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Instantons and the spectral function of electrons in the half-filled Landau level

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We calculate the instanton–anti-instanton action $S_{M\bar{M}}(\tau)$ in the gauge theory of the half-filled Landau level. It is found that $S_{M\bar{M}}(\tau)=(3-\eta)[\Omega_0(\eta)\tau]^{1/(3-\eta)}$ for a class of interactions $v(\mathbf{q})=V_0/q^\eta$ $(0\leq\eta<2)$ between electrons. This means that the instanton–anti-instanton pairs are confining so that a well-defined "charged" composite fermion can exist. It is also shown that $S_{M\bar{M}}(\tau)$ can be used to calculate the spectral function of electrons from the microscopic theory within a semiclassical approximation. The resulting spectral function varies as $e^{-[\Omega_0(\eta)/\omega]^{1/(2-\eta)}}$ at low energies.

The idea of composite fermions[1] has been used to explain the hierarchical structure of fractional quantum Hall (FQH) liquids. Using the fact that composite fermions in the filling fraction $\nu=\frac{1}{2}$ state feel zero magnetic field at the mean-field level and employing the fermionic Chern-Simons gauge theory of the FQH states,[2,3] Halperin, Lee, and Read[4] constructed a renormalized Fermi-liquid theory of the half-filled Landau level. As a direct consequence of a well-defined Fermi surface at the mean-field level, the integral quantum Hall effect of composite fermions can be viewed as an extreme form of the Shubnikov–de Haas effect.[4] Although several experiments[5–8] already appeared to support the existence of a well-defined Fermi surface, from the theoretical point of view, the strong gauge-field fluctuations and the resulting divergent effective mass of fermion reflect the difficulty of explaining the success of the mean-field theory.

It is well known that in (2+1)-dimensional compact Maxwell U(1) gauge theory or QED, the existence of the instanton solutions leads to the confinement of charges and significantly changes the infrared behavior of the theory.[9] This happens because instantons or monopoles effectively change the logarithmic interaction to a linear one.

Thus one may worry about the confinement of composite fermions in the compact gauge theory of the $\nu=\frac{1}{2}$ state. By calculating instanton–anti-instanton action, it is shown that instantons are confining in the gauge theory of the $\nu=\frac{1}{2}$ state so that well-defined composite fermions can exist. This problem is important because, if instantons were not confining, well-defined composite fermions would not exist and there would be no well-defined Fermi surface for composite fermions, which is necessary to explain experiments.

We also calculated one-electron Green's function of the $\nu=\frac{1}{2}$ state from the microscopic theory. Here we are in a completely different situation (compared to the usual case) that the electron operator not only creates a composite fermion but also creates flux quanta. Therefore we need to develop a method to calculate the correlation functions of electrons. It is found that, using the calculated instanton–anti-instanton action, one can compute the spectral function of electrons in a semiclassical approximation. The resulting spectral function shows a strong suppression at low energies which may explain a recent measurement of the low-temperature I-V tunneling characteristics of a double-layer FQH system near $\nu=\frac{1}{2}$.[10] It is worthwhile to mention that this highly suppressed spectral density in the infrared limit is not realized in the usual Fermi liquid. There are some numerical calculations of small size systems[11,12] and a phenomenological model based on the low-lying density fluctuations[11] successfully explained the experiment. However, the calculation presented here is a microscopic derivation which shows some deviations from the phenomenological construction.

In a recent paper (see also Refs. 13 and 14), Diamantini, Sodano, and Trugenberger[15] discussed the instanton effect in a (2+1)-dimensional compact U(1) gauge theory with the Chern-Simons term. It was found that the effect of the Chern-Simons term dominates the role of monopoles in the infrared limit so that the monopoles are linearly confining. Our problem is more delicate because there are also fermions in the theory and this fermionic degree of freedom generates particle-hole excitations across the Fermi surface which may affect the dynamics of the gauge field.[16,17] Recently, Nagaosa[17] investigated a dissipative U(1) gauge model that is a simplified version of the gauge theory of high-T_c superconductors. He found that the above-mentioned low-energy excitations give rise to a dissipative effect on the gauge field so that the confinement of charges is strongly suppressed.[17]

In the gauge theory of the $\nu=\frac{1}{2}$ state, both the low-energy particle-hole excitations and the Chern-Simons term exist. In this paper, it is shown that the effect of the low-energy particle-hole excitations dominates the effect of the Chern-Simons term for a class of interactions between electrons: $v(\mathbf{q})=V_0/q^\eta$ $(0\leq\eta<2)$. This leads to a confinement of instantons similar to that in the gauge theory of high-T_c superconductors.[17]

