Dielectric function and collective modes of two-dimensional interacting bosons

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We present a theoretical study of the dielectric response and collective excitations of a two-dimensional system of bosons interacting via a dipolar interaction. The model is designed to simulate the situation occurring in semiconductor double quantum well systems under strong electric fields perpendicular to the layers. The field produces a net polarization of photoexcited electron and hole carriers and favors the appearance of long-lived polarized excitons. This gas of interacting point dipoles shows interesting features and we study the effects of low dimensionality and statistics. Well-defined density-fluctuation excitations appear at low temperatures with a linear dispersion relation at long wavelengths. The low dimensionality of the system gives rise to an effective long-ranged potential with only a faster power-law decay.

I. INTRODUCTION

Extensive research of two-dimensional (2D) Fermi systems with electrons and/or holes has been undertaken in recent years. The subject has been studied both experimentally and theoretically, and a number of books and comprehensive reviews is available.[1] This whole area has greatly benefited by advances in materials fabrication, since charge carriers can be confined to move essentially on a plane, and a 2D degenerate electron (or hole) gas is readily achieved under real experimental conditions.

At the same time, Bose systems of reduced dimensionality have not attracted as much attention. Major experimental efforts have been directed to superfluid helium films,[2] while theoretical investigations have also dealt with other model systems. A charged Bose gas is perhaps one of the simpler and yet interesting models which have been studied in the past decades.[3–9] This system combines the well-known Coulomb interaction with peculiarities of Bose statistics and effects of different dimensionalities. Although there is no direct physical realization of such a model, this system has been useful as test ground in the development of various theoretical techniques. These studies have helped establish a connection with Fermi systems, especially in the case of reduced dimensionality.[5,7–9] Moreover, these studies illustrate the substantial difficulties in approaching the many-particle interacting Bose problem in a planar geometry.[10,11] Indeed, in the purely 2D case one cannot consider the two-body interaction potential merely in the Born approximation. The corresponding perturbation series for the renormalized potential has to be summed taking into account an infinite set of terms (to escape the divergence for low momenta in performing the Fourier transform). This divergence is purely quantum mechanical, and it is also responsible for the vanishing of the scattering t matrix for particles with low energies.[12] (A renormalization group approach has been found to be fruitful in the handling of this problem in low dimensionalities.[9,11]) Moreover, although classical phase fluctuations lead to a vanishing of the long-range order in two-dimensional Bose systems, a theory describing the transition to a superfluid state has been developed.[11,13] More recently, the problem of a 2D dilute Bose gas has also been addressed in the context of high-T_c superconductors.[14]

We address the present work to the question of effective screening in a 2D Bose gas with weak interparticle repulsion. The system of particular interest is a gas of electric dipoles, which are free to move on the x-y plane, and all of which are *polarized* in the z direction. Such a problem is closely related to the experiments of Fukuzawa and co-workers[15] and Kash and co-workers.[16,17] These authors studied a double quantum well system, and an electric field applied normal to the plane of free motion of electrons and holes (i.e., along the growth direction). If the field is strong enough, electrons and holes become spatially separated, although coupled via the Coulomb potential, so that they form excitons whose lifetime increases by up to three orders of magnitude compared to the zero-field case — as the lifetime is controlled by the strength of the applied electrical field (through the overlap of electron and hole wave functions).[16] For large fields ($\approx$ 30 kV/cm), so that the excitons can reach thermal equilibrium well within their lifetime, the system can be modeled by a weakly interacting Bose gas, as the typical interelectron or interhole separations are much larger than the electron-hole distance (or exciton radius)—see below. The original experiment was interpreted as a phase transition to a collective state at a fixed strength of electric field while the temperature was lowered.[15] Although detailed analysis has cast doubts on this identification, a similar experimental arrangement with improved samples would perhaps provide interesting and more definite results.[17]

Other possibilities for realization of a planar Bose system of excitons with spatially separated electrons and holes may include semiconductor heterojunctions of type II, based upon materials such as InAs, AlSb, and GaSb.

