^{1/T} d\tau' \beta(\tau-\tau')\sin\frac{\varphi(\tau)+\varphi(\tau')}{2}. \qquad (2.8)$$

All path integrals in this paper are understood to have the "thermodynamic boundary conditions," i.e., the integration extends over all paths with restriction $\varphi(0) = \varphi(1/T)$.

B. Adiabatic approximation

The junction capacitance C contributes to the effective action the "inertial" term

$$\int_0^{1/T} d\tau \, \frac{C}{2}\left(\frac{1}{2e}\frac{d\varphi}{d\tau}\right)^2, \tag{2.9}$$

which makes irrelevant those phase paths $\varphi(\tau)$ which vary rapidly on the time scale $1/E_C$. If the condition $\Delta \gg E_C$ holds, the remaining (relevant) paths change little on the scale $1/\Delta$. Since for $T = 0$ the kernels $\alpha(\tau)$ and $\beta(\tau)$ are sharply peaked for small values of $|\Delta\tau|$, one may approximate the first contribution to S_J by

$$\begin{aligned}\int d\tau_1 \int d\tau_2 \, \beta(\tau_1 - \tau_2) \cos\left(\varphi(\tau_1) - \frac{\varphi(\tau_1) - \varphi(\tau_2)}{2}\right) \\ \approx \int d\tau \int d\tau' \, \beta(\tau') \left[\cos\varphi(\tau) + \frac{\tau'}{2}\frac{d\varphi(\tau)}{d\tau}\sin\varphi(\tau) - \frac{\tau'^2}{8}\left(\frac{d\varphi(\tau)}{d\tau}\right)^2 \cos\varphi(\tau)\right] \\ = -E_J \int d\tau \cos\varphi(\tau) + \int d\tau \frac{1}{2}\delta C_\beta(\varphi(\tau)) \left(\frac{1}{2e}\frac{d\varphi(\tau)}{d\tau}\right)^2,\end{aligned} \tag{2.10}$$

with $E_J \equiv -\int d\tau \beta(\tau)$ and $\delta C_\beta(\varphi) \equiv -e^2 \cos\varphi \int d\tau \; \beta(\tau)\tau^2$. For the second contribution one finds that

$$\begin{aligned}-\int d\tau_1 \int d\tau_2 \, \alpha(\tau_1 - \tau_2) \cos\frac{\varphi(\tau_1) - \varphi(\tau_2)}{2} \\ \approx \int d\tau \, \frac{1}{2}\left(\int d\tau' \, \alpha(\tau')\tau'^2\right)\left(\frac{1}{2}\frac{d\varphi(\tau)}{d\tau}\right)^2 + \text{const} \\ = \int d\tau \, \frac{1}{2}\delta C_\alpha \left(\frac{1}{2e}\frac{d\varphi(\tau)}{d\tau}\right)^2 + \text{const},\end{aligned} \tag{2.11}$$

where $\delta C_\alpha \equiv e^2 \int d\tau \alpha(\tau)\tau^2$. This is the so-called *adiabatic approximation*, in which the nonlocal time dependence of the effective action is replaced by the local one. The α term simplifies to a contribution proportional to the squared phase velocity, which effectively increases the capacitance by $\delta C_\alpha = 0.75 C E_J E_C/\Delta^2$.[14] This capacitance renormalization plays an important role in an analysis of the superconducting-insulating phase transition in networks of Josephson junctions.[18] The β term reduces, to the lowest order in $d\varphi/d\tau$, to the Josephson potential $-E_J \cos\varphi(\tau)$, where the Josephson energy E_J is expressed in terms of Δ and R_T by the Ambegaokar-Baratoff formula $E_J = \pi\Delta/(4R_T e^2)$.[19] Expanding systematically to second order in $d\varphi/d\tau$, one also finds a contribution (ignored in Ref. 18) $\delta C_\beta(\varphi) = \cos\varphi \; \delta C_\alpha/3$ to the capacitance renormalization from the β term.[20] Because of its phase dependence — which corresponds to a position-dependent mass in a mechanical analog — the physical interpretation of $\delta C_\beta(\varphi)$ is less obvious than that of δC_α. Fortunately, this does not cause any problem if one considers the situation in which the phase makes only small excursions from its average value (see below).

