Multipole edge plasmons of two-dimensional electron-gas systems

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For a two-dimensional electron gas (2DEG) with a nonabrupt edge electron-density profile, edge modes, analogous to the higher-multipole surface modes of a 3DEG, are found in addition to the regular edge plasmon. Several simple model edge density profiles are investigated. For a linear edge profile the integral equation for the scalar potential is solved by expanding in a complete orthonormal set of functions. The resulting secular equation is truncated and solved numerically. For a simple double-step edge profile, which can be solved analytically, the multipole edge modes can be understood in a simple intuitive way.

In addition to the regular surface plasmons,[1] an electron gas with a diffuse surface-density profile can support higher-multipole surface-plasmon modes.[2] These modes can be thought of as bulk plasmons of the surface layer[3] whose spatially varying density is everywhere lower than the bulk electron density. The name higher multipole is appropriate for these modes because, in contrast to the regular surface plasmon, the integral (along the normal to the surface) of their charge density vanishes for every point on the surface. A number of theoretical approximations including hydrodynamic models,[2] the random-phase approximation,[4] and the time-dependent local-density approximations[5] were used to study higher-multipole modes before they were observed experimentally by Tsuei *et al.*[6] on K and Na surfaces.

Necessary conditions for the existence of higher-multipole modes include both a spatially varying electron density and dispersion of the bulk plasmon.[7] For a three-dimensional electron gas (3DEG) a nonlocal conductivity is required for dispersion of the bulk plasmon. Whether higher-multipole modes were an artifact of oversimplified treatment of nonlocal effects was a subject of some controversy. For a 2DEG the bulk-plasmon frequency is proportional to the square root of the wave vector q even in a simple local theory of conduction. The reason for this is that the electric field of a charged fluctuation on a 2D plane spreads into the third dimension, resulting in a restoring force (the in-plane component of **E**), which decreases with the increasing wavelength. Regular edge plasmons were first studied by Mast, Dahm, and Fetter[8] and by Glattli *et al.*[9] In the long-wavelength limit these modes occur at a frequency approximately 10% smaller than the frequency of a bulk plasmon of the same wavelength. There have been a number of studies[8–14] of regular magnetoplasma edge modes of a 2DEG and of a layered electron gas. However, until now[15] there has been no investigation of possible higher-multipole edge modes of a 2DEG with a nonabrupt edge profile.

Let us consider a self-sustaining density fluctuation $\delta n(\mathbf{r},t)$ of the form $\delta n(x)\exp(iqy-i\omega t)$ of a 2DEG from its equilibrium density $n(x)$. In a simple local approximation, the physics of the problem is contained in three basic equations which must be solved self-consistently. These equations are the Poisson's equation

$$\nabla\cdot[\epsilon\mathbf{E}(\mathbf{r})]=4\pi\delta n(\mathbf{r}) , \tag{1}$$

the equation of continuity

$$\nabla\cdot\mathbf{j}(\mathbf{r})=i\omega\delta n(\mathbf{r}) , \tag{2}$$

and the local equation relating the current density $\mathbf{j}$ to the electric field $\mathbf{E}$,

$$\mathbf{j}(\mathbf{r})=\hat{\sigma}(\mathbf{r})\cdot\mathbf{E}(\mathbf{r}) . \tag{3}$$

In these equations ϵ and $\hat{\sigma}$ are the background dielectric constant and the conductivity tensor. To simplify the treatment we assume a uniform dielectric constant ϵ.

In the absence of a magnetic field the conductivity tensor is diagonal and given by $\sigma(x)=ie^2n(x)/m\omega$ in the absence of collisions. In the electrostatic limit the electric field can be expressed as the gradient of scalar potential $\phi(\mathbf{r})$. By combining Eqs. (1)–(3), we can arrive at a self-consistent integral equation for the potential $\phi(x)$,

$$\phi(x)=\frac{4\pi e^2}{m\epsilon\omega^2}\int dx' L_q(x-x')\left\{n(x')\left[q^2-\frac{d^2}{d(x')^2}\right]-\frac{dn(x')}{dx'}\frac{d}{dx'}\right\}\phi(x') , \tag{4}$$

where the integration kernel L_q is given by

$$L_q(x)=\frac{1}{4\pi}\int dp\frac{\exp(ipx)}{\sqrt{p^2+q^2}} . \tag{5}$$

The nontrivial solutions of this equation correspond to the collective modes of the electron system. For an arbitrary electron density $n(x)$, Eq. (4) cannot be solved analytically. To facilitate numerical calculations, we transform the integral equation into a matrix equation by expanding the potential $\phi(x)$ in Laguerre polynomials,

$$\phi(x)=\exp(qx)\sum_{n=0}^{\infty} c_n L_n(-2qx) \ . \tag{6}$$

We find that if the electron-density profile near the edge takes the form of a polynomial in x, the computation is simplified considerably. The plasmon-dispersion relations are obtained by requiring that nontrivial solutions exist for the matrix equation.