The chemical potential there typically occurs in the energy gaps of both materials, producing neighboring layers of electrons and holes.[18]

Systems with spatially separated electrons and holes have been studied theoretically by several groups. The very idea was seemingly first proposed by Kogan and Tavger.[19] They considered a system of two semiconductor films with electron and hole carriers, respectively, separated by a dielectric slab. Pairing between the charge carriers was provided by the attractive Coulomb interaction. In this pioneering work they calculated the BCS-like energy spectrum of the system, the critical temperature of the corresponding transition to the collective state, and investigated the formation of itinerant excitations. It was also suggested that the Coulomb interaction between these spatially separated charge carriers provides the mechanism by which the system exhibits superconducting behavior. These results did not receive much attention until later, as the experiments on the double quantum well under the influence of strong electric fields were inspired by work of Lozovik and Yudson[20] and Shevchenko.[21] Unlike the problem we consider, those systems were essentially three dimensional—the charges were free to move in semi-infinite slabs of semiconductor or in slabs of finite thickness, and bulk screening between identical carriers was taken into account, as required by the typically higher carrier density considered. In our approach, the role of Bose statistics as well as the two dimensionality of the system is investigated.

In closely related developments, the behavior of spatially separated electrons and holes under strong magnetic fields has attracted much interest recently, mostly in connection with the fractional quantum hall effect (FQHE) and in the high magnetic field regime (such that electrons and holes occupy only a small fraction of the lowest Landau level).[22,23] Different aspects of the problem in high magnetic fields, such as Wigner crystallization, excitations of the incompressible quantum liquid responsible for FQHE in a double layer system, and an excitonic charge-density-wave instability have also been discussed in the literature.[24]

In this paper we calculate the dielectric response function $\epsilon(\vec{q},\omega)$, which provides a general description of various phenomena such as longitudinal collective excitations, and the screening effects in interacting systems. We shall consider the case of an ideal double quantum well without impurities or geometrical imperfections. In our model, excitons are represented by identical dipoles with only a z component (i.e., polarized excitons), which are free to move in the x-y plane. These Bose particles interact via a dipole-dipole potential as long as the system is dilute, such that higher multipolar fields are weak and the constituent nature of these excitons can be ignored. The problem for a general interaction potential can also be approached within the framework of the dielectric function formalism, as we describe below.

The structure of the paper is as follows. First, an interaction potential which reduces well to the known limiting cases is introduced: a Coulomb interaction between constituent particles, dominating at short distances between the composite "Bose particles," and a dipole-dipole interaction at large separation between polarized excitons. Next, we present our results on calculating the dielectric function in the self-consistent-field approximation. Then we compare the asymptotic form of our results with other available calculations: the various temperature limits of a planar Fermi system and a system of charged bosons in two dimensions. The overall behavior of the dielectric function is defined in quadratures and displayed for different temperatures. Moreover, using analytical expressions for the low-temperature limit, we find the dispersion relation for density-fluctuation excitations in this system. Finally, we discuss the qualitative consequences of screening of an external potential by this two-dimensional system of polarized dipoles.

II. APPROACH AND CALCULATIONS

A. Dipole interaction potential

In the next section we present the calculation of the dielectric function for a 2D Bose gas in the mean-field approximation for a general type of interaction. However, it is of special interest to consider the interaction potential between dipoles polarized in the z-direction, since such a problem is closely related to current experiments, as discussed in the Introduction. As usual, the dipole potential is given by

$$\varphi(r) = (\vec{p}\cdot\nabla)1/r\,, \tag{1}$$

where $\vec{p}$ is the dipole strength and r is the distance to the dipole. Correspondingly, for two point dipoles polarized in the z direction, the interaction becomes

$$\phi(r) = \frac{p^2\left(1-3\cos^2\theta\right)}{r^3}\,, \tag{2}$$

where θ is the angle between the line connecting the pair of point dipoles and their orientation. Moreover, for dipoles dynamically constrained to the x-y plane ($\theta=\pi/2$), the potential energy is particularly simple:

$$\phi = \frac{p^2}{r^3}\,. \tag{3}$$

Notice, however, that a somewhat more rigorous potential should include the interaction between the constituent electrons and holes, since their separation is finite for the experimental systems we envision. Still, in the dilute regime the short-range part of the potential should not play a decisive role and the composite nature of the particles can be neglected both in regard to the interaction as well as in regard to the statistics they obey. (The interesting question of nonbosonic behavior, particularly relevant for high densities, will be addressed in detail elsewhere; see Refs. 19–21.) Here we exploit a phenomenological approach, where the interaction becomes Coulomb-like at short distances, while preserving asymptotically a dipolar character. We then use the form

$$\phi(r) = \frac{p^2}{r^3}\left[1-\exp(-r^2/d^2)\right]\,, \tag{4}$$

where d represents the distance at which excitons become "aware" of their internal structure, and as such, d should be of the order of the exciton radius. A great advantage of this particular choice of potential is that it has a well-defined 2D Fourier transform, which can furthermore be calculated analytically:

$$\phi(q) = \frac{qp^2}{2\pi}\left\{-1 + \frac{qd\sqrt{\pi}}{4}\exp(-q^2d^2/8)\left[I_1\left(\frac{q^2d^2}{8}\right) + \left(1+\frac{4}{q^2d^2}\right)I_0\left(\frac{q^2d^2}{8}\right)\right]\right\}. \qquad (5)$$

Here I_n is the Bessel function of imaginary argument of nth order.[25] Notice that $\phi(q)$ has regular behavior in the long-wavelength limit, with the zeroth order term (for any finite d) given by

$$\phi(q\to 0) = \frac{p^2}{2\,d\,\sqrt{\pi}}, \qquad (6)$$

and is, therefore, ill defined at $d = 0$.

It is instructive to establish the correspondence between screening in Fermi and Bose systems from the outset. While typical systems consider charged particles, so that a restoring force is provided by a background of the opposite charge, there is nothing like this in this neutral dipolar Bose system. In contrast to the local charge neutrality condition in those systems, it is the *fixed* average dipole density which provides the screening in this case. As each particle repels the others without a local background restoring force, the dipoles cannot fly away if we insist on a fixed dipolar density. Local piling of these Bose dipoles, or a local shortage of them, results in an effective restoring force as we impose hard-wall boundary conditions on the edges of the system "box." Mathematically, this is equivalent to the requirement of cancellation of the long-wavelength limit of the interaction potential, just as it appears in a jellium model.[12]

B. Self-consistent-field dielectric function

In this self-consistent mean-field approximation (SCFA), the dielectric function is given by a Lindhard expression:[12,26]

$$\epsilon(q,\omega) = 1 - \frac{1}{S}\phi(q)\sum_{\vec{k}}\frac{f(E_{\vec{k}+\vec{q}}) - f(E_{\vec{k}})}{E_{\vec{k}+\vec{q}} - E_{\vec{k}} + \hbar\omega - i\delta}. \qquad (7)$$

Here, $\phi(q)$ is the 2D Fourier component of the given interaction potential, S is the area of the domain occupied by the particles, $f(E)$ is the Bose distribution function for the particles with energy spectrum E, and the exciton (center of mass) energy spectrum is taken to be isotropic and with total effective mass $m = m_e + m_h$:

$$E_k = \frac{\hbar^2\,k^2}{2\,m}. \qquad (8)$$

Correlation effects beyond this SCFA are neglected in this picture and should be accounted for differently. However, it should provide an adequate description of the system for not-too-low temperatures, so that one can ignore degenerate gas effects.[12]

In the following we assume the dilute regime. This is crucial for studying the problem in this approximation, since excitons are taken to be Bose particles, and their composite nature is neglected. Consequently, the average separation between excitons should be larger than the radius of such a "polarized exciton." The diluteness of the system is also used explicitly when calculating the chemical potential $\mu(n,T)$. To the lowest order, interactions are neglected and the chemical potential is assumed to be that of an ideal two-dimensional Bose gas, which can be written as

$$\mu = T\,\ln\left[1 - \exp\left(\frac{-2\,\pi\,\hbar^2\,n}{m\,T}\right)\right]. \qquad (9)$$

