General relation between carrier spatial distributions and the generation function in photoconductors

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A Fourier transform analysis is used to derive a general formula for the determination of the dependence of the electron and hole spatial distribution functions on the carrier-generation function, under a small-signal nonuniform illumination of a photoconductor. The suggested procedure also yields general basic theorems regarding the relation between any carrier-generation function and the resulting electron and hole concentration distributions in the steady state. We do not know of any previous attempt to formulate such general theorems. In particular, we prove that in the most general case, provided the Einstein relation holds, the only carrier-generation function that can yield functionally similar distributions for the two types of carriers is spatially sinusoidal. The present results explain well the uniqueness of the sinusoidal generation function and provide the rigorous basis for the recently suggested photocarrier grating method, which is used for the experimental determination of the ambipolar diffusion length in photoconductors.

I. INTRODUCTION

The phenomenon of charge-carrier generation by optical excitation in solids has drawn wide interest and has generated intensive research for more than a century because of both its interesting basic physics aspects and its numerous applications. The physics aspects are primarily associated with nonequilibrium statistics and carrier transport, while the practical applications are encountered daily. Conspicuous examples of the applications are photodetectors, photoelectric power generators, and xerographic copiers. It became quite clear at the onset of the understanding of photoconductivity[1] that electrical measurements on samples subjected to nonuniform illumination may yield important information concerning fundamental phototransport parameters, such as charge carriers' diffusion lengths[2] or mobility-lifetime products.[3] Indeed, various nonuniform illumination configurations have been suggested[1-4] for the study of phototransport parameters. Of these configurations the older ones consisted of a "slit,"[2,3] or a "spatial pulse" illumination,[1,3] while more recently periodic illumination configurations such as a spatial "square wave"[5] or a spatial "sinusoidal wave"[6] have been applied. While phototransport properties of the majority carrier (the carrier which dominates the photoconductivity) can be determined from the relatively simple measurement of the samples' conductivity under uniform illumination, the determination of the minority-carrier phototransport properties almost always requires the application of nonuniform illumination.[7] The relation between the spatial dependence of the nonuniform illumination, which here we call the carrier generation function, and the resulting steady-state spatial carrier distribution, has been studied thus far only for a few illumination configurations and mainly for majority carriers.[2-4,6] Each illumination configuration has been studied as a specific case, and no general solutions or theorems have been suggested for this relation.

The purpose of this paper is to show that for any given small-signal nonuniform carrier generation one has a general solution for the spatial distribution of both the majority and minority carriers. This general solution enables us to prove rigorously that the applied nonuniform carrier generation which became very popular recently for the study of minority-carrier properties, i.e., the sinusoidal small-signal configuration,[8,9] is the one and *only* configuration for which the two carriers have the *same* spatial distribution as the carrier-generation function. Such a similarity is not a trivial expectation since, for example, it is obvious that under a configuration of alternating illumination intensity, i.e., a "square-wave" generation,[5] the carrier distribution will be a "smeared" (rather than a "square wave") distribution due to diffusion. Moreover, under an applied electric field, this distribution is also expected to be asymmetric. The latter expectation is in contrast to the symmetric distribution suggested[8] for the sinusoidal generation even when an electric field is applied. Since the sinusoidal carrier generation is very easy to obtain experimentally by the interference of two coherent light beams (as is well known from many experiments in general[10] and in photoconductors in particular[6,8]), it is quite fortunate for the analysis of phototransport data[8,9] that such an experimental configuration yields a simple proportionality between the carrier-generation function and each carrier-distribution function. The simplicity of the analysis is manifested by the existence of a simple analytic relation between the measured quantities and the microscopic effective phototransport parameters which are to be determined.[8] This allows the deviation of those parameters without assumptions or preknowledge of other material parameters.[9] Indeed, since its introduction[8] small-signal sinusoidal generation became the most popular[9] tool for the study of minority-carrier phototransport properties, in materials for which previously available experimental methods[2,3] were found[11] to fail. The application of our present approach to the particular case of the sinusoidal grating is rigorous since it applies a *general* Fourier analysis and does not assume the *a priori* unjustified proportionality between and the carrier generation and the carrier distribution as done in the analyses reported thus far.[8,12–14]

The structure of the paper is as follows. In Sec. II we derive the general solution of the small-signal continuity equations in terms of the Fourier transforms of the two carriers' spatial distribution functions. In Sec. III we present the simplified *explicit analytic* solutions for the two carrier distributions which are obtained for the sinusoidal carrier generation. We note that such full explicit solutions have not been given previously for the *general case,* which includes the effect of the application of an electric field. In Sec. IV we prove general theorems regarding relations between the generation function and the *two carrier-distribution* functions. These theorems emphasize the uniqueness of the sinusoidal generation function. Finally, in Sec. V, we summarize and discuss the results and their application for the derivation of phototransport parameters from photoconductivity measurements on samples which are subjected to nonuniform illumination.