For the purpose of illustration we choose a model in which the electron density decreases linearly in the edge region, i.e.,

$$n(x)=\begin{cases} 0 \ \text{ if } x>0 \\ -n_0 x/a \ \text{ if } 0>x>-a \\ n_0 \ \text{ if } -a>x \ . \end{cases} \tag{7}$$

The plasmon dispersion is calculated by truncating the matrix at a finite order N. We find that the numerical results converge quickly with increasing N, and that choosing $N=18$ gives the desired numerical accuracy.

In Fig. 1, we plot the plasmon dispersion for the electron-density profile in the form of Eq. (7). It is important to point out that the figure remains the same for any value of a, the width of the edge layer, when the unit of the frequency is taken as $\Omega=(4\pi n_s e^2/\epsilon m a)^{1/2}$. Therefore we can choose $a>5a_0$, where a_0 is the effective Bohr radius, so that the nonlocal effect can be neglected when $qa\sim 1$. For small values of q, the system can support only one mode. This is the monopole edge plasmon mode first discussed in Refs. 8 and 9. At a value of q of the order of $0.8a^{-1}$ another mode appears, and at $q\simeq 1.5a^{-1}$ a third mode appears below the bulk-plasma frequency. These are the dipole and quadrupole modes, respectively. As the value of q is increased additional modes appear below the bulk-plasma frequency, but nonlocal effects in the conductivity become important for $qa_0\sim 1$. It is a universal feature of 2D systems that these multipole modes exist as stable edge excitations only at a finite value of q. In contrast, in 3D systems higher-multipole surface modes occur even in the limit of infinite wavelength. As the edge density profile becomes more diffuse, the values of q at which higher multipoles first appear become smaller.

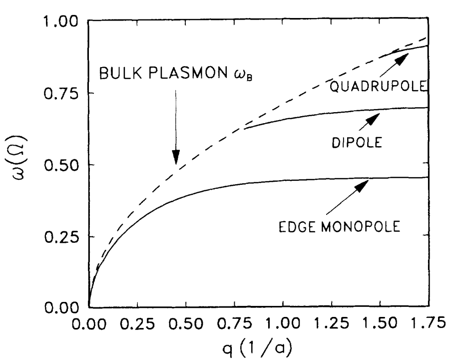

FIG. 1. Dispersion of edge plasmon modes for an electron-density profile decreasing linearly near the edge [see Eq. (7)]. Here a is the width of the edge layer and $\Omega=(4\pi n_s e^2/\epsilon m a)^{1/2}$. The 2D bulk-plasmon dispersion with an electron density n_0 is shown as a dashed line.

It is clear from Fig. 1 that the edge profile plays an essential role in determining the frequency of the new edge modes. To illustrate the physics of these new modes associated with the edge density profile, we study a simple situation in which the edge profile takes the form of a double-step function,

$$n(x)=n_B\Theta(-x-a)+n_S\Theta(-x)\Theta(x+a) \ , \tag{8}$$

where Θ is the single-step function. If we replace L_q in Eq. (3) by an approximate kernel $L_q=2^{-3/2}\exp(-\sqrt{2}q|x|)$, as was done in Ref. 8, Eq. (4) can be solved analytically by using the Wiener-Hopf technique. With the requirement that the potential be continuous at both the boundaries $x=0$ and $-a$, the plasmon modes are found to be solutions of the equation

$$(\alpha_S-\alpha_B)\cosh(\gamma_S qa)+(\alpha_S\alpha_B-1)\sinh(\gamma_S qa)=0 \ , \tag{9}$$

where $\gamma_\nu^2=2(\omega_\nu^2-\omega^2)/(2\omega_\nu^2-\omega^2)$ with $\nu=B$ or S, $\alpha_B=\gamma_B(2\omega_B-\omega^2)/\gamma_S(2\omega_S^2-\omega^2)$, $\alpha_S=\sqrt{2}\omega^2/\gamma_S(2\omega_S^2-\omega^2)$, and $\omega_\nu=2\pi n_\nu e^2 q/\epsilon m$ is the bulk-2D-plasmon frequency for a system with density n_ν.