Here T is temperature, and n is the average 2D density of excitons. Interaction effects should be taken into account in the chemical potential to go beyond the mean-field approximation. Unfortunately, the approach similar to the three-dimensional case is not applicable here, since for the long-range interaction potential the 2D scattering length is not well defined and cannot be used as a parameter in the perturbation expansion in two dimensions.[9]

The Lindhard expression leads to the following quadrature formulas for the real and imaginary parts of the frequency-dependent dielectric function:

$$\mathrm{Re}\ \epsilon(\vec{q},\omega) = 1 + \phi(q)\frac{m}{2\pi\hbar^2 q}\left[\gamma_+\int_0^1 dy\left(-1+\exp\frac{1}{T}\left[-\mu + \frac{\gamma_+^2\hbar^2(1-y^2)}{2m}\right]\right)^{-1} + \gamma_-\int_0^1 dy\left(-1+\exp\frac{1}{T}\left[-\mu+\frac{\gamma_-^2\hbar^2(1-y^2)}{2m}\right]\right)^{-1}\right], \qquad (10)$$

$$\mathrm{Im}\ \epsilon(\vec{q},\omega) = -\phi(q)\frac{m}{2\pi\hbar^2 q}\left[\gamma_+\int_0^\infty dy\left(-1+\exp\frac{1}{T}\left[-\mu+\frac{\gamma_+^2\hbar^2(1+y^2)}{2m}\right]\right)^{-1} - \mid\gamma_-\mid\int_0^\infty dy\left(-1+\exp\frac{1}{T}\left[-\mu+\frac{\gamma_-^2\hbar^2(1+y^2)}{2m}\right]\right)^{-1}\right]. \qquad (11)$$

Here, the functions $\gamma_\pm$ are defined by

$$\gamma_\pm = \frac{q}{2} \pm \frac{m\,\omega}{\hbar\,q}\,. \tag{12}$$

These expressions allow us to explore the form of the dielectric function analytically in various limits, and to study its qualitative behavior by plotting the results of numerical calculation for both parts of $\epsilon(q,\omega)$.

III. RESULTS AND DISCUSSION

To check for the diluteness condition we need to find the effective radius of a z-polarized 2D exciton in its lowest energy state. For a simple estimate we minimize the total energy as described by the Hamiltonian in the center-of-mass reference system:

$$\hat{H} = \frac{\hat{p}^2}{2\mu} - \frac{e^2}{\kappa\sqrt{d^2+r^2}}\,, \tag{13}$$

where κ is the background dielectric constant and μ is the reduced mass for the electron-hole system, $\mu = m_e m_h/(m_e+m_h)$. Using the uncertainty relation yields the following extremum condition for the exciton radius:

$$1 + \frac{r^2}{d^2} = K\left(\frac{r}{d}\right)^{\frac{8}{3}}, \tag{14}$$

where d is the separation between the neighboring layers of charge carriers. In the two limits, when the parameter $K = (d/a_0\,\kappa)^{2/3}$ is either small or large (with a_0 being the Bohr radius, $a_0 = \hbar^2/\mu e^2$), one finds the effective radius and the corresponding energy for the ground state as follows.

(i) If $K \ll 1$, that is, when the separation between layers is small compared to the Bohr radius of an exciton, one obtains $r \approx \kappa a_0$, and we have the three-dimensional result $E/\mathrm{Ry}^* = -\left[1-(d/a_0\kappa)^2\right]$, where the usual energy unit is defined as $\mathrm{Ry}^* = \mu e^4/2\hbar^2\kappa^2$.

(ii) If $K \gg 1$, then $r = (d^3 a_0 \kappa)^{1/4}$ and $E = \left(1-\sqrt{a_0\kappa/d}\right)e^2/\kappa d$.