II. THE GENERAL SOLUTION OF THE PHOTOCARRIER GRATING

We start by considering the two basic small-signal continuity equations which already include the corresponding Poisson equation. Since the general derivation of these equations is well known in general,[2,3] and it has been discussed recently in detail for the small-signal case,[8,14] we do not repeat their derivation here. Let us just mention that we assume a uniform background of electrons and holes, which determines the carriers' recombination times,[15] that the spatial variation of the illumination intensity is along the x axis, that the light absorption into the bulk of the material (which is in the perpendicular direction, say, along the z axis) is uniform, and that the quantum efficiency for the carrier generation is 1. The quantities which we wish to calculate are the nonuniform excess (above background) concentration of electrons, Δn, and the excess concentration of holes, Δp. We assume that the background density of electrons, n_0, and holes, p_0, is the same. The well-known steady-state equations are then[8,14]

$$D_h\partial^2(\Delta p)/\partial x^2-\mu_h E_0\partial(\Delta p)/\partial x-(q\mu_h p_0/\epsilon)(\Delta p-\Delta n)-(\Delta p/2t_p+\Delta n/2t_n)=-\Delta G(x) \quad (2.1)$$

and

$$D_e\partial^2(\Delta n)/\partial x^2+\mu_e E_0\partial(\Delta n)/\partial x+(q\mu_e n_0/\epsilon)(\Delta p-\Delta n)-(\Delta p/2t_p+2n/2t_n)=-\Delta G(x)\ . \quad (2.2)$$

Here $D_h(D_e)$ is the diffusion coefficient of the holes (the electrons), $\mu_h(\mu_e)$ is the mobility of the holes (the electrons), $p_0(n_0)$ is the large, uniform, background concentration of the holes (the electrons) with $n_0=p_0$, ϵ is the dielectric constant, q is the absolute value of the electronic charge, $t_p(t_n)$ is the small signal recombination time[8] of the holes (the electrons), E_0 is the applied electric field, and $\Delta G(x)$ is the excess carrier-generation function. The above equations can be reorganized into the forms

$$D_h\partial^2(\Delta p)/\partial x^2+b_p\partial(\Delta p)/\partial x+c_p\Delta p+d_p\Delta n=-\Delta G(x) \quad (2.3)$$

and

$$D_e\partial^2(\Delta n)/\partial x^2+b_n\partial(\Delta n)/\partial x+c_n\Delta n+d_n\Delta p=-\Delta G(x), \quad (2.4)$$

where we define

$$b_p=-\mu_h E_0\ ,\quad c_p=-(q\mu_h p_0/\epsilon+1/2t_p)\ ,$$
$$d_p=q\mu_h p_0/\epsilon-1/2t_n$$

and

$$b_n=\mu_e E_0\ ,\quad c_n=-(q\mu_e n_0/\epsilon+1/2t_n)\ ,$$
$$d_n=q\mu_e n_0/\epsilon-1/2t_p\ .$$

In order to solve $\Delta p(x)$ and $\Delta n(x)$ for a given $\Delta G(x)$, let us consider these three quantities in terms of their corresponding Fourier transforms,[16] i.e., by the equations

$$\Delta G(x)=\int_{-\infty}^{\infty}G_k\exp(ikx)dk\ , \quad (2.5)$$

$$\Delta p(x)=\int_{-\infty}^{\infty}p_k\exp(ikx)dk\ , \quad (2.6)$$

and

$$\Delta n(x)=\int_{-\infty}^{\infty}n_k\exp(ikx)dk\ , \quad (2.7)$$

where k belongs to a complete set of values which can be chosen according to the problem at hand, and where the G_k's are determined by the inverse relation of (2.5). We note immediately that $G_k^*=G_{-k}$, since $\Delta G(x)$ is real, and thus each term in the expansion $\Delta G(x)=\int_{-\infty}^{\infty}[G_k\exp(ikx)+G_{-k}\exp(-ikx)]dk$ is real. Substituting Eqs. (2.5)–(2.7) into Eqs. (2.3) and (2.4) then yields the following algebraic equations for the excess hole and the excess electron concentrations:

$$(-D_hk^2+b_pik+c_p)p_k+d_pn_k=-G_k \quad (2.8)$$

and

$$(-D_ek^2+b_nik+c_n)n_k+d_np_k=-G_k\ . \quad (2.9)$$

Solving these equations simultaneously, we find that

$$p_k=G_k(D_ek^2-b_nik+d_p-c_n)/(Ak^4-Bik^3-Ck^2+Dik+E) \quad (2.10)$$

and

$$n_k=G_k(D_hk^2-b_pik+d_n-c_p)/(Ak^4-Bik^3-Ck^2+Dik+E)\ , \quad (2.11)$$

where $A=D_hD_e$, $B=(D_eb_p+D_hb_n)$, $C=(D_hc_n+D_ec_p$

$+b_n b_p$), $D=c_n b_p+c_p b_n$, and $E=c_n c_p-d_n d_p$. Equations (2.10) and (2.11) yield closed-form expressions for p_k and n_k and thus *unique solutions* for Eqs. (2.1) and (2.2).

The form of p_k and n_k in Eqs. (2.10) and (2.11) provides an immediate answer, in principle, to the question of the spatial dependence of Δp and Δn. Clearly, $\Delta p(x)$ has the same functional form as $\Delta G(x)$ if these functions are proportional to each other, and this is possible only if all their Fourier coefficients are proportional to each other. Equations (2.10) and (2.11) show, however, that the proportionality factors between p_k or n_k and G_k are k dependent. Hence in *general* $\Delta p(x)$ and $\Delta n(x)$ are *not* proportional to $\Delta G(x)$.

Following the fact that $G_k=G^*_{-k}$, we can conclude immediately from Eqs. (2.10) and (2.11) that $p_k=p^*_{-k}$ and that $n_k=n^*_{-k}$. Then we have that $p_k\exp(ikx)+p_{-k}\exp(-ikx)$ and $n_k\exp(ikx)+n_{-k}\exp(-ikx)$ are real quantities. Defining real quantities $\phi_{G,k}$, $A_{p,k}$, $\delta_{p,k}$, $A_{n,k}$, and $\delta_{n,k}$, such that $G_k=|G_k|\exp(i\phi_{G,k})$, $p_k=(A_{p,k}/2)\exp(i\delta_{p,k})$, and $n_k=(A_{n,k}/2)\exp(i\delta_{n,k})$, we see from Eqs. (2.6) and (2.7) that $\Delta p(x)$ and $\Delta n(x)$ can be written as

$$\Delta p(x)=\int_{-\infty}^{\infty} A_{p,k}\cos(kx+\delta_{p,k})dk \qquad (2.12)$$

and

$$\Delta n(x)=\int_{-\infty}^{\infty} A_{n,k}\cos(kx+\delta_{n,k})dk \ . \qquad (2.13)$$

After rearranging terms in Eqs. (2.10) and (2.11) we obtain a more concise form of the equations for the four quantities $A_{p,k}$, $\delta_{p,k}$, $A_{n,k}$, and $\delta_{n,k}$. The results are

$$A_{p,k}=2|G_k|\{[(D_e k^2+d_p-c_n)^2+k^2 b_n^2]/[(Ak^4-Ck^2+E)^2+k^2(D-Bk^2)^2]\}^{1/2} \ , \qquad (2.14)$$

$$\tan(\delta_{p,k}-\phi_{G,k})=k[(Bk^2-D)(D_e k^2+d_p-c_n)-b_n(Ak^4-Ck^2+E)]/[(D_e k^2+d_p-c_n)(Ak^4-Ck^2+E)+k^2 b_n(Bk^2-D)] \ , \qquad (2.15)$$

$$A_{n,k}=2|G_k|\{[(D_h k^2+d_n-c_p)^2+k^2 b_p{}^2]/[(Ak^4-Ck^2+E)^2+k^2(D-Bk^2)^2]\}^{1/2} \ , \qquad (2.16)$$

and

$$\tan(\delta_{n,k}-\phi_{G,k})=k[(Bk^2-D)(D_h k^2+d_n-c_p)-b_p(Ak^4-Ck^2+E)]/[(D_h k^2+d_n-c_p)(Ak^4-Ck^2+E)+k^2 b_p(Bk^2-D)] \ . \qquad (2.17)$$

