Fractal properties of a one-dimensional film surface

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The fractal dimension of films formed by random deposition of particles onto a surface was investigated by means of computer simulation. Unit-square particles are deposited randomly onto a one-dimensional lattice of unit cells, forming clusters. The relation between the density of particles, proportional to the thickness of the film, and the fractal dimension of the clusters was obtained for a static case of a constant average density of particles. The results show that the fractal dimensions are less than one in agreement with Tanaka's quantitative predictions. For small densities of particles the dimensions are significantly less than one, while for larger clusters the dimensions tend to unity and the fractal properties disappear.

A model of a one-dimensional film deposited onto a surface has been presented by Nakamura.[1–3] The surface is divided into N unit cells which are randomly filled with unit-square particles. As a result, a pattern of clusters is formed on the surface. Each cluster may be described by its area A (i.e., the number of particles in the cluster), its bottom length B, and its surface length L, as shown in Fig. 1. Then the fractal dimension d may be defined either as

$$L = c_0 B^{d_0} \tag{1}$$

or

$$L = c_1 A^{d_1/2}. \tag{2}$$

Fractal dimension d_0 defined by Eq. (1) describes the relation between the cluster's bottom and surface lengths, while d_1 defined by Eq. (2) describes the relation between its area and surface length. However, the fractal dimension d is expected to depend on the average number γ of particles per cell. In other words, γ is the thickness of the film and d is a function of γ.

The statistical character of the exponential law (1) or (2) means that d values depend also on the conditions of the film growth. Tanaka[4] obtained an analytical expression for $L(\gamma, B)$. He showed that d is greater than one if $L(\gamma, B)$ is averaged over uniformly distributed γ, when resulting $L(B)$ is used to fit Eq. (2). This may correspond to results of Nakamura[3] who obtained $d_0 = 1.05$ and $d_1 = 1.86$ for clusters growing as the deposition proceeds when γ grows too. In case of a single γ value, dimension d is less than one as it was shown by Tanaka.[4] In this static case we consider a group of clusters with a constant γ and $L(B)$ is obtained as a configurational average over the ensemble.

In our simulation $K = \gamma N$ particles were randomly deposited with a uniform distribution into N unit cells. Then A, B, and L of resulting clusters were analyzed

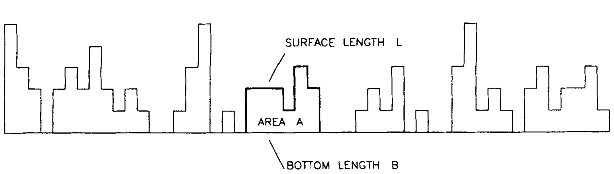

FIG. 1. Example of cluster patterns formed on a surface.

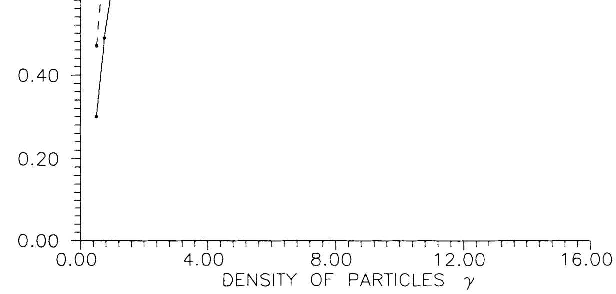

FIG. 2. Plot of fractal dimension d_0 (solid line) and correlation coefficient (dashed line) against γ.