In case of the GaAs-based double-well excitons in Ref. 15, which are formed by electrons and (heavy) holes with effective masses $m_e = 0.067 m_0$ and $m_h = 0.377 m_0$, respectively (where m_0 is the free electron mass), one gets $\mu = 0.057 m_0$, and for a typical separation $d = 50$ Å, and background dielectric constant $\kappa = 12$, it all results in a Bohr radius $a_0 = 9.1$ Å, and $K \approx 0.6$. The exciton radius is equal to the interwell separation d within a numerical factor of order 1. The typical exciton density achieved by optical pumping in these systems is $n \le 10^{11}$ cm^{-2}, and the diluteness condition is definitely satisfied for these parameters: $n^{-1/2} \gg d \approx r$.

In this dilute exciton limit one can study various q and ω limiting cases of Eqs. (10) and (11), as discussed in the following sections.

A. Static limit: $\omega = 0$

In the static limit, the expression for the dielectric function simplifies to

$$\epsilon(q) = 1 + \phi(q)\frac{m}{2\pi\hbar^2}\int_0^1 dy \times \left(-1+\exp\frac{1}{T}\left[-\mu+\frac{\hbar^2 q^2}{8\,m}\left(1-y^2\right)\right]\right)^{-1}. \tag{15}$$

From this, the static dielectric function is readily obtained in the long-wavelength limit ($\hbar q/\sqrt{m\,T} \ll 1$):

$$\epsilon(q\to 0,\omega=0) = 1 + \phi(q)\frac{m}{2\pi\hbar^2}\frac{1}{[-1+\exp(-\mu/T)]}\,. \tag{16}$$

Using the chemical potential of the ideal (noninteracting) Bose gas, Eq. (9), the dielectric function for low momenta can be expressed as

$$\epsilon(q\to 0,\omega=0) = 1 + \phi(q)\frac{m}{2\pi\hbar^2}\left[\exp\left(\frac{2\pi\hbar^2 n}{mT}\right)-1\right]. \tag{17}$$

Thus, physically distinct limits in this case are the high-temperature limit ($2\pi\hbar^2 n/mT \ll 1$):

$$\epsilon(q\to 0,\omega=0) \approx 1 + \frac{n\phi(q)}{T}\,, \tag{18}$$

and the low-temperature limit ($2\pi\hbar^2 n/mT \gg 1$):

$$\epsilon(q\to 0,\omega=0) \approx 1 + \phi(q)\frac{m}{2\pi\hbar^2}\exp\left(\frac{2\pi n\hbar^2}{m\,T}\right). \tag{19}$$

The high-temperature limit is independent of statistics — Eq. (18) is exactly the same for a 2D fermion system, calculated for example in the mean-field approximation with the chemical potential of the ideal Fermi gas (see Ref. 1). The high-temperature limit corresponds also to the classical Debye problem, which is independent of the statistics, and does not contain the Planck constant. The inverse temperature dependence corresponds to the fact that thermal motion of the particles prevents them from effectively screening the external potential. On the contrary, at low temperatures the external potential is strongly screened in the two-dimensional Bose-system via the $\exp(1/T)$ dependence (see Sec. III C). We emphasize, however, that this result was obtained for the chemical potential of the noninteracting Bose gas. The low-temperature behavior of the chemical potential can be changed by interparticle interactions, which should impose an upper limit on the dielectric function as $T \to 0$, for dipole interactions.

On the other hand, the high momentum behavior of the static dielectric function is seen to have the following dependence ($\hbar q/2\sqrt{mT} \gg 1$):

$$\epsilon(q,\omega=0) = 1 + \phi(q)\,\frac{4mn}{\hbar^2 q^2}\,. \tag{20}$$

Here, the limit is somewhat similar to the high-temperature dependence, but the physical role of the temperature is played by the kinetic energy of the exciton with a given momentum $\hbar q$. Note that this expression also describes the static dielectric function for all momenta in the extreme case of zero-temperature limit.