The adiabatic approximation becomes exact in the *adiabatic limit* $\Delta/E_C \to \infty$. In what follows we shall explore situations in which the adiabatic approximation is *not* always justified.

III. VARIATIONAL PRINCIPLE AND SELF-CONSISTENT HARMONIC APPROXIMATION

We start from the variational principle for the free energy,[21]

$$F \le F^* = F_{\text{tr}} + T\langle S - S_{\text{tr}}\rangle_{\text{tr}}, \tag{3.1}$$

where F and F_{tr} are the free energies that are evaluated for the true action S and for an arbitrary trial action S_{tr}, respectively. The symbol $\langle A\rangle_{\text{tr}}$ indicates that the average of a quantity A is to be taken with the trial action, i.e.,

$$\langle A\rangle_{\text{tr}} = \frac{\int \mathcal{D}\varphi A[\varphi] \exp(-S_{\text{tr}}[\varphi])}{\int \mathcal{D}\varphi \; \exp(-S_{\text{tr}}[\varphi])}. \tag{3.2}$$

We take a Gaussian trial action S_{tr} which well approximates the true action S if the essentially contributing paths $\varphi(\tau)$ undergo only small fluctuations around some average path φ_0. Since we are considering a dc supercurrent, φ_0 must be independent of τ. An appropriate trial action can be obtained by expanding S_J to second order in $\varphi_1 \equiv \varphi - \varphi_0$ and replacing the kernels α and β by $\tilde\alpha$ and $\tilde\beta$ that are to be determined so as to minimize F^*:

$$\begin{aligned}S_J[\varphi] \to \int_0^{1/T} d\tau_1 \int_0^{1/T} d\tau_2 \Bigg\{ \tilde\beta(\tau_1 - \tau_2)\Big[-\frac{\varphi_1(\tau_1) + \varphi_1(\tau_2)}{2}\sin\varphi_0 \\ -\frac{1}{2}\Big(\frac{\varphi_1(\tau_1) + \varphi_1(\tau_2)}{2}\Big)^2 \cos\varphi_0\Big] + \tilde\alpha(\tau_1 - \tau_2)\frac{1}{2}\Big(\frac{\varphi_1(\tau_1) - \varphi_1(\tau_2)}{2}\Big)^2\Bigg\},\end{aligned} \tag{3.3}$$

where terms independent of φ_1 have been omitted. Since the average $\langle\varphi_1\rangle_{\text{tr}}$ is zero by definition, the total trial action including S_x should not contain terms linear in φ_1; thus from Eqs. (6) and (14) we find that

$$I_x = \left[-2e\int_0^{1/T}\tilde{\beta}(\tau)d\tau\right]\sin\varphi_0 \equiv I_c^{\text{eff}}\sin\varphi_0. \tag{3.4}$$

This relation is different from the usual Josephson relation in that the current amplitude is renormalized compared with the classical value $I_c \equiv 2eE_J$, and that the effective "critical" current I_c^{eff} depends on φ_0; that is, the functional form of the Josephson relation is modified. This was pointed out for the adiabatic limit in Ref. 22.

