Quenching mechanisms of nonlocal transport in laterally confined two-dimensional systems

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Non-Ohmic and sample-size-dependent transport effects [i.e., Shubnikov–de Haas (SdH) and quantum Hall effect] of mesoscopic two-dimensional (2D) systems prove the occurrence of nonlocal contributions to the electronic conductance in these systems. However, this nonlocal regime accompanied by a nonequilibrium population of the edge states with respect to the 2D bulk state is quenched at rather low values of external electric fields or flowing currents, respectively. Beyond this quench, the bulk state is coupled to the edge by an increasing amount of electron transitions between the corresponding states. We analyze the non-Ohmic behavior of SdH oscillations at GaAs/$Ga_xAl_{1-x}As$ quantum Hall conductors on the basis of a model including edge and bulk transport. We deduce the current-dependent nonequilibrium population of edge and bulk states quantitatively. Further, we give estimates for the current ranges in which transitions of electrons between edge and bulk states due to elastic and inelastic scattering are relevant. The change of the typical nonequilibrium parameters as the equilibration length and the maximal difference of chemical potentials of edge and bulk states in tilted magnetic fields are also discussed.

The electron transport in laterally confined two-dimensional (2D) systems in quantizing magnetic fields (i.e., in 2D electron layers of samples with Hall-bar geometry) is dominated by nonlocal edge states if the number of scatterers and the sample current are sufficiently low.[1] In this case, local conductivity or resistivity tensor components retain their meaning for the uppermost quantum level (bulk level) only.[2–4] The complete filling factor dependence of longitudinal and Hall resistance of such high-mobility samples can then be given by a description containing nonlocal edge currents and a bulk current. In the presence of nonideal current contacts and at very low currents, nonequilibrium population of edge and bulk states can occur.[5] This is experimentally observable in a variety of transport effects as, i.e., the nonlocal Shubnikov–de Haas (SdH) effect,[6] and the behavior of quantum point contacts[7] and gate-controlled barrier structures.[8–12]

The first attempts of modeling this behavior treated edge and bulk states independently.[13] Later, the partial coupling between these states was taken into account.[3,4,14–16] In our previous works, we focused on the dependence of the edge-to-bulk coupling at half-filled bulk Landau levels on the external electric fields applied to or the currents driven through Hall bars made from high-mobility GaAs/$Ga_xAl_{1-x}As$ wafers.[14–16] The current and spin-dependent non-Ohmic behavior of SdH peaks observed by several authors (for an overview, see Ref. 16) and ourselves could be explained by the assumption of an equilibration length λ exponentially decreasing with the electric Hall field at the sample.[14] This description is equivalent to a current-dependent coupling between edge and bulk states.

In a previous paper,[16] we have given a qualitative argument for the existence of an upper limit of the noncoupling (i.e., nonlocal) regime. We now supplement this by giving quantitative account for the Hall-field-dependent nonequilibrium population of edge and bulk states in this paper. The basic assumptions and equations developed in Ref. 14 are briefly sketched here to explain the calculation procedure of the nonequilibrium populations deduced from experimental data.

The transition probability of an electron from edge to bulk states or vice versa $P_{e\leftrightarrow b}$ over a distance δx in current flow direction can be expressed by introducing an equilibrium length λ by $P_{e\leftrightarrow b}=\delta x/\lambda$.[17,4] This can be used to relate the difference between the chemical potential of the edge states $\mu_e(x)$ and that of the bulk $\mu_b(x)$ to the potential drop along the current flow direction (giving the longitudinal resistance signal of the sample)[14]

$$\mu_e(x)-\mu_e(x+\delta x)=-\frac{\delta x}{\lambda N}[\mu_e(x)-\mu_b(x)] , \qquad (1)$$

where N is the number of the quantum levels including spin splitting at or below E_F.

Here, a complete equilibration of the $(N-1)$ edge states leading to a joint chemical potential μ_e is assumed.[13,18] Far enough from the nonideal current contacts, a quasistationary state develops due to both the finite level of coupling between edge and bulk states and the finite conductivity of the bulk level. This state is

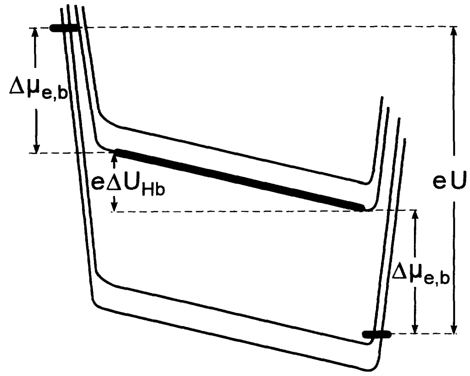

FIG. 1. Energy scheme of the coupling between edge and bulk states for a laterally confined 2D system. $\Delta\mu_{e,b}$—difference of the chemical potentials of edge and bulk states. eU_H—entire Hall-field energy. $e\Delta U_{\mathrm{Hb}}$—bulk component of the Hall-field energy.