B. Finite frequency: $\omega \neq 0$

The overall behavior is changed at finite frequencies, as it is seen in the figures. Qualitatively, Fig. 1 shows that a nonzero imaginary part of the dielectric function exists, which narrows as $T \to 0$ and turns into a sharp δ-like function. The real part, on the other hand, has a resonancelike behavior in the zero-temperature limit, with the "resonance" occurring at $\hbar\omega = \hbar^2 q^2/2m$. The temperature has the effect of smearing this picture in the vicinity of the jump, providing smooth and continuous connection of the two branches. The height of both peaks decreases with growing temperature, while they become wider. In particular, this behavior means that longitudinal collective excitations are possible in this system at low enough temperature, when the low-q branch of Re ϵ crosses through zero and the width of the imaginary part becomes less than the separation between the position of this zero and the position of the "resonance." In Fig. 2, the function $\mathrm{Im}[\epsilon^{-1}(q,\omega)]$, which provides a measure of the "oscillator strength" of the collective excitations, and for example describes the signal intensity for an electron energy-loss experiment, shows a peak which becomes more pronounced and shifts towards the $T \to 0$ result, indicating the existence of well-defined collective modes for $q/q_0 \approx 1$. Here, and for all the curves, the units for the momentum, frequency, particle density, and temperature are chosen naturally for the experimental conditions, such that $2\pi\hbar^2 n/mT \approx 1$ at $T = 1$ K, when the density n_0 is taken equal to 10^{10} cm^{-2}. Similarly $q_0^2 \approx mT/\hbar^2$ provides units for the wave vector $q_0 = 10^6$ cm^{-1}, and for the frequency: $\omega_0 \approx q_0^2\hbar/2m = 10^{12}$ s^{-1}.

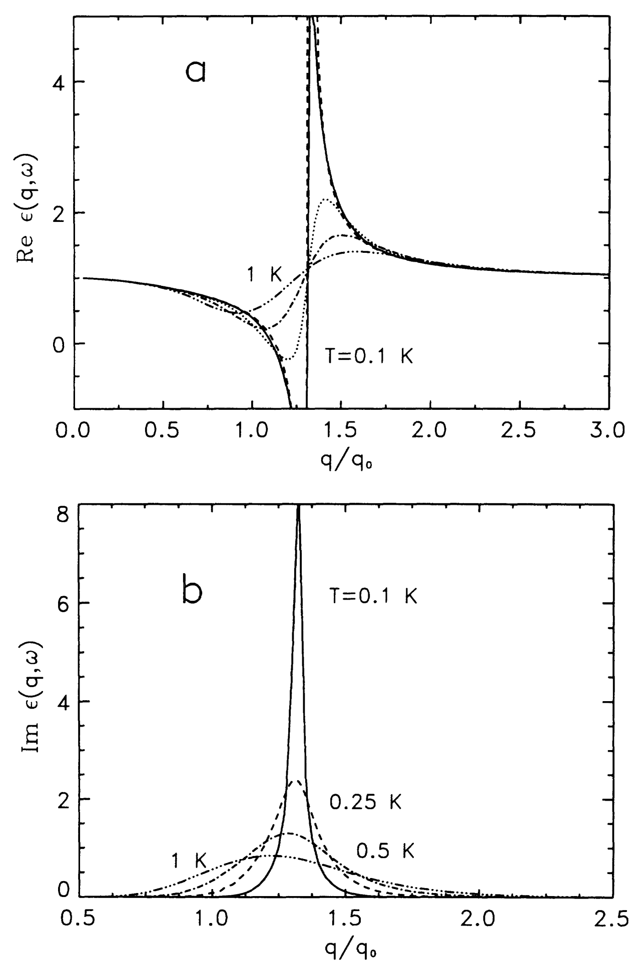

FIG. 1. (a) Real and (b) imaginary part of the dielectric function for Bose dipole system. For low temperatures Re $\epsilon(q,\omega)$ has a resonance-type behavior, which is smeared at finite temperatures around the jump position $\hbar\omega = \hbar^2 q^2/2m$. (Dashed lines shows $T = 0$ limit; solid curve is for 0.1 K.) Imaginary part has δ-like behavior for the same frequency, with increasing broadening at higher temperatures. $q_0 = 10^6$ cm^{-1} is defined in text.