The trial action is thus given by $S_{\text{tr}} = \tilde{S} + S_0$, where

$$\tilde{S}[\varphi] = \frac{1}{2}\int_0^{1/T}d\tau_1\int_0^{1/T}d\tau_2\left[-\tilde{\beta}(\tau_1-\tau_2)\Big(\frac{\varphi_1(\tau_1)+\varphi_1(\tau_2)}{2}\Big)^2\cos\varphi_0 + \tilde{\alpha}(\tau_1-\tau_2)\Big(\frac{\varphi_1(\tau_1)-\varphi_1(\tau_2)}{2}\Big)^2\right]$$
$$\equiv \frac{1}{2}\int_0^{1/T}d\tau_1\int_0^{1/T}d\tau_2\;\varphi_1(\tau_1)K(\tau_1-\tau_2)\varphi_1(\tau_2), \tag{3.5}$$

where the last equation defines the kernel $K(\tau)$ $[= K(-\tau) = K(1/T-\tau)]$. With this trial action the averages in Eq. (3.1) can be expressed in terms of the Green function $G(\tau) \equiv -\langle\mathcal{T}\varphi_1(\tau)\varphi_1(0)\rangle_{\text{tr}}$ as follows:

$$\langle S_J\rangle_{\text{tr}} = \frac{1}{T}\int_0^{1/T}d\tau[\beta(\tau)\cos\varphi_0\, e^{[G(\tau)+G(0)]/4} - \alpha(\tau)e^{[-G(\tau)+G(0)]/4}], \tag{3.6}$$

$$\langle\tilde{S}\rangle_{\text{tr}} = -\frac{1}{2T}\int_0^{1/T}d\tau K(\tau)G(\tau). \tag{3.7}$$

The kernel K of the trial action is determined by the requirement that the variation of F^* be zero:

$$0 = \delta F^* = \delta F_{\text{tr}} + T\delta\langle S_J + S_x - \tilde{S}\rangle_{\text{tr}} = T\Big(\langle\delta\tilde{S}\rangle_{\text{tr}} + \delta\langle S_J\rangle_{\text{tr}} - \delta\langle\tilde{S}\rangle_{\text{tr}}\Big)$$
$$= \int_0^{1/T}d\tau\left[\beta(\tau)\cos\varphi_0\,\delta e^{[G(\tau)+G(0)]/4} - \alpha(\tau)\delta e^{[-G(\tau)+G(0)]/4} + \frac{1}{2}K(\tau)\delta G(\tau)\right]$$
$$= T\sum_\nu\int_0^{1/T}d\tau[\beta(\tau)\cos\varphi_0\, e^{[G(\tau)+G(0)]/4}(e^{-i\omega_\nu\tau}+1) - \alpha(\tau)e^{[-G(\tau)+G(0)]/4}(-e^{-i\omega_\nu\tau}+1) + 2K(\tau)e^{-i\omega_\nu\tau}]\frac{\delta G(\omega_\nu)}{4}. \tag{3.8}$$

We thus obtain

$$K(\omega_\nu) = \frac{1}{2}\int_0^{1/T}d\tau\;[-\beta(\tau)\cos\varphi_0\, e^{[G(\tau)+G(0)]/4}(1+e^{-i\omega_\nu\tau}) + \alpha(\tau)e^{[-G(\tau)+G(0)]/4}(1-e^{-i\omega_\nu\tau})]. \tag{3.9}$$

This equation is closed by the expression of G in terms of K,

$$G(\omega_\nu) = \left[G_0^{-1}(\omega_\nu) - K(\omega_\nu)\right]^{-1}, \tag{3.10}$$

where $G_0(\omega_\nu)$ is given in Eq. (5). We observe that the nonadiabatic effect emerges as a frequency dependence of the self-energy for the Green function of the phase. The supercurrent amplitude defined in Eq. (3.4) is obtained from Eq. (3.5) as

$$I_c^{\text{eff}} = \frac{2eK(\omega=0)}{\cos\varphi_0}. \tag{3.11}$$

This formula can also be obtained by directly taking the average of the Josephson current (2.8) with respect to the trial action S_{tr}, and using the relation (3.9).

We have solved the coupled nonlinear equations (3.9) and (3.10) iteratively on a computer, in the limit of vanishing temperature. Usually, convergence to within an accuracy of 1 in 1000 was achieved in four iterations. We now discuss the obtained results.

