Degenerate Landau bands with interband disorder: A semiclassical picture

D.K.K. Lee

Department of Physics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139

(Received 18 April 1994)

Localization in a pair of degenerate Landau levels, coupled by random interlevel matrix elements, is discussed in a semiclassical picture appropriate to the case of smooth disorder. It is shown that there are two distinct energies on either side of the band center where delocalization occurs. Each transition belongs to the same universality class as that of a nondegenerate Landau level in a random potential. Using a simple physical picture for the eigenstates, I will further argue that, unique to this model, the localization length also diverges at the band center, in accordance with recent results of Hikami, Shirai, and Wegner for white-noise disorder [Nucl. Phys. B **408**, 415 (1993)].

There has been recent interest in the delocalization transition in integer quantum Hall systems where the samples are sufficiently disordered that the broadening of the orbital Landau levels exceeds the Zeeman energy and the levels become spin unresolved. It has been suggested that this transition belongs to a different universality class from the transition in spin-split systems. It is therefore of theoretical interest to study the effect of the mixing of degenerate Landau levels on the localization behavior of the system. In this paper, I study the simplest of such models[1,2] and develop a simple physical picture for the eigenstates of the system in the limit of smooth disorder. I will show how the universality class found in Ref. 1 is special to this model.

The integer quantum Hall effect is modeled, in the simplest case, as spin-split Landau levels in the presence of a random potential. This system represents the best characterized example to date of critical behavior at a metal-insulator transition. The dependence on energy E of the localization properties of the eigenstates has been discussed in the context of a scaling theory.[3,4] Within each disorder-broadened Landau level, almost all eigenstates are Anderson localized, but the localization length $\xi(E)$ diverges at a critical energy E_c (which lies at the band center in the case of a symmetric potential): $\xi(E) \sim |E - E_c|^{-\nu}$. This scaling behavior has indeed been observed as this zero-temperature critical point is approached, as functions of temperature, sample size, and frequency,[5] yielding a value of $\nu = 2.3 \pm 0.1$. This value for the critical exponent has also been obtained from numerical simulations, using a variety of models and techniques.[6–8] The possible existence of a universality class for spin-unresolved Landau levels has been suggested by experimental studies which, assuming that the localization length diverges only at one energy, have obtained a larger value for critical exponent: $\nu \simeq 4.8$.[9,10] However, recent theoretical work[11,12,2] on the "spin-degenerate" limit, where the Zeeman energy vanishes, suggests the existence of two delocalized energies, in contrast to the scaling diagram proposed in Ref. 10.

Hikami, Shirai, and Wegner[1] recently introduced a model of two degenerate Landau levels with random interlevel transitions (but without potential disorder) and studied the limit of vanishing correlation length for the disorder. They obtained a finite longitudinal conductance ($\sigma_{xx} = e^2/\pi h$) at the band center and suggested that this signified a delocalization transition belonging to a universality class different from that of a spin-split Landau level. I will argue that this delocalization at the band center can also be understood in the opposite limit of smooth disorder where the random matrix elements are correlated over distances large compared to the magnetic length. Experience with the spin-split Landau level has indicated that the question of the existence of a delocalization transition (and the associated critical behavior) does not depend on the details of the microscopic Hamiltonian, such as the correlation length of the disorder.[6–8]

Using a semiclassical picture appropriate to the limit of smooth disorder, I will show that the states near the band center have significant amplitudes only around a set of isolated *points* in space where the interband coupling vanishes. Although one might initially expect such states to be typically localized, it will be argued that the delocalization transition of Hikami *et al.* is possible due to an exact reflection symmetry of the spectrum which ensures significant hybridization among these states. I will also show that there are two other delocalization transitions on either side of the band center in the same universality class as that found in spin-split Landau levels (Fig. 1), similar to the results of Refs. 11, 12, and 2 for the spin-degenerate Landau levels.