In the fermionic Chern-Simons gauge theory of FQH states,[2–4] the problem of interacting electrons in a uniform magnetic field can be transformed to an equivalent system in which a fermion is minimally coupled to a statistical gauge field $a_\mu(\mathbf{r})$ as well as the uniform magnetic field. The fermion operator $\psi^\dagger$ is related to the electron operator $\psi_e^\dagger$ (Refs. 2 and 4) as

$$\psi_e^\dagger=\psi^\dagger\exp\left[i\tilde{\phi}\int d^2r'\,\varphi(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')\right], \tag{1}$$

where $\varphi(\mathbf{r}-\mathbf{r}')$ is the angle between $\mathbf{r}-\mathbf{r}'$ and the x axis, and $\rho(\mathbf{r})$ is the electron or fermion density operator at the point $\mathbf{r}$. For fermionic theory, $\tilde{\phi}$ should be even integers and especially $\tilde{\phi}=2$ for the $\nu=\frac{1}{2}$ state. In this fermionic language, the Hamiltonian can be written as[4]

$$H=H_0+V,$$

$$H_0=\frac{1}{2m^*}\int d^2r\ \psi^\dagger(-i\nabla-\Delta\mathbf{a})^2\psi, \qquad (2)$$

$$V=\tfrac{1}{2}\int d^2r\, d^2r'\, v(\mathbf{r}-\mathbf{r}'):\rho(\mathbf{r})\rho(\mathbf{r}'):,$$

where the colons represent the normal ordering and m^* is the effective mass of the fermion. Here we assume that interaction between electrons takes a form (in Fourier space): $v(\mathbf{q})=V_0/q^\eta$ $(0\leq\eta<2)$. $\Delta\mathbf{a}(\mathbf{r})=\mathbf{a}(\mathbf{r})-e\mathbf{A}(\mathbf{r})$ is the fluctuation above the mean-field configuration $\mathbf{a}(\mathbf{r})=e\mathbf{A}(\mathbf{r})$ and $\mathbf{a}(\mathbf{r})=\tilde{\phi}\int d^2r'\,\nabla\varphi(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')$.

In the rest of the paper, $a_\mu(\mathbf{r})$ means the fluctuation above the mean-field configuration, i.e., $\Delta a_\mu(\mathbf{r})$. We will use the Euclidean functional-integral formalism and choose the temporal gauge in which $a_0=0$. The effective action of the gauge field can be obtained after integrating out the fermions in the original action. Since only the transverse fluctuation of the gauge field is important in the low-energy limit,[16,17] we will drop the longitudinal fluctuation from now on. It turns out that the gauge-field propagator is not renormalized by the fluctuations beyond the random-phase approximation.[18] Therefore, we can employ the same gauge-field fluctuation as that of Ref. 4. The effective gauge-field action can be written as the following:[16,17]

$$S_{\text{eff}}=S_0+S_{cs},$$

$$S_0=\frac{1}{2}\int\frac{d^2q}{(2\pi)^2}\frac{d\omega}{2\pi}[\varepsilon(\mathbf{q},\omega)e_\alpha(\mathbf{q},\omega)e_\alpha(-\mathbf{q},-\omega)$$
$$+\mu(\mathbf{q},\omega)b(\mathbf{q},\omega)b(-\mathbf{q},-\omega)], \qquad (3)$$

$$S_{cs}=-i\int d\tau\, d^2r\,\frac{m}{4}\,\epsilon_{\mu\nu\lambda}a_\mu f_{\nu\lambda},$$

where $e_\alpha=\partial_0 a_\alpha$ $(\alpha=1,2)$, $b=\partial_1 a_2-\partial_2 a_1$, $m=1/(2\pi\tilde{\phi})$, and $f_{\mu\nu}=\partial_\mu a_\nu-\partial_\nu a_\mu$. The dielectric function $\varepsilon(\mathbf{q},\omega)$ and the magnetic permeability $\mu(\mathbf{q},\omega)$ (for the statistical gauge field) are given by $\varepsilon(\mathbf{q},\omega)=\nu_1/|\omega|q$ $(\nu_1=2n_e/m^*v_F)$ and $\mu(\mathbf{q},\omega)=1/12\pi m^*+v(\mathbf{q})/(2\pi\tilde{\phi})^2$.