IV. RESULTS

A. Adiabatic limit

To provide a reference point for our results, we first discuss the supercurrent calculated from the SCHA in the adiabatic limit $\Delta/E_C \to \infty$ at zero temperature. We consider a circuit in which a Josephson junction having capacitance C is shunted by an Ohmic resistor with resistance R. The total impedance is given by $Z_0(\omega) = R/(1-i\omega RC)$. In the adiabatic limit, the coupled integral equations for G and K reduce to algebraic equations, from which we obtain

$$\frac{I_c^{\text{eff}}}{I_c} = \Theta(w)\exp\left(-\frac{1}{\alpha_s w^{1/2}}\ln\frac{1+w^{1/2}}{1-w^{1/2}}\right)$$
$$+\Theta(-w)\exp\left(-\frac{2}{\alpha_s|w|^{1/2}}\text{arccot}|w|^{-1/2}\right), \tag{4.1a}$$

$$w = 1 - \left(\frac{2\pi^2}{\alpha_s^2}\frac{E_J}{E_C}\cos\varphi_0\right)\frac{I_c^{\text{eff}}}{I_c}, \tag{4.1b}$$

where $I_c \equiv 2eE_J$ is the classical value of the critical current. The solution of these equations for I_c^{eff}/I_c is plotted in Fig. 1. For large values of E_J/E_C and/or $\alpha_s \equiv R_Q/R$, the effective supercurrent is suppressed only slightly below its classical value. If both E_J/E_C and α_s are small, on the other hand, the supercurrent vanishes. For an intermediate region, there is a boundary between the superconducting and insulating phases. This is the dissipative phase transition.[6]

Solving Eqs. (4.1) for $I_c^{\text{eff}}/I_c \to 0$ (and $\varphi_0 = 0$) yields

$$\frac{I_c^{\text{eff}}}{I_c} = \left(\frac{\pi^2}{2\alpha_s^2}\frac{E_J}{E_C}\right)^{\frac{1}{\alpha_s-1}} . \tag{4.2}$$

The SCHA thus predicts that as long as $E_J/E_C < 2/\pi^2$, the phase boundary between the regions with $I_c^{\text{eff}} = 0$ and $I_c^{\text{eff}} > 0$ lies at $\alpha_s = 1$, independent of E_J/E_C. For larger values of E_J/E_C, however, the SCHA predicts a first-order phase transition along a phase boundary bending from $\alpha_s = 1$ to smaller values of α_s (see Fig. 1).

In the limit $E_J/E_C \to 0$, a perturbative renormalization group (RG) analysis[13] produces the same phase boundary as the SCHA. The opposite case $E_J/E_C \to \infty$ can also be treated by a perturbative RG analysis,[13,23] with the aid of the duality transformation introduced by Schmid.[6] The phase boundary, however, is found again at $\alpha_s = 1$ in disagreement with the result of the SCHA when applied to the *original* action. Although it is not unusual for mean-field theory such as the SCHA to overestimate the (phase) ordered region, *prima facie*, it takes us by surprise that the SCHA fails where it is expected to work best, for large E_J/E_C, where the potential energy for the phase has deep minima, so that phase fluctuations should be small and the quadratic approximation to the effective action should be appropriate. In fact, the disagreement with the RG result stems from the neglect of very infrequent large fluctuations that cause the phase to hop from one local minimum of the Josephson potential to a neighboring one. In thermal equilibrium, this hopping leads to complete delocalization of the phase, thereby destroying the supercurrent. In some experimentally accessible regions, however, large fluctuations occur so rarely that the SCHA result makes more sense than the thermodynamically correct RG result. For instance, for $E_J = 1$ K, $\alpha_s = 0.5$, and $I_x = 0$, the lifetime of a state with a localized phase was estimated[7] to be 30 ns for $E_J/E_C = 3$, 1 min for $E_J/E_C = 30$, and 10^{26} years for $E_J/E_C = 300$. We also remark that, if one uses the SCHA after the duality mapping, then the resulting phase boundary for large E_J/E_C —of course— agrees with the RG result.