To be precise, I consider the motion of a particle in a plane in the presence of a strong uniform magnetic field B perpendicular to the plane. This particle has an internal degree of freedom which can be regarded as an $S = 1/2$ pseudospin. Disorder is incorporated as local scattering between the two degenerate Landau levels but not within each level. In the pseudospin language, the pseudospin has a Zeeman coupling to a random external pseudofield in the xy plane: $\mathbf{h}(\mathbf{r}) = (h_x(\mathbf{r}), h_y(\mathbf{r}))$. I will use the magnetic length $l_B = (\hbar/eB)^{1/2}$ as the unit of length and the cyclotron energy $\hbar\omega_c = e\hbar B/m$ as the unit of energy. The Hamiltonian can be written as

$$H = \tfrac{1}{2}(-i\boldsymbol{\nabla} + \mathbf{A})^2 + \mathbf{h}(\mathbf{r}) \cdot \boldsymbol{\sigma} , \qquad (1)$$

where $\mathbf{A}$ is the vector potential due to the uniform field

0163-1829/94/50(11)/7743(4)/$06.00

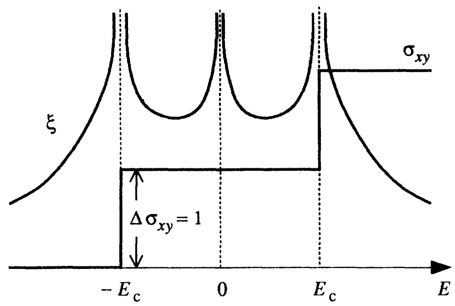

FIG. 1. Schematic diagram of the energy dependence of the localization length ξ and the Hall conductance σ_{xy} (in units of e^2/h). Energy is measured from the center of the Landau band.

B, and $\sigma_{x,y}$ are the Pauli matrices. The random pseudofield has a Gaussian distribution with

$$\overline{h_{x,y}(\mathbf{r})} = 0 \ ,$$
$$\overline{h_x(\mathbf{r})h_x(\mathbf{r}')} = \overline{h_y(\mathbf{r})h_y(\mathbf{r}')} = \Delta^2 \, e^{-|\mathbf{r}-\mathbf{r}'|/\lambda} \ . \tag{2}$$

I consider here the "semiclassical" limit, defined as the case when the correlation length λ is much larger than the magnetic length l_B. I will also require that the disorder is weak ($\Delta \ll \hbar\omega_c$) so that the mixing of orbital Landau levels can be ignored.

The XY coupling to the pseudofield has the property that it changes sign under the unitary transformation which transforms the spinor ψ to $\sigma_z\psi$ (since $\sigma_z\sigma_{x,y}\sigma_z = -\sigma_{x,y}$). It will be shown explicitly [see Eq. (5)] that this gives rise to an important symmetry of the spectrum after the projection onto a given orbital Landau level: if one can find an eigenstate $(\psi_\uparrow, \psi_\downarrow)$ at an energy of $(n+\frac{1}{2})\hbar\omega_c+E$, then $(\psi_\uparrow, -\psi_\downarrow)$ is also an eigenstate of the system with an energy of $(n+\frac{1}{2})\hbar\omega_c - E$. This reflection symmetry in the spectrum is important to the description of the states near the center of each Landau band.

I will now argue that this system can be understood to a large extent, using arguments borrowed from the semiclassical theory of a nondegenerate orbital Landau level with scalar disorder.[13–15] In the latter system, electron motion can be separated into two components: a rapid cyclotron orbit and a slow drift of the guiding center along equipotential lines. Delocalization coincides with the percolation of the guiding-center trajectory. In the present problem, one might expect that the pseudospin of the particle would follow the smoothly varying local field $\mathbf{h}(\mathbf{r})$, so that the particle would drift along contours of constant Zeeman energy. The energy of the corresponding eigenstate, measured from the unperturbed Landau level, should be proportional to $\pm|\mathbf{h}|$. (Note that this picture respects the reflection symmetry in the spectrum.) The analogy of the semiclassical picture of the present system with that of the Landau level in scalar disorder suggests that the two systems possess similar localization properties. In particular, I argue that, near the delocalization transition, both systems can be mapped onto the network model of Chalker and Coddington.[6] Hence, the lowest Landau level should have delocalization transitions at a pair of energies $E_c = \pm h_c$ (as measured from the unperturbed Landau level), when the electron trajectories on the contour $|\mathbf{h}| = h_c \sim O(\Delta)$ percolate through the system. This transition should be accompanied by a change in the Hall conductance of $\Delta\sigma_{xy} = e^2/h$, and belongs to the same universality class as that found in a spin-split Landau level with a localization length exponent of $\nu \simeq 2.3$.