Before the calculation of instanton–anti-instanton action and the demonstration of confinement of instantons, we would like to show a relation between the electron Green's function and the instanton–anti-instanton action. The one-electron Green's function $G_+(\tau)=\langle\psi_e(\mathbf{0},\tau)\psi_e^\dagger(\mathbf{0},0)\rangle$ can be calculated semiclassically in the spirit of the WKB approximation.[19] In the functional integral approach, $G_+(\tau)$ can be written as

$$G_+(\tau)=\int D\psi^\dagger D\psi Da_\mu\psi_e(\tau)\psi_e^\dagger(0)e^{-S(\psi^\dagger,\psi,a_\mu)}$$
$$=\int D\psi^\dagger D\psi Da_\mu\psi(\tau)\psi^\dagger(0)\,\delta(M\bar{M})e^{-S(\psi^\dagger,\psi,a_\mu)}, \qquad (4)$$

where $S(\psi^\dagger,\psi,a_\mu)$ is the action given by Eq. (2). Notice that, since an electron is a fermion plus two flux quanta, the creation and annihilation of electrons at times zero and τ not only create and annihilate a fermion, they also create and annihilate flux quanta. The creation and annihilation of flux quanta is represented by a singular boundary condition on the gauge field and $\delta(M\bar{M})$ represents this boundary condition. Notice that the creation (annihilation) of a flux quantum corresponds to inserting a monopole (antimonopole) in space-time. Formally, Eq. (4) can be written as

$$G_+(\tau)=\int Da_\mu\delta(M\bar{M})\langle\psi(\tau)\psi^\dagger(0)\rangle_a e^{-S_{\text{eff}}(a_\mu)}, \qquad (5)$$

where

$$\langle\psi(\tau)\psi^\dagger(0)\rangle_a=\int D\psi^\dagger D\psi\psi(\tau)\psi^\dagger(0)e^{-S(\psi^\dagger,\psi,a_\mu)}/e^{-S_{\text{eff}}(a_\mu)},$$
$$e^{-S_{\text{eff}}(a_\mu)}=\int D\psi^\dagger D\psi e^{-S(\psi^\dagger,\psi,a_\mu)}. \qquad (6)$$

Notice that both $\langle\psi(\tau)\psi^\dagger(0)\rangle_a$ and $e^{-S_{\text{eff}}(a_\mu)}$ are not gauge invariant. Let us introduce $S_{\text{eff}}(a_\mu,j_\mu)=S_{\text{eff}}(a_\mu)-\int d^3r\,a_\mu j_\mu$, where j_μ is the fermion current corresponding to the straight-line path and has the following form:

$$j_\mu=[\theta(x_0-\tau)-\theta(x_0)]\delta(x_1)\delta(x_2)\delta_{\mu0}. \qquad (7)$$

Now we can write the integrand of the functional integral as a product of two gauge-invariant objects:

$$G_+(\tau)=\int Da_\mu\delta(M\bar{M})\Big[\langle\psi(\tau)\psi^\dagger(0)\rangle_a$$
$$\times\exp\Big(-\int d^3r\,a_\mu j_\mu\Big)\Big]e^{-S_{\text{eff}}(a_\mu,j_\mu)}, \qquad (8)$$

where $\langle\psi(\tau)\psi^\dagger(0)\rangle_a\exp(-\int d^3r\,a_\mu j_\mu)$ and $e^{-S_{\text{eff}}(a_\mu,j_\mu)}$ are gauge invariant. Notice that the two terms in $S_{\text{eff}}(a_\mu,j_\mu)$ are not gauge invariant, respectively, due to the presence of the monopoles for the first term and nonconservation of the current j_μ for the second term (an electron is created and annihilated). However, the total effective action $S_{\text{eff}}(a_\mu,j_\mu)$ is gauge invariant. Notice also that, in the semiclassical limit, the paths of the fermions are close to the straight-line path in a given gauge-field background, thus the factor $\exp(-\int d^3r\,a_\mu j_\mu)$ is almost compensated by the contribution from the fermions $\langle\psi(\tau)\psi^\dagger(0)\rangle_a$. Therefore, the saddle point of the integrand is dominated by $e^{-S_{\text{eff}}(a_\mu,j_\mu)}$. By taking out the saddle-point value $S_{\text{eff}}(\bar{a}_\mu,j_\mu)$ in which the boundary condition $\delta(M\bar{M})$ should be incorporated, one can do the following semiclassical approximation:

$$G_+(\tau)\approx e^{-S_{\text{eff}}(\bar{a}_\mu,j_\mu)}\int D\delta a_\mu\langle\psi(\tau)\psi^\dagger(0)\rangle_a$$
$$\times\exp\Big(-\int d^3r\,a_\mu j_\mu\Big)e^{-S_{\text{eff}}(\delta a_\mu)}, \qquad (9)$$

where δa_μ is the fluctuation around the saddle point and $S_{\text{eff}}(\delta a_\mu)$ can be taken as a quadratic expansion in δa_μ. Combined with the boundary condition on the gauge field, j_μ of Eq. (7) is exactly the source of the instanton–anti-instanton (or monopole-antimonopole) solution of the effective gauge-field action.[15] In other words, the monopole and antimonopole are connected by a string of source j_μ in Euclidean space. From these arguments, we can identify

$S_{\rm eff}(\bar{a}_\mu, j_\mu)$ as the monopole-antimonopole action $S_{M\bar{M}}$. Therefore, the electron Green's function can be written as

$$G_+(\tau) \approx G_0(\tau) e^{-S_{M\bar{M}}(\tau)}, \tag{10}$$

where $G_0(\tau)$ is at most an algebraically decaying function of τ (Ref. 20) because of the above-mentioned compensation effect in the semiclassical approximation. It will be shown that $e^{-S_{M\bar{M}}(\tau)}$ is the dominant suppression factor of low-energy electron spectral function.

Now let us concentrate on the evaluation of $S_{M\bar{M}}(\tau)$ in our model. We will use the same idea of Ref. 15 to calculate the monopole-antimonopole action from an equivalent self-dual model.

First of all, the equations of motion that are derived from the action given in Eq. (3) are found to be

$$\begin{aligned} &\varepsilon(\mathbf{q},\omega) q_\alpha e_\alpha(\mathbf{q},\omega) + m b(\mathbf{q},\omega) = 0, \\ &\varepsilon(\mathbf{q},\omega)\omega \epsilon_{\alpha\beta} e_\beta(\mathbf{q},\omega) + \mu(\mathbf{q},\omega) q_\alpha b(\mathbf{q},\omega) - m e_\alpha(\mathbf{q},\omega) = 0, \end{aligned} \tag{11}$$

where $\alpha=1,2$. Let us define f_μ as the dual of the field strength tensor $f_{\mu\nu}$: $f_\mu = \epsilon_{\mu\nu\lambda} f_{\nu\lambda}/2$.[15,21] The Euclidean partition function of an equivalent dual theory can be written as

$$Z = \int Df_\mu Df_\mu^* e^{-S_E(f_\mu, f_\mu^*)},$$

$$\begin{aligned} S_E(f_\mu, f_\mu^*) = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3} \bigg[&\mu(\mathbf{q},\omega) f_0^* f_0 - \frac{1}{m}\mu(\mathbf{q},\omega) f_0^* \varepsilon(\mathbf{q},\omega)(q_1 f_2 - q_2 f_1) + \varepsilon(\mathbf{q},\omega) f_1^* f_1 - \frac{1}{m}\varepsilon(\mathbf{q},\omega) f_1^*[\mu(\mathbf{q},\omega) q_2 f_0 \\ &- \varepsilon(\mathbf{q},\omega)\omega f_2] + \varepsilon(\mathbf{q},\omega) f_2^* f_2 - \frac{1}{m}\varepsilon(\mathbf{q},\omega) f_2^*[\varepsilon(\mathbf{q},\omega)\omega f_1 - \mu(\mathbf{q},\omega) q_1 f_0] \bigg], \end{aligned} \tag{12}$$

where $f_\mu^*(\mathbf{q},\omega) = f_\mu(-\mathbf{q},-\omega)$. One can easily check that the above action gives the same equations of motion as Eq. (11). In the lattice version of the action, as a result of the appropriate regularization, we can separate out singularities from f_μ.[15] Now we can define the regularized dual field strength tensor $f_\mu^{\rm reg}$ as $f_\mu = f_\mu^{\rm reg} - (1/m) j_\mu$, where the string singularity j_μ is given by Eq. (7). This singularity acts as a source for $f_\mu^{\rm reg}$ (Ref. 15) and the corresponding equations of motion for $f_\mu^{\rm reg}$ can be written as

$$\begin{aligned} &f_0^{\rm reg} - \frac{1}{m}\varepsilon(\mathbf{q},\omega)(q_1 f_2^{\rm reg} - q_2 f_1^{\rm reg}) = \frac{1}{m} j_0, \\ &f_1^{\rm reg} - \frac{1}{m}[\mu(\mathbf{q},\omega) q_2 f_0^{\rm reg} - \varepsilon(\mathbf{q},\omega)\omega f_2^{\rm reg}] = 0, \\ &f_2^{\rm reg} - \frac{1}{m}[\varepsilon(\mathbf{q},\omega)\omega f_1^{\rm reg} - \mu(\mathbf{q},\omega) q_1 f_0^{\rm reg}] = 0. \end{aligned} \tag{13}$$