It is worth noticing that even for $\alpha_s > 1$ supercurrent can only be metastable, despite the fact that I_c^{eff} does not vanish for $\varphi_0 = 0$. The reason is simple: Since a nonzero supercurrent requires $\varphi_0 \neq 0$ and $I_x \neq 0$, macroscopic quantum tunneling of the phase has a nonvanishing probability, and will eventually give rise to an — albeit small — dc voltage. For $\alpha_s \gg E_J/E_C$, one can use a result obtained by Korshunov[24] to estimate the lifetime of the supercurrent. For example, with $E_J = 1$ K, $I_x = 0.1I_c$, and $E_J/E_C = 1$, one finds a lifetime of only 0.2 ms for $\alpha_s = 5$, but already 2 days for $\alpha_s = 10$, and 10^7 years for $\alpha_s = 15$. In view of these facts, the boundary of the dissipative phase transition is not very significant as far as supercurrent is concerned. We also note that the lifetime of supercurrent depends very sensitively on E_J/E_C and α_s, and that it is long enough to be observed in a parameter region where there is still a sizable influence (about 10%) of quantum phase fluctuations.

B. Nonadiabatic case

We now turn to the new results we have obtained for the case in which the energy gap Δ is *not* large compared to E_C. Figure 2 shows the nonadiabatic SCHA result for I_c^{eff}/I_c as a function of Δ/E_C, together with the SCHA result in the adiabatic approximation (obtained from Eqs. (4.1) with E_C replaced by $e^2/2[C + \delta C(\varphi_0)]$). For large Δ/E_C the adiabatic limit is recovered. With decreasing ratio Δ/E_C the effective critical current increases, which implies that quantum fluctuations of the phase are suppressed. This can be understood in the adiabatic approximation, where Δ/E_C only enters via the capacitance renormalization by $\delta C = 0.75C(E_J/E_C)(E_C/\Delta)^2[1 + \cos\varphi_0/3]$. The in-

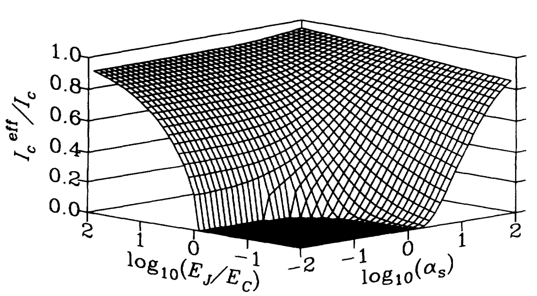

FIG. 1. I_c^{eff} calculated in the adiabatic limit $\Delta/E_C \to \infty$ for $\varphi_0 = 0$, normalized by the classical value $I_c = 2eE_J$.

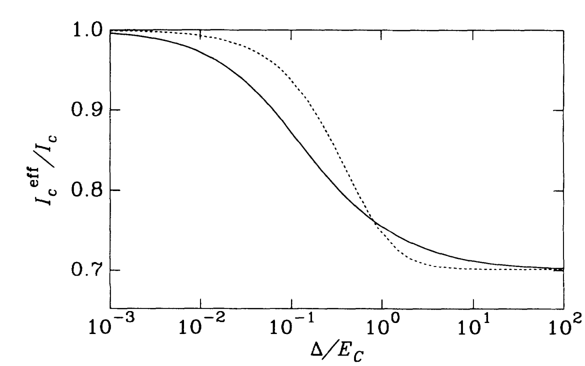

FIG. 2. Dependence of I_c^{eff}/I_c on the ratio of energy gap Δ to charging energy E_C, for $E_J/E_C = 1$, $\alpha_s = 10$, and $\varphi_0 = 0$. The solid curve is the nonadiabatic SCHA result, and the dashed curve the adiabatic approximation.