This simple picture of trajectories on contours of $|\mathbf{h}|$ ignores the precession of the pseudospin around the local pseudofield which should affect the quantization of the electron orbit. I argue here that this is justified in the limit of smooth disorder. Formally, one can write, as in Ref. 2, $\mathbf{h} = |\mathbf{h}|\eta^\dagger\boldsymbol{\sigma}\eta$, where $\eta(\mathbf{r}) = (\eta_\uparrow, \eta_\downarrow)$ is a spinor chosen such that $\mathbf{h}$ lies in the xy plane: $|\eta_\uparrow|^2 = |\eta_\downarrow|^2 = 1/2$. Performing a local SU(2) gauge rotation

$$U = \begin{pmatrix} \eta_\uparrow & \eta_\downarrow^* \\ \eta_\downarrow & -\eta_\uparrow^* \end{pmatrix} , \tag{3}$$

the Hamiltonian becomes

$$H = \tfrac{1}{2}(-i\boldsymbol{\nabla} + \mathbf{A} - iU^\dagger\boldsymbol{\nabla}U)^2 + \ |\mathbf{h}(\mathbf{r})|\,\sigma_z \ . \tag{4}$$

The semiclassical picture discussed above assumes that the term $U^\dagger\boldsymbol{\nabla}U$ in the covariant derivative can be neglected in our limit since the spatial variation of U is small on the scale of the magnetic length. The effect of this term on an open trajectory can be illustrated in a simple example where the pseudofield $\mathbf{h}$ has a fixed magnitude but rotates along the x direction: $\mathbf{h} = |\mathbf{h}|(\cos Qx, \sin Qx)$. It can be shown that, after projection onto the lowest Landau level, the eigenstates have energies of $\pm|\mathbf{h}|\exp(-\frac{1}{2}Q^2)$, as measured from the unperturbed Landau level. Since $Ql_B \sim l_B/\lambda \ll 1$ in the limit of smooth disorder, it is reasonable to neglect the exponential correction factor for the energy.

A further objection to the neglect of the SU(2) gauge term in (4) is that this term gives rise to a nontrivial Berry phase if the electron trajectories form closed orbits. This is not important to the argument for the delocalization transitions discussed above because the relevant trajectories percolate through the system. Nevertheless, small closed orbits are found in the vicinity of the zeros of the pseudofield $\mathbf{h}$. These orbits correspond to states near the center of the Landau band because the Zeeman energy is small in these regions. I will now concentrate on this part of the spectrum, since, as already mentioned, a delocalization transition has been found at the band center for the case of a short correlation length.[1]

The zeros of the pseudofield are sparsely distributed around the system with a density of $1/\lambda^2$. One might therefore have reason to expect the electron orbits around these points to be *localized.* To reconcile this argument with the results of Hikami *et al.*,[1] one has to consider these eigenstates near the band center in greater detail. I will, from now on, project onto the lowest Landau level and measure energy from the center of the band, $\frac{1}{2}\hbar\omega_c$. The projected Hamiltonian, written out as a 2×2 matrix in the pseudospin space, is then

$$H_P = \begin{pmatrix} 0 & h_P \\ h_P^\dagger & 0 \end{pmatrix} \quad , \quad h_P = P(h_x(\mathbf{r}) + ih_y(\mathbf{r}))P, \quad (5)$$

where $P = \sum_m |\psi_m\rangle\langle\psi_m|$ is the projection operator onto the orbital wave functions ψ_m of the lowest Landau level. The off-diagonal form of the Hamiltonian ensures that the eigenenergies of H_P occur in $(E, -E)$ pairs for each realization of the disorder. It should be noted that this reflection symmetry in the spectrum is destroyed if the Hamiltonian (1) contains a random potential or a Zeeman coupling to a third component of the pseudofield, h_z.