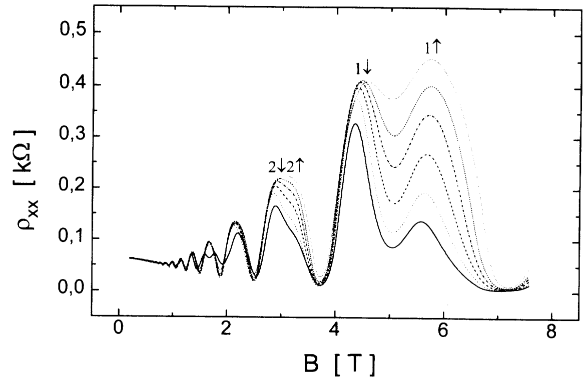

FIG. 2. Longitudinal resistivity $\rho_{xx}=\epsilon_x/j_x$ vs magnetic field up to 8 T for several fixed channel currents (sample CS50/7, $T=1.3$ K). $I=1$ μA (solid); $I=1.5$ μA (dot); $I=2$ μA(dash); $I=2.5$ μA (dash dot); $I=3$ μA (short dash); $I=4$ μÅ (short dot).

characterized by a constant difference $\Delta\mu_{e,b}=\mu_e-\mu_b=e\Delta U_{e,b}$ between edge and bulk levels and a constant electric field ϵ_x in current flow direction[14]

$$\epsilon_x=\frac{1}{\lambda N}\Delta U_{e,b}\ . \tag{2}$$

Applying current conservation and the relations between Hall voltage U_H and $\Delta U_{e,b}$

$$U_H=\epsilon_{yb}w+2\Delta U_{e,b}\ , \tag{3}$$

where ϵ_{yb} is the Hall field in the bulk state only (see Fig. 1) and w is the sample width, the longitudinal "resistivity" ρ_{xx} (which is not a tensor component of local meaning but a value characterizing a Hall-bar segment of length l and w) can be calculated

$$\rho_{xx}=\frac{\epsilon_x}{j_x}=\frac{\rho_0}{1+N\frac{2\lambda}{w}\frac{e^2}{h}(N-1)\rho_0}\ , \tag{4}$$

where $j_x=I/w=I_e+I_b/w$, j_x is the average value of current density, I_e is the edge current, and I_b is the bulk current, with ρ_0 being a saturation value in the limit of complete edge-to-bulk coupling ($\lambda\rightarrow0$). This corresponds to the limit of high Hall fields or high sample currents, respectively.[14] Assuming an exponential decay of the equilibrium length λ with increasing Hall field,

$$\lambda=\lambda_0\exp\left\{-\frac{|U_H|}{\beta}(N-1)\right\}\ , \tag{5}$$

where β is the empirical parameter and λ_0 is the equilibration length, the experimentally observed suppression of SdH peak values and spin dependence of this suppression is easily explainable.[14–16] This model[14] is similar to that of Richter, Wheeler, and Sacks[4] for complete edge-to-edge coupling ($P_{e\leftrightarrow e}=1$) and the further assumption of a current-dependent edge-to-bulk coupling $P_{e\leftrightarrow b}=f(I)$. Both models yield much longer equilibration lengths λ_0 (in the zero current limit) for μ_b situated in spin-split quantum levels of odd numbers (spin-up states) due to the cyclotron energy $\Delta E_1=\hbar\omega_c$ being much higher in comparison to the spin splitting $\Delta E_S=g^*\mu_bB$ in GaAs.[4,14]

The larger equilibration length for spin-up bulk states explains the stronger suppression of the high-field-side SdH peaks of spin-split Landau levels and their more pronounced current dependence (see Fig. 2).

In the noncoupling limit ($\lambda\rightarrow\infty$), the current would be carried by the $(N-1)$ edge states only. However, this regime is limited to currents below a certain value I_c corresponding to the condition

$$\Delta\mu_{e,b}^{\uparrow}\leq\Delta E_1-\Delta E_s\ , \tag{6a}$$

for μ_b in a spin-up level, and

$$\Delta\mu_{e,b}^{\downarrow}\leq\Delta E_s\ , \tag{6b}$$

for μ_b in a spin-down level.