crease in C by δC effectively shifts up the ratio of the Josephson energy to charging energy, and thus leads to a larger value of I_c^{eff}/I_c (see Fig. 1). Although the adiabatic approximation could not, *a priori*, be expected to hold unless $\Delta/E_C \gg 1$, comparison with the nonadiabatic result in Fig. 2 reveals that the adiabatic approximation gives *qualitatively* correct results even where this condition is not met. This suggests that the quenching of quantum phase fluctuations for small Δ/E_C may be ascribed mainly to capacitance renormalization.

Quantitatively, however, the adiabatic approximation is not correct unless $\Delta \gg E_C$, as is obvious from Fig. 2. In Fig. 3 we show the relative deviation of the effective critical current $I_{c,\text{a.a.}}^{\text{eff}}$ in the adiabatic approximation from the nonadiabatic result I_c^{eff} over a wide range of E_J/E_C and α_s, for $\Delta/E_C = 1/10$ and $\phi_0 = 0$. This deviation is quite large where I_c^{eff} is small, that is, near the (mean-field) boundary of the dissipative phase transition.

For large E_J/E_C and/or large α_s, where the suppression of the supercurrent by quantum phase fluctuations is small, the interesting quantity is the correction $\delta I_c \equiv I_c^{\text{eff}} - I_c$ to the classical value I_c rather than I_c^{eff} itself. The relative deviation of δI_c in the adiabatic approximation from the nonadiabatic result for δI_c is displayed in Fig. 4, for the same ratio $\Delta/E_C = 1/10$. Again we see that the deviation can be quite large, about 50%.

The dependence of I_c^{eff} on the ratio E_J/E_C and the shunt conductance α_s is exemplified in Figs. 5 and 6. We see that the difference between adiabatic and nonadiabatic results, as well as the suppression of supercurrent itself, vanishes more rapidly with increasing E_J/E_C than with increasing α_s. In view of the lifetime estimates in the previous section, parameters in the vicinity of $E_J/E_C = 1$, $\alpha_s = 10$, and $\Delta/E_C = 0.1$ seem most appropriate for the experimental investigation of the suppression of supercurrent. There this suppression amounts to about 14%, compared with 6% in the adiabatic approximation. The jump of I_c^{eff} at the first-order phase boundary predicted by the SCHA can be seen in Figs. 5 and 6, but, as discussed previously, it is of little experimental relevance.

Finally we examine the current-phase relation. In Fig. 7 we plot $[I_c^{\text{eff}}(\varphi_0)/I_c^{\text{eff}}(0)]\sin\varphi_0$ versus the (aver-

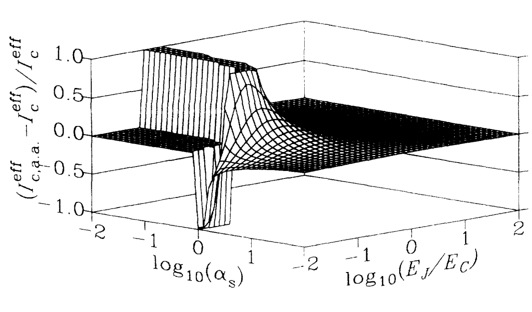

FIG. 3. Relative deviation of the critical current in the adiabatic approximation, $I_{c,\text{a.a.}}^{\text{eff}}$, from the nonadiabatic result I_c^{eff} as a function of E_J/E_C and α_s for $\Delta/E_C = 0.1$ and $\varphi_0 = 0$. The graph is clipped at ± 1.