In the limiting case of $n_S\to n_B$ or $a\to 0$, the system becomes the single-step model studied in Refs. 8 and 9. In that case Eq. (9) reduces to the correct approximate result $\omega=\sqrt{2/3}\omega_B$ for the regular edge plasmon derived previously. For $n_B\gg n_S$ and $qa\gg 1$ the dispersion relation for the higher-multipole edge modes can be expressed as

$$\omega_n^2\simeq\frac{2\pi n_S e^2}{\epsilon m}\sqrt{q^2+(n\pi/a)^2} \ , \tag{10}$$

where $n=1,2,3,\ldots$. From Eq. (10) it is clear that the higher-multipole edge modes are essentially bulk 2D plasmons of the edge region with an effective wave number $q_{\text{eff}}=\sqrt{q^2+(n\pi/a)^2}$. Physically these modes are standing-wave plasmons with an integer number of half-wavelengths in the x direction fitting into the low-electron-density edge region. For $\omega_n(q)$ smaller than ω_B the nth higher multipole cannot propagate into the region $x<-a$ and is therefore trapped in the low-density edge region.

It is interesting to note that as $q\to 0$ the solution of Eq. (9) approaches the limit $\omega=\sqrt{\frac{2}{3}}\omega_B$, as if the electron-density profile played no role in determining the frequency of the regular edge plasmon. The reason for this is that the electric field associated with the edge monopole penetrates a distance of the order of q^{-1} into the bulk. When the distance is very large compared to a, the small edge region where the electron density varies from its bulk value does not significantly affect the restoring force or the regular edge plasmon frequency. The same effect occurs for the 3D surface monopole plasmon; its frequency is $\omega=\Omega_0/\sqrt{2}$ in the long-wavelength limit independent of the density profile.

In Fig. 2, we show the plasmon dispersion calculated from Eq. (9) for the double-step density profile with $a>5a_0$ and $n_S/n_B=0.6$. Similar to the situation in Fig. 1, for small values of q, only the monopole edge mode ex-

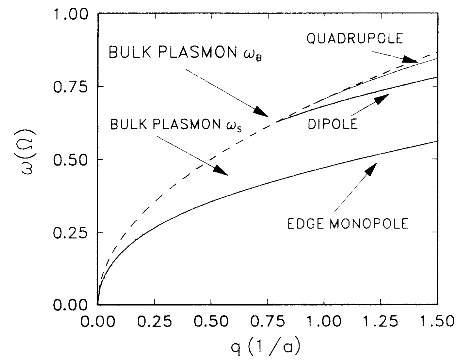

FIG. 2. Dispersion of edge plasmon modes for a double electron-density profile [see Eq. (8)] obtained with the approximate kernel. Here a is the width of the edge layer, $n_S/n_B=0.6$, and $\Omega=(4\pi n_s e^2/\epsilon m a)^{1/2}$. The 2D bulk plasmons with an electron density n_B and n_S are also shown as dashed and dotted lines, respectively.

ists. Its frequency is given by $\omega=(\frac{2}{3})^{1/2}\omega_B(q)$ for very small values of q, slightly above the bulk-plasma frequency $\omega_S(q)=(0.6)^{1/2}\omega_B(q)$ of the edge region whose density is 0.6 of n_B. As the value of q increases higher multipole modes appears at $q\sim 0.8a^{-1}$ and $1.0a^{-1}$. With larger values of a, multipole modes appear at smaller values of q. All the multipole modes approach the bulk plasmon of the low-density region as q becomes very large.

It appears feasible to prepare samples of semiconductor quantum-well structures, which simulate 2D electron-gas systems with controllable electron-density profiles near the edge.[16] Such systems provide a realistic opportunity to experimentally study multipole edge plasmons.

In summary, we have studied the edge excitations arising from a nonabrupt electron-density profile near the edge of a 2DEG system. We find a sequence of higher-multipole edge modes, which, unlike their 3D counterparts, exist as well-defined edge excitations only at finite values of q, the wave number along the edge.

In the presence of a magnetic field **B** perpendicular to the 2D layer, a series of low-frequency edge modes[17] with $\omega \propto q$ is found for a smooth profile and a single localized interedge[18] mode is found for a two-step profile, in addition to the high-frequency higher-multipole modes. In both cases these low-frequency modes propagate in only one direction along the edge for a given direction of **B**. The effect of an applied magnetic field and of more realistic density profiles will be discussed in a separate publication.

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