Different q dependence is accounted for by the functions $\gamma_\pm$, so that a qualitatively different limit occurs when the frequency is comparable to or higher than the momentum such that $\hbar^2 q^2/2m \approx \hbar\omega$. At low temperature one can get the following expression for the dielectric function:

$$\mathrm{Re}\ \epsilon(q,\omega) = 1 - 2n\phi(q)\,\frac{E_q}{(\hbar\omega)^2 - E_q^2}\,, \tag{21}$$

with $E_q = \hbar^2 q^2/2m$. This result turns into the expression for the dielectric function obtained for charged bosons in two dimensions for $T \approx 0$ (see, for example, Ref. 5), for $\phi(q)$ corresponding to the Coulomb potential. Equation (21) allows us to calculate the dispersion of the longitudinal collective excitations possible in the system of polarized excitons for sufficiently low temperature, so that they exist as a well-defined feature in the spectral function $\mathrm{Im}\,\epsilon^{-1}$,

$$\omega = q\left[\frac{n\phi(q)}{m} + \left(\frac{\hbar q}{2m}\right)^2\right]^{1/2}. \tag{22}$$

From this equation, it is clear that the dispersion relation for collective excitations in this 2D dipole gas system is acoustic (linear q dependence) at small momenta, since

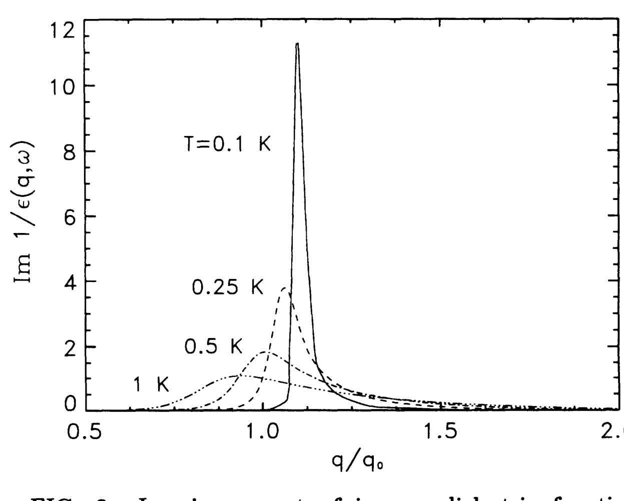

FIG. 2. Imaginary part of inverse dielectric function. Peak position indicates frequency of collective mode, which becomes damped at high temperatures.

$\phi(q \to 0)$ is a constant [see Eq. (6)]. Notice also that a transition from linear to quadratic q dependence occurs at $q^2 \approx ne^2dm/\hbar^2 \approx 10^{12}$ cm^{-2}, and the temperature is required to be less than $\min[n\phi(q); \hbar\omega]$ for the validity of these analytical expressions. The picture for higher temperatures is smeared around the "resonance point" $\hbar\omega = \hbar^2q^2/2m$, with the same qualitative behavior, until the "oscillator strength" of the mode eventually vanishes.

Similarly, the imaginary part of the dielectric function nearly vanishes everywhere in the zero-temperature limit and behaves like a δ function:

$$\text{Im } \epsilon(\omega, q) = \frac{m\phi(q)}{2\hbar^2}\, \delta\!\left(\frac{1}{T}\left(\frac{\hbar^2q^2}{2m} - \hbar\omega\right)\right), \tag{23}$$

so that at low enough temperatures the collective excitations are not suppressed by thermal effects.