Equations (2.15) and (2.17) reveal the fact that there is a competition between the space-charge term (the two carriers' attraction, which is the first term in the numerator) and the field term (the two carriers' separation, which is the second term in the numerator). Hence the general situation (which has not been previously discussed) is that $\delta_{n,k}-\phi_{G,k}$ can have the same or the opposite sign of $\delta_{p,k}-\phi_{G,k}$. The intuitively expected case of the opposite sign will be obtained for high enough fields[14] (see below) or long enough dielectric relaxation times.[8]

As usual in various physical situations, such cumbersome expressions can be simplified under extreme conditions.[12,14] In our problem the extreme cases are of course that of the zero field and that of the very high electric field (to be defined below). In the first case we have that $E_0=0$ and therefore that $b_n=b_p=0$. Correspondingly we have that $A=D_e D_h$, $B=0$, $C=D_h c_n+D_e c_p$, $D=0$, and $E=c_n c_p-d_n d_p$. With these coefficients we obtain that $\delta_{p,k}-\phi_{G,k}=\delta_{n,k}-\phi_{G,k}=0$, as is to be expected from the directional symmetry of the problem. For the corresponding transform amplitudes, we then obtain from Eqs. (2.14) and (2.16) that

$$A_{p,k}=2|G_k|(D_e k^2+d_p-c_n)/[D_h D_e k^4-(D_h c_n+D_e c_p)k^2+c_n c_p-d_n d_p] \ , \qquad (2.18)$$

and that

$$A_{n,k}=2|G_k|(D_h k^2+d_n-c_p)/[D_h D_e k^4-(D_h c_n+D_e c_p)k^2+c_n c_p-d_n d_p] \ . \qquad (2.19)$$

Turning to the very-high-electric-field case we note that the diffusion contribution to the transport is manifested by the $D_h D_e k^4$ term. The relative importance of the drift (the $b_n b_p$) term should then be considered with respect to this diffusion term. Hence we define the very-high-electric-field regime as the regime in which the geometric means of the drift length (for either carrier) and the wavelengths of all the contributing k components are much larger than the diffusion length of either carrier.[14] Correspondingly, the very high field means that $b_n, b_p \gg D_h k$, $D_e k$, c_p/k, c_n/k, d_p/k, and d_n/k, for all values of k for which the terms contribute significantly to Eqs. (2.12) and (2.13). Using our abbreviated notations, this yields that $B \gg Ak$, $C\approx b_n b_p \gg Bk$, $D/k, E/k^2$, and hence that $Ck^2 \gg Ak^4$. These relations yield, in the denominators of Eqs. (2.14)–(2.17) that $(Ak^4-Ck^2+E)^2\approx C^2k^4$ and also that $C^2k^4 \gg k^2(D-Bk^2)^2$. On the other hand, we have that $k^2 b_p^2 \gg (D_h k^2+d_n-c_p)^2$ and $k^2 b_n^2 \gg (D_e k^2+d_p-c_n)^2$. Using these inequalities, we still have that the above amplitudes are rational functions of E_0, but we can easily obtain the leading terms. The values obtained are

$$A_{p,k}=2|G_k k b_n/k^2 C|\approx 2|G_k|/k\mu_h E_0 \qquad (2.20)$$

and

$$A_{n,k}=2|G_k kb_p/k^2C|\approx 2|G_k|/k\mu_e E_0 \ . \quad (2.21)$$

We then see that in this regime the amplitudes $A_{p,k}$ and $A_{n,k}$ are smaller than G_k by a factor proportional to $1/k$. This means that the information conveyed by large wave numbers k disappears rapidly as k increases. Since large values of k are related to smaller length scales, the $1/k$ factor in the amplitudes $A_{p,k}$ and $A_{n,k}$ means that $\Delta p(x)$ and $\Delta n(x)$ will be "smeared" with respect to $\Delta G(x)$, in the sense that the finer details of the generation function tend to disappear. Clearly, $\Delta p(x)$ and $\Delta n(x)$ are proportional to $1/E_0$ in this case, so that for large electric fields the excess hole or electron distribution amplitude is very small. For the phase of the k component of the hole distribution, we obtain from (2.15) that

$$\tan(\delta_{p,k}-\phi_{G,k})=kC/(k^2D_h b_n-c_p b_n-b_p d_p) \ . \quad (2.22)$$

This latter result can be also expressed in terms of the basic physical variables [see Eqs. (2.1) and (2.2)], yielding the explicit expressions

$$\begin{aligned}\tan(\delta_{p,k}-\phi_{G,k})\\ &=-k\mu_e\mu_h E_0/[\mu_e(D_h k^2+q\mu_h p_0/\epsilon+1/2t_p)\\ &\quad+\mu_h(q\mu_h p_0/\epsilon-1/2t_n)] \ . \quad (2.23)\end{aligned}$$