Inverting these equations, we can get the following solutions:

$$f_\mu^{\rm reg}(\mathbf{q},\omega) = \frac{1}{m} G_{\mu\nu}(\mathbf{q},\omega) j_\nu(\mathbf{q},\omega), \tag{14}$$

where

$$G_{\mu\nu}(\mathbf{q},\omega) = (A^{-1})_{\mu\nu}(\mathbf{q},\omega),$$

$$A_{\mu\nu}(\mathbf{q},\omega) = \begin{pmatrix} 1 & q_2\varepsilon(\mathbf{q},\omega)/m & -q_1\varepsilon(\mathbf{q},\omega)/m \\ -q_2\mu(\mathbf{q},\omega)/m & 1 & \omega\varepsilon(\mathbf{q},\omega)/m \\ q_1\mu(\mathbf{q},\omega)/m & -\omega\varepsilon(\mathbf{q},\omega)/m & 1 \end{pmatrix}. \tag{15}$$

Equation (14) represents the monopole-antimonopole solution of the effective gauge theory.

The monopole-antimonopole action can be obtained from Eqs. (12) and (14),[15]

$$\begin{aligned} S_{M\bar{M}}(\tau) &= \frac{1}{2}\int \frac{d^3q}{(2\pi)^3}\frac{1}{m^2} j_0(-\mathbf{q},-\omega) \times \mu(\mathbf{q},\omega) G_{00}(\mathbf{q},\omega) j_0(\mathbf{q},\omega) \\ &= \frac{1}{2}\int \frac{d^3q}{(2\pi)^3}\frac{1}{m^2} j_0(-\mathbf{q},-\omega) \times \frac{[m^2 + \varepsilon^2(\mathbf{q},\omega)\omega^2]\mu(\mathbf{q},\omega)}{m^2 + \varepsilon(\mathbf{q},\omega)[\varepsilon(\mathbf{q},\omega)\omega^2 + \mu(\mathbf{q},\omega) q^2]} j_0(\mathbf{q},\omega). \end{aligned} \tag{16}$$

Notice that the appearance of m^2 in the fractional expression reflects the screening effect of the Chern-Simons term. In the Maxwell-Chern-Simons theory, $\varepsilon = \mu = 1$ so that the m^2 term dominates in the infrared limit and this screening effect confines the monopole-antimonopole pairs.[15] However, in our model, ε and μ are divergent (μ is divergent for $\eta > 0$) in the infrared limit so that the m^2 term becomes irrelevant. Therefore, the contributions from the dielectric function and the magnetic permeability due to the particle-hole excitations dominate the screening effect of the Chern-Simons term. Now we can safely set $m^2 = 0$ in the numerator and the denominator of the fractional expression in Eq. (16); then $S_{M\bar{M}}(\tau)$ can be written as

$$S_{M\bar{M}}(\tau) = \int \frac{d^2q}{(2\pi)^2}\int \frac{d\omega}{2\pi}\frac{(2\pi\tilde{\phi})^2}{\omega^2}\frac{\varepsilon(\mathbf{q},\omega)\mu(\mathbf{q},\omega)\omega^2}{\varepsilon(\mathbf{q},\omega)\omega^2 + \mu(\mathbf{q},\omega) q^2} \times [1 - \cos(\omega\tau)], \tag{17}$$

where $\tilde{\phi} = 2$ for $\nu = \frac{1}{2}$. The above result can be easily understood once we realize that the Chern-Simons term is irrelevant in the infrared limit. After dropping the Chern-Simons term, the effective action $S_{\rm eff}(a_\mu)$ [see Eq. (3)] is essentially the Maxwell theory with frequency and momentum-dependent dielectric function $\varepsilon(\mathbf{q},\omega)$ and magnetic permeability $\mu(\mathbf{q},\omega)$. (17) is just the monopole-antimonopole action in this generalized Maxwell theory.[16]