C. Screened potential

The static dielectric function provides all the necessary information to calculate the effective interaction potential between the particles as modified by screening effects:

$$\phi_{\text{eff}}(r) = 2\pi \int_0^\infty dq\, q\, J_0(qr) \left(\frac{1}{\phi(q)} + \chi(q)\right)^{-1}. \tag{24}$$

Here, the dielectric function is expressed as

$$\epsilon(q) = 1 + \phi(q)\chi(q)\,, \tag{25}$$

where ϕ_{eff} is the dressed or effective potential, and the polarization part $\chi(q)$ does not depend on the particular choice of the interacting potential:

$$\chi(q) = \frac{m}{2\pi\hbar^2} \int_0^1 dy \times \left(\exp \frac{1}{T}\left[-\mu + \frac{\hbar^2q^2}{8m}(1-y^2)\right] - 1\right)^{-1}. \tag{26}$$

Asymptotic behavior of the bare potential is readily recovered [setting $\chi(q) = 0$ in the equation above], for large distances:

$$\phi(r \gg d) \simeq \frac{p^2}{r^3}\,, \tag{27}$$

and for short distances:

$$\phi(r \ll d) \simeq \frac{p^2}{d^2}\frac{1}{r}\,. \tag{28}$$

[These results are obtained after two integrations by parts, and the use of the various limiting expressions for $\phi(q)$, as well as the relation for Bessel functions $xJ_0 = d(xJ_1)/dx$.] As usual, the asymptotic form of the screened potential at large distances is governed by the long-wavelength behavior of the Fourier component of the bare potential, but the low-temperature limit depends strongly on the order in which these two limits are taken. For $T = 0$, the dielectric function is singular at $q \to 0$ $(q \ll 1/d)$, which leads to a faster decay of the screened potential [where Eq. (20) has been used in the $T \to 0$ limit]

$$\phi(r \gg d) \simeq \frac{225\pi}{4}\frac{a_0^2}{n^2d^2}\frac{p^2}{r^7}\,. \tag{29}$$

This result is similar to the calculations on the screened potential for charged bosons,[5] where similar change of the bare potential to a rapid power-law falloff is found. Under realistic experimental conditions, however, this $T \to 0$ limit is not a suitable one: the characteristic length corresponding to thermal motion of the excitons at $T = 1$ K is comparable to the charge separation $(l \simeq \sqrt{\hbar^2/8mT} \approx d)$, and one should then look at the zero-q limit for low but finite temperatures. In this case the result is drastically different:

$$\phi(r) = \frac{p^2}{r^3}\left(\frac{a_0}{d}\right)^2 8\pi^2 \Bigg/ \left(\exp\left[\frac{2\pi\hbar^2 n}{mT}\right] - 1\right)^2. \tag{30}$$

The functional form of the screened potential remains the same, and screening effects only reduce the strength of the interaction. As the temperature is lowered, the prefactor becomes vanishingly small and the crossover into the r^{-7} dependence is eventually reached.

Quite generally, from these results one can observe a major qualitative difference in the screening by systems of reduced dimensionality: just as in the 2D charged Fermi systems the screening does not lead to a short-range effective interaction potential, characterized by an exponential decay at large distances.[1] At best, the screened potential decays as an inverse power law with some characteristic length or scale, which depends upon temperature and particle density. However, at the same time, the screening is more pronounced in Bose systems (for a given interaction potential), as accounted for by the singular behavior of the polarizability at $q \to 0$.

IV. CONCLUSION

In summary, we have presented calculations of the dielectric function for the 2D interacting dilute Bose gas in the self-consistent mean-field approximation. It is shown that the obtained analytical expressions are consistent with available asymptotic limits, such as the high-temperature regime for the two-dimensional electron gas and the low-temperature limit for charged bosons. It is shown that in the low-temperature limit the collective excitations are always possible for repulsive interaction between the particles (such that the two-dimensional Fourier component of the interacting potential is non-negative for all q), and the corresponding dispersion relation is found. For long wavelengths, an acoustic dispersion is obtained for the case of polarized dipoles. Results of numerical evaluation of the obtained quadrature formulas are also shown for both real and imaginary parts of the dielectric function, as well as for the quantity Im $\epsilon^{-1}(q,\omega)$. We also show that screening in this system of spatially separated electrons and holes does not lead to an effectively short-range interaction potential, but only to a faster power-law decay.

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