Similarly,

$$\begin{aligned}\tan(\delta_{n,k}-\phi_{G,k})\\ &=kC/(k^2D_e b_p-c_n b_p-b_n d_n)\\ &=k\mu_e\mu_h E_0/[\mu_h(D_e k^2+q\mu_e n_0/\epsilon+1/2t_n)\\ &\quad+\mu_e(q\mu_e n_0/\epsilon-1/2t_p)] \ . \quad (2.24)\end{aligned}$$

We see immediately that for $E_0\to\infty$, we obtain that $\delta_{n,k}-\phi_{G,k}\to-\pi/2$ and $\delta_{n,k}-\phi_{G,k}\to\pi/2$. This is to be physically expected due to the shift of the two carries' steady-state distributions with respect to their common spatial generation.

For all values of E_0 a simple proportionality between the Fourier generation coefficients G_k and the Fourier carrier coefficients p_k and n_k is obtained from Eqs. (2.10) and (2.11) for the sinusoidal carrier-generation function, which hereafter we will call the sinusoidal grating. In this special case $|G_k|=|G_0|$ for some $k=\pm k_0$, and $G_k=0$ for all other k's. We have then that the illumination period is $\Lambda=2\pi/k_0$, and all the n_k and p_k coefficients, except those of the $k=\pm k_0$ components, are zero. Correspondingly, for this case we should simply replace k by k_0 in Eqs. (2.14)–(2.24), recalling that this is the only component in the "Fourier expansions" Eqs. (2.12) and (2.13) that does not vanish. Comparison of the results (2.18)–(2.24) with the straightforward solutions of Eqs. (2.3) and (2.4), which were obtained[14] with the *a priori* assumption of sinusoidal carrier distributions yields the expected agreement.

To conclude this section we note that the solutions obtained above are general for any k component of the generation function, and thus we have a systematic procedure for every generation function [see Eqs. (2.5)–(2.7)]. Furthermore in order to derive the solution *we do not have to make any a priori* assumption regarding its functional form. As a particular consequence we have also proved that the *sinusoidal generation grating yields always a sinusoidal photocarrier grating for both carriers,* thus providing an *a posteriori* justification for the theories which dealt only with that case.[8,12,14] However, we did not prove thus far that the reverse is true, i.e., that the sinusoidal grating is the only grating which has that property. This will be done in Sec. IV.

III. THE SPECIAL CASE OF THE SINUSOIDAL GRATING

As we have pointed out already, in the sinusoidal generation case all the G_k's but two (e.g., G_{k_0} and G_{-k_0}) are zero. The Fourier transform of $\Delta G(x)$ has the form

$$G_k=(G_0/2)\delta(k-k_0)+(G_0^*/2)\delta(k+k_0) \ , \quad (3.1)$$

where $G_0=|G_0|\exp(i\phi_G)$. Equation (3.1) means that $\Delta G(x)$ will be given by

$$\Delta G(x)=|G_0|\cos(k_0x+\phi_G) \ . \quad (3.2)$$

Hence, since $\Delta G(x)$ is sinusoidal and since $\Delta p(x)$ and $\Delta n(x)$ will be of the same functional form, from Eqs. (2.12), (2.14), and (3.1) we obtain that

$$\Delta p(x)=|\Delta p|\cos(k_0x+\phi_p+\phi_G) \quad (3.3)$$

and

$$\Delta n(x)=|\Delta n|\cos(k_0x+\phi_n+\phi_G) \ , \quad (3.4)$$

where

$$|\Delta p|=|G_0|\{[(D_e k_0^2+d_p-c_n)^2+k_0^2b_n^2]/[(Ak_0^4-Ck_0^2+E)^2+k_0^2(D-Bk_0^2)^2]\}^{1/2} \quad (3.5)$$

and

$$|\Delta n|=|G_0\{[(D_h k_0^2+d_n-c_p)^2+k_0^2b_p^2]/[(Ak_0^4-Ck_0^2+E)^2+k_0^2(D-Bk_0^2)^2]\}^{1/2} \quad (3.6)$$

are the amplitudes of the excess carrier concentrations, $\phi_p=\delta_{p,k}-\phi_G$ and $\phi_n=\delta_{n,k}-\phi_G$ are the phase shifts of the corresponding carrier-concentration gratings with respect to the carrier-generation grating, and