Up to this limit, the entire Hall voltage is related to the $(N-1)$ edge states only[16]

$$eU_H=2\Delta\mu_{e,b}^{\uparrow\downarrow}=\frac{h}{e(N-1)}I\ . \tag{7}$$

Above this limit, bulk electrons at μ_b start to populate the edge states of the lower populated edge, which degenerate spatially with the bulk state.[16] This marks the onset of a bulk current flow diminishing $\Delta\mu_{e,b}$ (switch of coupling probability $P_{e\leftrightarrow b}$ from 0 to 1). In real systems, the equilibration length has some finite and current-dependent value leading to a softer decay of $\Delta\mu_{e,b}$ near the current limit. The current dependence of $\Delta\mu_{e,b}$ in the presence of edge-to-bulk coupling can be explicitly calculated using Eq. (4) with ϵ_x according to Eq. (2) and λ according to Eq. (5)

$$\Delta\mu_{e,b}=\frac{eNI\rho_0}{\frac{w}{\lambda_0}\exp\left\{\frac{|U_H|}{\beta}(N-1)\right\}+2N(N-1)\frac{e^2}{h}\rho_0}\ . \tag{8}$$

The saturation value ρ_0, the equilibration parameters

λ_0 and β, and the Hall voltage U_H can be taken from the current-dependent transport measurements (see, i.e., Refs. 14 and 16). The determination of the current dependence of U_H in the low current regime is somewhat complicated, because there is no tensor component ρ_{xy} of local sense due to the partially nonlocal transport. A change of $P_{e\leftrightarrow b}$ from 0 to 1 by increasing the current would correspond to an alteration of the Hall resistance R_{xy} from $R_{xy}^0 = h/[(N-1)e^2]$ to $R_{xy}^E = h/[(N-\frac{1}{2})e^2]$ for μ_b in the center of the Nth quantum level.

In reality, the change of R_{xy} is less dramatic due to the finite dependence of edge-to-bulk coupling. We have taken this into account by assuming an exponential approach of R_{xy} to the bulk-dominated saturation value R_{xy}^E taken from experiment using the same exponent as for the current dependence of the equilibration length.

Figures 3(a) and 3(b) show the current dependence of nonequilibrium edge-state population represented by $\Delta\mu_{e,b}$ for the two levels $N=3$, $N=4$ and $N=5$, $N=6$ corresponding to the traces of Fig. 2. The measured sample data relevant for the calculations are listed in Table I.

The sample analyzed is a Hall bar of 10-μm channel width and three pairs of potential probes equally spaced with 550 μm. For all levels, $\Delta\mu_{e,b}$ is zero for vanishing current (corresponding to thermodynamical equilibrium between edge and bulk states at any finite degree of coupling, $P_{e\leftrightarrow b}>0$). As the current increases, $\Delta\mu_{e,b}$ increases almost linearly as the Hall voltage. Here, nearly the entire current is carried by the edge states. That is equivalent to the condition $U_H = 2\Delta\mu_{e,b}/e$ and a vanishing bulk-state current (complete nonlocal transport,[16] see also Fig. 4 showing the Hall voltage in the bulk in dependence on the current). At higher currents, $\Delta\mu_{e,b}$ reaches a maximum and then drops again towards zero due to the exponentially increasing amount of coupling between edge and bulk. At these currents, the entire Hall voltage drops along the bulk state, and the edge currents on both sample edges compensate each other. Here, the edge currents become meaningless for the conduction process, which becomes a local one.

TABLE I. Sample data relevant for the calculations (see text).

n_s (10^{11} cm^{-2})	μ_H (10^5 cm^2/V s)	N	λ_0 (μm)	β (V)	ρ_0 (kΩ)
		3	620	0.013	0.47
3.6	3.0	4	19	0.011	0.41
		5	165	0.011	0.22
		6	20	0.016	0.22