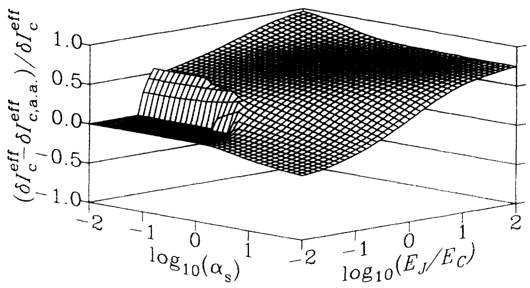

FIG. 4. Relative deviation of the suppression of the critical current in the adiabatic limit, $\delta I_{c,\text{a.a.}} \equiv I_{c,\text{a.a.}}^{\text{eff}} - I_c$, from the nonadiabatic SCHA result $\delta I_c \equiv I_c^{\text{eff}} - I_c$, for $\Delta/E_C = 0.1$ and $\varphi_0 = 0$.

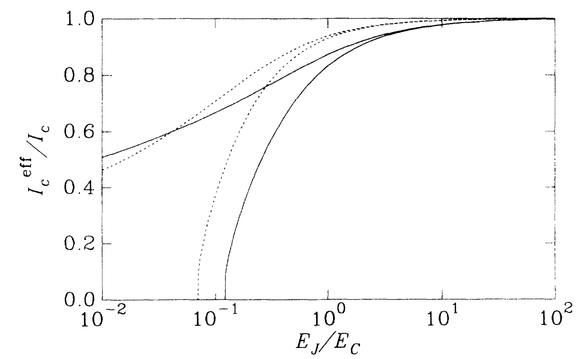

FIG. 5. Dependence of I_c^{eff} on the ratio of Josephson energy and charging energy, for $\Delta/E_C = 0.1$ and $\varphi_0 = 0$. The solid curve is the nonadiabatic SCHA result, and the dashed curve the adiabatic approximation; $\alpha_s = 10$ for the upper pair of curves, and $\alpha_s = 0.5$ for the lower pair.

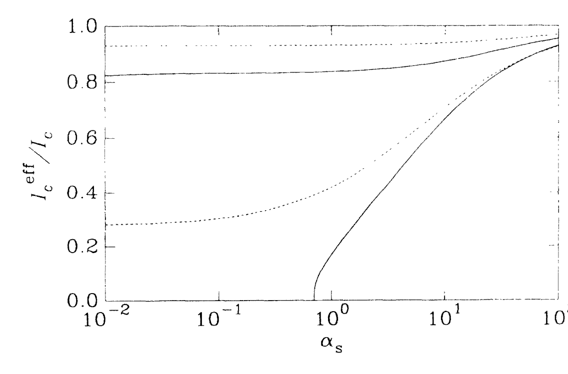

FIG. 6. Dependence of I_c^{eff} on the shunt conductance for $\Delta/E_C = 0.1$, and $\varphi_0 = 0$. The solid curve is the nonadiabatic SCHA result, and the dashed curve the adiabatic approximation; $E_J/E_C = 1$ for the upper pair of curves, and $E_J/E_C = 0.1$ for the lower pair.

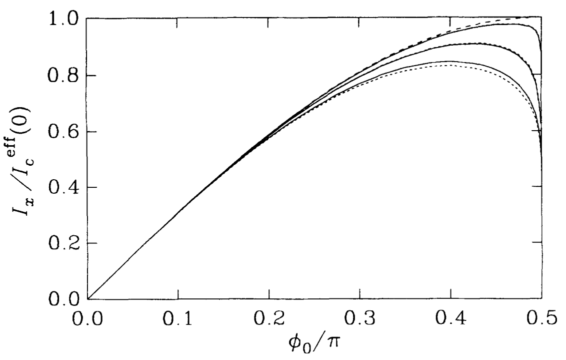

FIG. 7. Supercurrent-phase relation, normalized by $I_c^{\mathrm{eff}}(\varphi_0 = 0)$. The solid curves are the nonadiabatic SCHA results, the dashed curves the adiabatic approximation. The three pairs of curves correspond to (from top to bottom) $E_J/E_C = 10$, $E_J/E_C = 1$, and $E_J/E_C = 0.1$; $\alpha_s = 10$ and $\Delta/E_C = 0.1$ for all pairs. The topmost curve (with long dashes) is the classical Josephson relation $I_x/I_c = \sin\varphi_0$.

age) Josephson phase φ_0. For phases close to $\varphi_0 = \pi/2$ the current-phase relation deviates significantly from the simple sine function that is valid in the limit $E_J/E_C \to \infty$. Most of the difference between the nonadiabatic result and the adiabatic approximation, however, has been taken into account by $I_c^{\mathrm{eff}}(0)$; the remaining differences prove to be small.