Consider first an *isolated* $\mathbf{h} = \mathbf{0}$ point at the origin. Typically, the pseudofield has a vortexlike configuration with a winding number of unity around this point. A linearized configuration, for the region within a correlation length from the origin, can be written in the general form

$$\mathbf{h} = \frac{A}{\sqrt{2}} \begin{pmatrix} e^{\theta}[y\sin(\Phi + \phi) + x\cos(\Phi + \phi)] \\ e^{-\theta}[-x\sin(\Phi - \phi) + y\cos(\Phi - \phi)] \end{pmatrix} . \quad (6)$$

For instance, a field configuration with circular symmetry is given by $\theta = 0$ and ϕ equal to 0 or $\pi/2$, depending on the sense of circulation of the vortex configuration [i.e., the rotation of $\mathbf{h}(\mathbf{r})$ as $\mathbf{r}$ is taken around the origin]. The parameter θ is a measure of the difference in the gradients of the field in the x and y directions. The angle between the lines of $h_x = 0$ and $h_y = 0$ is $2\phi + \frac{1}{2}\pi$.

Working in the symmetric gauge where the vector potential $\mathbf{A} = \frac{1}{2}B(y, -x)$, the basis states $\{\psi_m\}$ are of the form $z^m \exp(-z^* z)$ $(m = 0, 1, 2, \ldots)$. These states are generated by the creation and annihilation operators

$$b = \tfrac{1}{\sqrt{2}}(\partial_z + z^*) \qquad \text{and} \qquad b^\dagger = \tfrac{1}{\sqrt{2}}(-\partial_{z^*} + z), \quad (7)$$

where $z = \frac{1}{2}(x + iy)$. The projection onto the lowest Landau level is given by the procedure[16] $z \to b^\dagger/\sqrt{2}, z^* \to b/\sqrt{2}$. Thus

$$h_P = A\left[e^{-i\Phi}\cosh(\theta - i\phi)b^\dagger + e^{i\Phi}\sinh(\theta + i\phi)b\right] . \quad (8)$$

From now on, Φ is set to zero without loss of generality by rotating spin space relative to real space. A zero-energy state in this vortex configuration is annihilated by either h_P or its Hermitian conjugate. This is an eigenstate of σ_z, with eigenvalue -1 and $+1$, respectively. The spatial part of the wave function can be written as $\psi = f(z)\exp(-z^* z)$. For the $\sigma_z = -1$ state (annihilated by h_P), $f = f_\downarrow$ satisfies

$$[\sinh(\theta + i\phi)\partial_z + 2\cosh(\theta - i\phi)z]f_\downarrow(z) = 0 \quad (9)$$

so that

$$f_\downarrow(z) \sim \exp\left[-\frac{\cosh(\theta - i\phi)}{\sinh(\theta + i\phi)}\, z^2\right] . \quad (10)$$

For a spin-up state, $f_\uparrow$ is obtained from $f_\downarrow$ by the exchange cosh $\leftrightarrow$ sinh. Only one of these wave functions is normalizable. There is therefore only one zero-energy state associated with this vortex configuration. It is spin down if $|\tanh(\theta + i\phi)| > 1$, which requires $\cos 2\phi < 0$. Otherwise, the state is spin up. This condition on ϕ simply says that σ_z is determined by the sense of circulation of the pseudofield. (The wave function becomes very elongated and becomes unnormalizable as $\cos 2\phi \to 0$. This occurs in the rare event when the lines of $h_x = 0$ and $h_y = 0$ coincide with each other.)