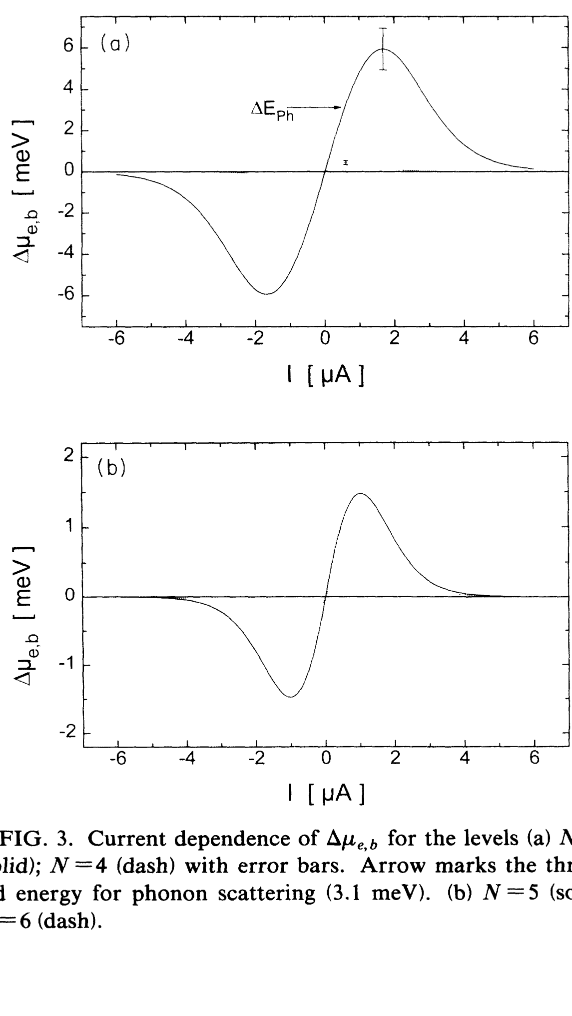

FIG. 3. Current dependence of $\Delta\mu_{e,b}$ for the levels (a) $N=3$ (solid); $N=4$ (dash) with error bars. Arrow marks the threshold energy for phonon scattering (3.1 meV). (b) $N=5$ (solid); $N=6$ (dash).

As visible in Fig. 3, both the maximal amount of $\Delta\mu_{e,b}$ ($\Delta\mu_{e,b}^{\max}$) and the sample current $I_{\max}$ corresponding to $\Delta\mu_{e,b}^{\max}$ strongly depend on the quantum state of the bulk level. For both Landau levels $l=1$ and $l=2$ ($N=3,4$; $N=5,6$) investigated in this study, $\Delta\mu_{e,b}^{\max}$ and $I_{\max}$ are higher for the spin-up levels. This is because for the $N=3$ and $N=5$ bulk levels the edge-to-bulk separation scales with $\Delta E_1 - \Delta E_s$, which is about 20 times higher than the spin-splitting energy ΔE_s. In all cases, $\Delta\mu_{e,b}^{\max}$ has to be definitely lower than the corresponding energy difference between the Nth and $(N-1)$th level [see Eqs. (6a) and (6b)]. That means the evaluation of $\Delta\mu_{e,b}^{\max}$ for spin-down levels (determined by ΔE_s) provides an estimate of the lower limit of the Landé factor g^*. This g^* factor is filling factor dependent and can be estimated for the $N=4$ ($l=2\downarrow$, $\nu=3.5$) level to be about 1.75 (in good agreement with data reported by Nicholas *et al.*[19]).

In a previous paper[16] we published measurements of the non-Ohmic behavior of SdH peaks in dependence on the tilt angle φ between the magnetic field and the 2D

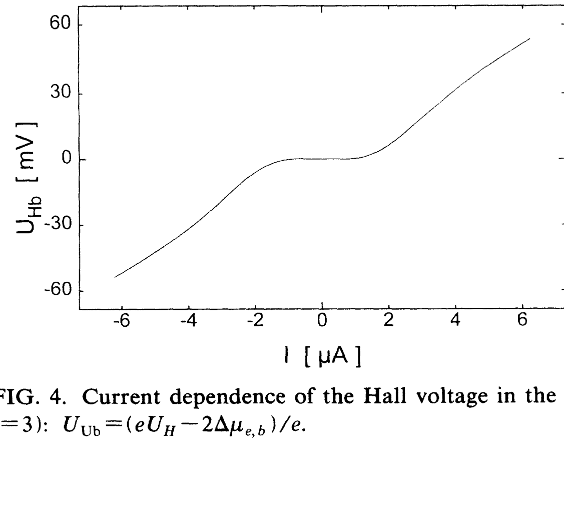

FIG. 4. Current dependence of the Hall voltage in the bulk ($N=3$): $U_{\text{Ub}} = (eU_H - 2\Delta\mu_{e,b})/e$.