V. SUMMARY

We have examined how quantum phase fluctuations influence the dc supercurrent through the junction with and without recourse to the adiabatic approximation. Here we summarize the main results.

(1) The adiabatic approximation is found to yield a qualitatively correct result even for low ratios of the superconducting energy gap to the elementary charging energy. Quantitatively, however, the reduction in the Josephson current predicted by the adiabatic approximation is wrong by 50% in some experimentally accessible regions.

(2) The effective critical current, in general, depends on the average phase, but the current-phase relations obtained here reveal that differences between the adiabatic and nonadiabatic results are small, irrespective of the value of E_J/E_C.

(3) For large E_J/E_C, the SCHA and perturbative RG analysis give different superconducting-insulating phase boundaries. A prescription of how to obtain agreement is discussed.

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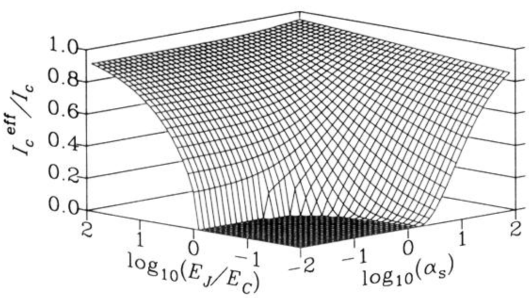

FIG. 1. $I_c^{\rm eff}$ calculated in the adiabatic limit $\Delta/E_C \to \infty$ for $\varphi_0 = 0$, normalized by the classical value $I_c = 2eE_J$.

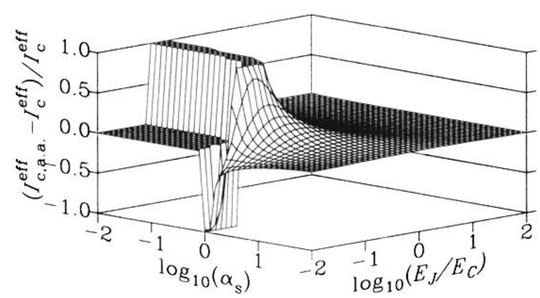

FIG. 3. Relative deviation of the critical current in the adiabatic approximation, $I^{\rm eff}_{\rm c,a.a.}$, from the nonadiabatic result $I^{\rm eff}_c$ as a function of E_J/E_C and α_s for $\Delta/E_C = 0.1$ and $\varphi_0 = 0$. The graph is clipped at ± 1.

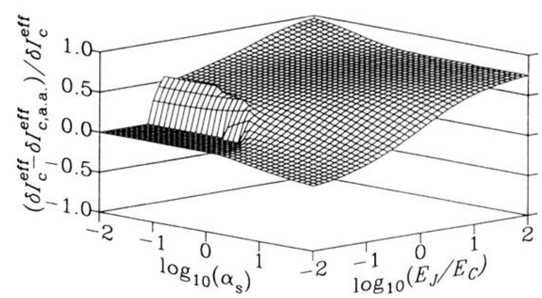

FIG. 4. Relative deviation of the suppression of the critical current in the adiabatic limit, $\delta I_{c,a.a.} \equiv I_{c,a.a.}^{eff} - I_c$, from the nonadiabatic SCHA result $\delta I_c \equiv I_c^{eff} - I_c$, for $\Delta/E_C = 0.1$ and $\varphi_0 = 0$.