We can also study the rest of the spectrum on these linearized vortex configurations, by diagonalizing h_P using a Bogoliubov transformation to new operators, c and $c^\dagger$:

$$\begin{pmatrix} c \\ c^\dagger \end{pmatrix} = \frac{1}{\alpha(\psi)} \begin{pmatrix} \cosh\psi & \sinh\psi \\ \sinh\psi^* & \cosh\psi^* \end{pmatrix} \begin{pmatrix} b \\ b^\dagger \end{pmatrix} , \quad (11)$$

where $\alpha(\psi) = [|\cosh\psi|^2 - |\sinh\psi|^2]^{1/2}$ and $[c, c^\dagger] = 1$. Choosing $\psi = \theta + i\phi$, we can write h_P as

$$h_P = A(\cos 2\phi)^{1/2} c^\dagger . \quad (12)$$

The eigenenergies of these "vortex states" are therefore $E = 0, \pm E_m \sim O(l_B \Delta/\lambda)$ $(m = 1, 2, \ldots)$, where

$$\pm E_m = \pm A|\cos 2\phi|^{1/2} m^{1/2} . \quad (13)$$

The $m^{1/2}$ dependence of the energy is easy to understand. Consider, for instance, the circularly symmetric case of $\theta = \phi = 0$ (so that the Bogoliubov transformation is unnecessary). The zero-energy state is $|m = 0, \uparrow\rangle$ and the states away from $E = 0$ are $(|m, \downarrow\rangle \pm |m + 1, \uparrow\rangle)/\sqrt{2}$. These states have significant amplitudes at a radius of $R \sim m^{1/2}$ from the origin. Noting that $|\mathbf{h}| \propto R$, we see that these eigenstates are well described, at least for large m, by the semiclassical picture in which the wave functions are peaked on contours of $|\mathbf{h}|$.

Before drawing general conclusions about these vortex states, one should examine how they are affected by deviations from the simple field configuration (6). In addition to corrections to the functional form of $\mathbf{h}(\mathbf{r})$, another correction is the presence of nonlocal terms in the projected Hamiltonian arising from spatial variations in $\mathbf{h}(\mathbf{r})$ over length scales smaller than the magnetic length l_B. One can see that these perturbations are weak in the limit of $l_B \ll \lambda$. As long as the matrix elements due to these perturbations are nonsingular, one may switch on these terms adiabatically. The states with small but nonzero m will then be shifted in energy by an amount small compared to the level spacing. (States at large m can be strongly admixed because the level spacing decreases as $m^{-1/2}$.) However, the $E = 0$ state is not shifted in energy at all in this process because no adiabatic shift of this single state at $E = 0$ can give rise to a pair of states at $\pm E \neq 0$, as required by the reflection symmetry of the spectrum. Thus, I argue that, in the regime where $\lambda \gg l_B$, there is always a single $E = 0$ state associated with each zero of the pseudofield $\mathbf{h}$.

In general, there are more than one zero in the pseudofield so that these zero-energy states are degenerate and will hybridize with each other. One can envisage an effective Hamiltonian with long-range hopping for these states near the band center:

$$H_{\text{eff}} = \sum_{i,j} t_{ij} |\mathbf{r}_i\rangle\langle\mathbf{r}_j| , \quad (14)$$

where $\{|\mathbf{r}_i\rangle\}$ is the set of zero-energy states associated with the zeros of the pseudofield $\mathbf{h}(\mathbf{r}_i) = 0$. Due to the exact degeneracy of these states, they will hybridize in spite of the exponentially small overlap between their wave functions. Thus a delocalization transition analogous to that of Hikami *et al.*[1] may be found at the band center. (This transition is not accompanied by any change in the Hall conductance of the system — the total change in σ_{xy} of $2e^2/h$ across this pair of Landau levels has been taken up by the pair of mobility edges at $E = \pm E_c$ discussed above. See Fig. 1.) This is in sharp contrast to systems without the reflection symmetry in the spectrum. In such systems, the probability for finding two localized states sufficiently similar in energy to give significant hybridization is very small and conduction is only possible with the assistance of processes at finite temperature or frequency.[17]