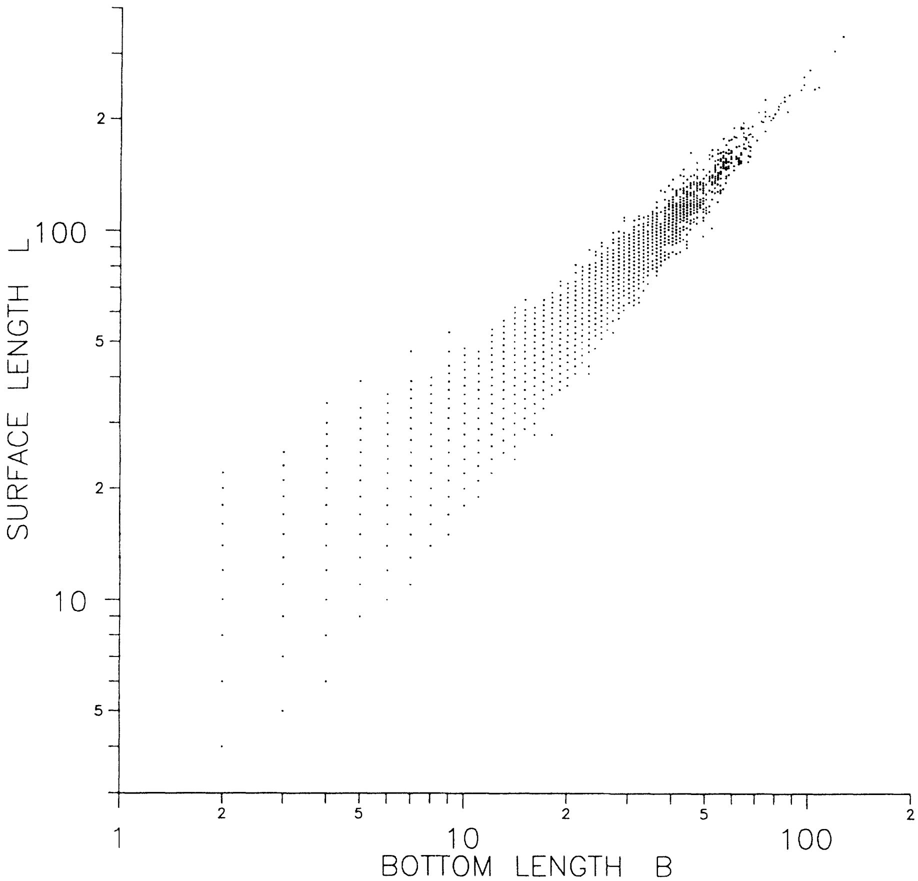

FIG. 3. Typical scatter plot of cluster's surface length L against bottom length B.

and parameters c_0, d_0 (or c_1, d_1) were fitted according to Eqs. (1) and (2). In this procedure γ was scanned from 0.5 to 15.

The fractal dimension d_0 plotted against γ is shown in Fig. 2 as a solid line. The correlation coefficient of the fit is displayed as a dashed line. Figure 3 shows a typical scatter plot of L against B in a log-log scale, obtained for $\gamma = 2.5$. A similar picture was obtained for A vs L dependence. It is seen that we get the correlation coefficient of 0.98 or more for γ larger than about 2.5, so that values for d_0 are accurate for γ larger than about 2.5. Actually, for $\gamma = 2.5$ we get $d_0 = 0.89$. For γ below 2.5 data points are more scattered and d_0 is obtained with smaller accuracy. For the smallest $\gamma = 0.5$ used in calculations the values of d_0 obtained for several runs differed as much as 0.1. However, it is clearly seen from Fig. 2 that the fractal dimension d_0 increases monotonically with increasing γ and approaches one for large γ. We conclude from our results that fractal dimensions are below one are in agreement with Tanaka[4] predictions in the case of constant γ. This is a static case with the configurational average over ensemble of randomly distributed clusters. Nakamura's $d > 1$ fractal dimensions correspond to different statistics when clusters grow by random deposition and so grows γ. For large clusters, dimensionality tends to unity, and the fractal properties disappear.

Similar results were obtained for d_1. We get $d_1 = 1.81$ for $\gamma = 2.5$ and d_1 increases for increasing γ. Again we get an integer $d_1 = 2$ in the limit of large γ.

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