For the Coulomb interaction $v(\mathbf{q})=2\pi e^2/\epsilon q$, μ can be approximated as $[2\pi e^2/\epsilon(2\pi\tilde{\phi})^2](1/q)$:

$$S_{M\bar{M}}(\tau)=\frac{e^2}{2\pi\epsilon}\sqrt{\tau/\beta}\int_0^\infty dx\int_0^\infty dy\,\frac{1}{xy^{3/2}}\,\frac{1}{1+x}[1-\cos(xy)]$$
$$\equiv 2\sqrt{\Omega_0(1)\tau}, \qquad (18)$$

where $\beta=e^2 l_c/4\epsilon$ (l_c is the magnetic length) (Ref. 11) and $\Omega_0(1)=\pi e^2/\epsilon l_c$. Therefore, the monopole-antimonopole pair is confining but the action is proportional to the square root of the distance between monopole and antimonopole which is different from the linearly confining monopole-antimonopole solution of the Maxwell-Chern-Simons theory. The confinement of monopoles means the existence of a well-defined "charged" particle or composite fermion.

The same calculation can be done for a class of interactions $v(\mathbf{q})=V_0/q^\eta$ $(0\leq\eta<2)$. Using the fact that $\mu\approx[V_0/(2\pi\tilde{\phi})^2](1/q^\eta)$ $(\eta>0)$ and $\mu=1/12\pi m^*+V_0/(2\pi\tilde{\phi})^2$ $(\eta=0)$, we get $S_{M\bar{M}}(\tau)=(3-\eta)[\Omega_0(\eta)\tau]^{1/(3-\eta)}$ with

$$\Omega_0(\eta\neq 0)=\frac{V_0 l_c}{4\pi}\left[\frac{2\pi}{(3-\eta)^2 l_c}\,\frac{1}{\Gamma\left(\dfrac{4-\eta}{3-\eta}\right)}\times\csc\left(\frac{\pi}{2(3-\eta)}\right)\csc\left(\frac{\pi}{3-\eta}\right)\right]^{3-\eta}, \qquad (19)$$

$$\Omega_0(\eta=0)=\frac{2^{11}\pi^4}{3^{17/2}\Gamma^3(\frac{4}{3})}\,\frac{\tilde{\chi}}{l_c^2},$$

where $\tilde{\chi}=1/12\pi m^*+V_0/(2\pi\tilde{\phi})^2$ is the effective diamagnetic susceptibility of the fermions. Therefore, the monopole-antimonopole pair is still confining.

From (10) and (19), we can see that the electron Green's function has a form $G_+(\tau)\approx G_0(\tau)\exp\{(3-\eta)[\Omega_0(\eta)\tau]^{1/(3-\eta)}\}$. After the relatively unimportant factor G_0 is dropped, it has the same functional form as the result of He, Platzman, and Halperin[11] in the case of the Coulomb interaction ($\eta=1$). Notice that our $\Omega_0(1)$ is two times larger than ω_0 they obtained in a similar expression.[11] The low-frequency behavior of the corresponding spectral function $A_+(\omega)$, which is the inverse Laplace transform of G_+,[11] is given by $\exp\{-[\Omega_0(\eta)/\omega]^{1/(2-\eta)}\}$. It was pointed out that the exponential suppression of the spectral density leads to the strong suppression of the tunneling current at low voltage biases.[11] These results show that the one-electron Green's function has very different behavior compared to that in the usual Fermi liquid although two-particle correlation functions may be Fermi-liquid-like.[18,22]

Recently, Bonesteel[23] extended the analysis of Ref. 4 to the double-layer system near $\nu=\frac{1}{2}$. It was found that the dynamics of the gauge-field fluctuations in two layers separates into out-of-phase mode and in-phase mode between two layers. The out-of-phase mode behaves as if there is no Coulomb interaction.[23] The tunneling between two layers corresponds to the creation of a monopole in one layer and an antimonopole in the other layer which only couple to the out-of-phase mode of the gauge field. Thus the tunneling current will be directly proportional to $e^{-S_{M\bar{M}}(\tau)}$ with $\eta=0$ (for short-range interaction). Replacing $\tilde{\chi}$ by the appropriate effective diamagnetic susceptibility[23] of out-of-phase current fluctuations, we get $I(V)\sim e^{-[8\Omega_0(0)/eV]^{1/2}}$ where the factor 8 comes from the existence of two layers. We expect the above to be valid at low biases where the interlayer screening becomes important.

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