I have argued that the semiclassical picture for this system can be reconciled with the expectation (from the results of Ref. 1) of a delocalized state at the band center. Indeed, the physical picture for the eigenstates near the band center clarifies the importance of the exact symmetry of the spectrum to the nature of the wave functions. Conversely, one expects this delocalization transition to be a fragile phenomenon: even weak disorder which is diagonal in the pseudospin space would destroy it by lifting the special degeneracy of the $E = 0$ states associated with the zeros of the pseudofield.

In summary, I have studied a model of a pair of degenerate Landau levels with random interlevel mixing in the semiclassical limit. As summarized in Fig. 1, delocalization transitions are found at a pair of energies displaced symmetrically from the center of the band. Each of these transitions belongs to the same universality class as that found in spin-split quantum Hall systems and should be robust to the introduction of other forms of disorder. An additional delocalization transition may be found at the band center but this critical point is unstable to the introduction of other scattering processes, such as a random potential in the modeling of a spin-degenerate Landau band.

I would like to thank J. T. Chalker, B. L. Altshuler, and X. G. Wen for valuable discussions. This work is supported by the Lindemann Trust and NATO.

[1] S. Hikami, M. Shirai, and F. Wegner, Nucl. Phys. B **408**, 415 (1993).
[2] Z. Wang, D.H. Lee, and X.G. Wen, Phys. Rev. Lett. **72**, 2454 (1994).
[3] D.E. Khmel'nitzkii, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 454 (1983) [JETP Lett. **38**, 552 (1983)].
[4] H. Levine, S.B. Libby, and A.M.M. Pruisken, Phys. Rev. Lett. **51**, 1915 (1983); A.M.M. Pruisken, *ibid.* **61**, 1297 (1988).
[5] H.P. Wei, D.C. Tsui, M.A. Paalanen, and A.M.M. Pruisken, Phys. Rev. Lett. **61**, 1294 (1988); S. Koch, R.J. Haug, K. von Klitzing, and K. Ploog, *ibid.* **67**, 883 (1991); L.W. Engel, D. Shahar, C. Kurdak, and D.C. Tsui, *ibid.* **71**, 2638 (1993).
[6] J.T. Chalker and P.D. Coddington, J. Phys. C **21**, 2665 (1988).
[7] B. Huckestein and B. Kramer, Phys. Rev. Lett. **64**, 1437 (1990).
[8] Y. Huo and R.N. Bhatt, Phys. Rev. Lett. **68**, 1375 (1992).
[9] H.P. Wei, S.W. Hwang, D.C. Tsui, and A.M.M. Pruisken, Surf. Sci. **229**, 34 (1990).
[10] S.W. Hwang, H.P. Wei, L.W. Engel, D.C. Tsui, and A.M.M. Pruisken, Phys. Rev. B **48**, 11 416 (1993). See the case of $g_{\rm eff} = 0$ in Fig. 4 of this reference.
[11] D.E. Khmelnitskii, Helv. Phys. Act. **65**, 164 (1992).
[12] D.K.K. Lee and J.T. Chalker, Phys. Rev. Lett. **72**, 1510 (1994).
[13] S.V. Iordansky, Solid State Commun. **43**, 1 (1982).
[14] S. Luryi and R.F. Kazarinov, Phys. Rev. B **27**, 1386 (1983).
[15] S.A. Trugman, Phys. Rev. B **27**, 7539 (1983).
[16] S.M. Girvin and J. Tach, Phys. Rev. B **28**, 4506 (1983).
[17] N.F. Mott and E.A. Davis, *Electronic Processes in Noncrystalline Materials* (Clarendon, Oxford, 1971), Chap. 2.