plane normal vector. We argued that, in case of only one electrical subband populated, the parallel component of the magnetic field changes neither the subband population nor the position of spin-degenerate SdH peaks in $B_\perp = B\cos\varphi$ due to the lack of diamagnetic carrier redistribution. Hence, by tilting the sample, we change the spin splitting for constant Landau-level separation. As to be expected, this leads to a decrease of λ_0 for spin-up levels and an increase of λ_0 for spin-down levels with increasing tilt angle.[16] This result is beautifully supported by the angular dependence of $\Delta\mu_{e,b}$ calculated in this study. Figures 5(a) and 5(b) show the results for the levels $N=3$ and $N=4$, respectively. Whereas $\Delta\mu_{e,b}^{\max}$ of the $N=3$ level decreases from 6 to 4.8 meV while turning the sample from $\varphi=0°$ to $\varphi=43.8°$, the corresponding value of the $N=4$ level increases from 0.4 to 0.8 meV in the angular range from 0° to 60.3°. It should be noted that the angular dependence of $\Delta\mu_{e,b}^{\max}$ is crucially determined not only by the level separation, but also by the angular dependence of the equilibration parameters [see Eq. (8)]. Therefore, $\Delta\mu_{e,b}^{\max}$ does not scale with the corresponding level separation, but follows the same trend and is always smaller than the latter one.

To make the transition of electrons between edge and bulk states possible, a momentum transfer between the electrons and impurity scattering centers or (and) acoustical phonons is necessary to obey momentum conservation. The interaction of electrons, which move across to the main current-flow direction (represented by a wave-vector component k_x) and change their cyclotron-orbit position by an amount of Δy due to the spatial quantum state separation near the sample edge, with scatterers can be either elastic (impurity scattering) or inelastic (acoustical phonons). An electron, changing its position by Δy in the edge region of the sample, has to change its momentum $p_x = \hbar k_x$ due to the change Δy of the cyclotron-orbit position

$$\Delta k_x = \frac{\Delta y}{\lambda_M^2}\ , \tag{9}$$

where $\lambda_M = \sqrt{\hbar/eB}$ is the magnetic length, and due to the change of the drift velocity v_D given by the electrical field related to the confinement potential $V(y)$ and the magnetic field B

$$v_D^i = \frac{1}{eB}\frac{\partial E_i(y)}{\partial y}\ , \tag{10}$$

with $E_i(y)$ being the eigenenergy of ith quantum state near the sample edge. Assuming a parabolic confinement potential of the form[20]

$$V(y) = \tfrac{1}{2}m^*\omega_0^2 y^2 \tag{11}$$

within the edge zones, both Δy and $\partial E_i(y)/\partial y$ can be calculated for a transfer from the $(N-1)$th (inner edge state) level to the Nth (bulk state) level

$$\Delta k = \frac{eB}{\hbar}(y_{N-1} - y_N) + \frac{m^*}{\hbar}(v_D^{N-1} - v_D^N)\ . \tag{12}$$

For simplicity, we set $E_0 = 0$ (bulk-level energy) and $y_N = 0$ (transition point between bent and flat region of the bulk level). Using Eq. (11) for the confinement potential, we obtain for the change of momentum $\Delta p = \hbar\Delta k$ necessary for an electron transfer from the edge to the bulk state

$$\Delta k = \frac{eB}{\hbar}\sqrt{2\Delta E/m^*\omega_0^2} + \frac{m^*}{\hbar}\frac{1}{eB}\sqrt{2m^*\omega_0^2\Delta E}\ ,$$
$$\Delta k = \frac{1}{\hbar}\left[\frac{\omega_c}{\omega_0} + \frac{\omega_0}{\omega_c}\right]\sqrt{2m^*\Delta E}\ , \tag{13}$$

with $\Delta E = \Delta E_1 - \Delta E_s$ or $\Delta E = \Delta E_s$.

Equation (13) is equivalent to the relation between cyclotron-orbit coordinate y and wave number k in a quantum wire

$$y = \frac{\omega_c^2}{\omega_0^2 + \omega_c^2}\lambda_M^2 k \tag{14}$$

in the presence of parabolic confinement.[21]

The inelastic energy loss of the electron due to an emitted phonon can be calculated

$$\Delta E_{\mathrm{Ph}} = \hbar c_s \Delta k\ , \tag{15}$$

where c_s is the sound velocity if Δk is far enough from the Brillouin-zone border.

To make an electron transfer mediated by phonon emission possible, the condition $\mu_e \geq \mu_b + \Delta E_{\mathrm{Ph}}$ must

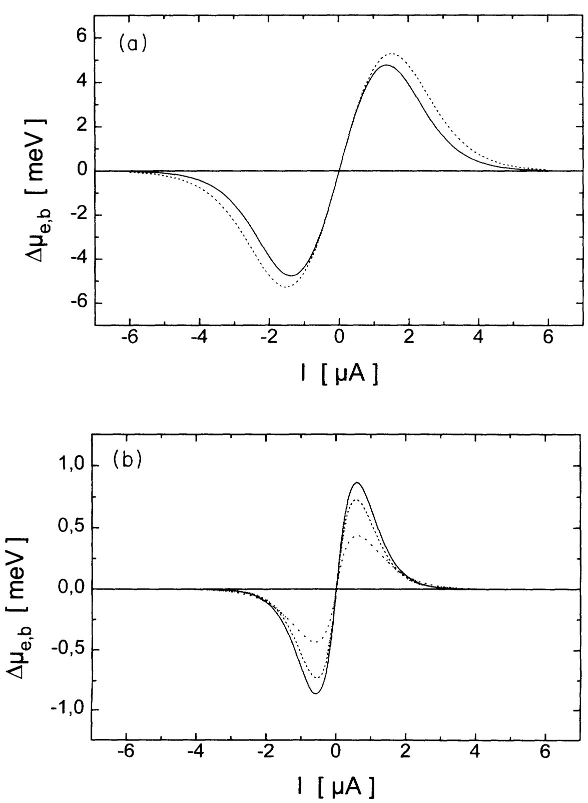

FIG. 5. Angular dependence of $\Delta\mu_{e,b}$ vs I. (a) $N=3$; $\varphi=0°$ (dot); $\varphi=25.4°$ (dash); $\varphi=43.8°$ (solid). (b) $N=4$, $\varphi=0°$ (dash dot dot); $\varphi=25.4°$ (dot); $\varphi=49.3°$ (dash); $\varphi=60.3°$ (solid).

hold to provide an empty state for the scattered electron. Hence, a certain value of $\Delta\mu_{e,b}=\mu_e-\mu_b\geq\Delta E_{\text{Ph}}$ is necessary to make the inelastic scattering possible. Using Eqs. (13) and (15), the lower limit for phonon emission can be given with respect to $\Delta\mu_{e,b}$ and the corresponding current value. This limit is marked in Fig. 3 and is higher for spin-up states as to be expected from Eq. (13). For example, the minimum current for phonon-mediated edge-to-bulk transfer of the $N=3$ bulk level is 0.55 μA for our sample using $c_s=5.2\times10^3$ m/s (Ref. 22) and $\omega_0=2.3\times10^{12}$ s^{-1}.[23] Below this limit, elastic scattering at impurities dominates the edge-to-bulk transfer.

To summarize, we have calculated the nonequilibrium population of edge and bulk states from experimental data (current-dependent SdH measurements) in dependence on the sample current, the bulk quantum number, and the tilt angle of the 2D system with respect to the magnetic-field direction. In all cases, the difference of the chemical potentials of edge and bulk $\Delta\mu_{e,b}$ bears qualitatively the same current dependence. Below a certain current limit, $\Delta\mu_{e,b}$ increases as half the Hall voltage. In this regime, the edge-to-bulk coupling is low and the current transport is of predominantly nonlocal character. Above this current limit, $\Delta\mu_{e,b}$ reaches a maximum, which is of the order but below the energetical separation of the uppermost bulk level and the next lower one. Hence, this maximum is observed to be dependent on the spin of the bulk level and the tilt angle. For spin-up states, the maximum of $\Delta\mu_{e,b}$ is of the order of the cyclotron energy reduced by spin splitting and decreases with the tilt angle. For spin-down states, the maximum is limited by the spin splitting, which is increasing with the tilt angle due to the shift of SdH peaks towards higher values of the total magnetic field B.

At higher currents, $\Delta\mu_{e,b}$ decreases again due to the exponential decrease of the equilibration length λ with the Hall voltage. This is equivalent to a transition to a bulk dominated, local conduction.

The phonon energy loss related to inelastic edge-to-bulk transitions requires a minimal current value, which is higher for spin-up levels. Below this limit, inelastic scattering is obstructed by the Pauli principle, and the momentum transfer necessary for an edge-to-bulk transition has to be realized via elastic impurity scattering.

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