$$\begin{aligned}\tan\phi_p=\{&k_0(Bk_0^2-D)(D_e k_0^2+d_p-c_n)\\ &-k_0b_n(Ak_0^4-Ck_0^2+E)\}/\{(Ak_0^4-Ck_0^2+E)(D_e k_0^2+d_p-c_n)-k_0^2b_n(D-Bk_0^2)\} \ , \quad (3.7)\end{aligned}$$

and

$$\tan\phi_n=\{k_0(Bk_0^2-D)(D_h k_0^2+d_n-c_p) -k_0 b_p(Ak_0^4-Ck_0^2+E)\}/\{(Ak_0^4-Ck_0^2+E)(D_h k_0^2+d_n-c_p)-k_0^2 b_p(D-Bk_0^2)\} . \tag{3.8}$$

Examining Eqs. (2.14)–(2.17) we see that not only have we proven that the sinusoidal generation yields functionally similar carrier gratings, but that we have the full solution for this *commonly applied* excitation.[6,11–13] Having derived the solution of the sinusoidal photocarrier grating (PCG), we note that, to our knowledge, this is the first time that a *full explicit analytic solution* for the carrier amplitudes in the presence of an electric field of any magnitude (not just for extreme cases[11–13]) is presented for the sinusoidal PCG. The present derivation is algebraically easy and is by far simpler than previous attempts[11–13] to derive these solutions. Moreover, the previous attempts were based on the *a priori* unjustified assumption of sinusoidal solutions and their straightforward substitution into the differential equations (2.1) and (2.2). The procedure which led to the corresponding algebraic equations for Δp, ϕ_p, Δn, and ϕ_n is however justified now. We also note that the previously described algebraic procedures[11–13] to find the explicit expressions of the above parameters are by far more tedious than the procedure given here.

Using results (3.5)–(3.8) we can now easily derive (by the procedure outlined above) the solutions for the extreme cases which were derived already in the literature.[12,14] For example, for the $E_0=0$ case we already saw that $b_n=b_p=0$, $A=D_eD_h$, $B=0$, $C=D_hc_n+D_ec_p$, $D=0$, and $E=c_nc_p-d_nd_p$, and thus that $\phi_p=\phi_n=0$. This means of course that the removal of the electric field also removes the asymmetry of the electrons' and holes' gratings with respect to the generation grating. The result for $|\Delta p|$ is then reduced to

$$|\Delta p|=|G_0|[D_e k_0^2+d_p-c_n]/[D_eD_h k_0^4-(c_nD_h+c_pD_e)k_0^2+c_nc_p{}_-d_nd_p] , \tag{3.9}$$

which in terms of the well-defined physical parameters in Eqs. (2.1) and (2.2) can be written as

$$|\Delta p|=|G_0|[D_e k_0^2+qn_0(\mu_e+\mu_h)/\epsilon]/\{D_eD_h k_0^4+k_0^2[q_1n_0(\mu_eD_h+\mu_hD_e)/\epsilon+D_h/2t_n+D_e/2t_p] +qn_0(\mu_e+\mu_h)(1/2t_n+1/2t_p)\} , \tag{3.10}$$

where the recall that we have assumed in Eqs. (2.1) and (2.2) that $n_0=p_0$, in accordance with the common experimental conditions which usually prevail[9] or essentially prevail[17] under the application of the sinusoidal PCG. Solution (3.10) is exactly the solution derived previously[14] for the same case when the sinusoidal solution has been assumed *a priori*. Note, however, that here, unlike Ref. 14, we have used the more appropriate definition of the small-signal lifetimes of Refs. 8 and 12 [see Eqs. (2.1) and (2.2)], i.e., $(1/2t_n+1/2t_p)$ instead of the $(1/t_n+1/t_p)$ term used there. The result for Δn, as can be seen from the above symmetric derivation (see Sec. II), is (except for the replacement of D_e by D_h in the numerator) much the same.

The other extreme case which has been considered previously[12,14] is that of the dominant electric-field term. For this high-electric-field case [see Eqs. (2.20)–(2.24)] we obtain that

$$\tan\phi_p=k_0b_nb_p/[D_hk_0{}^2b_n-(d_pb_p+c_pb_n)] . \tag{3.11}$$

Also, the general expression for $|\Delta p|$ is simplified to yield

$$|\Delta p|=k_0b_n|G_0|/(Ck_0^2)\approx|G_0|/|k_0b_p| . \tag{3.12}$$

Writing (3.11) and (3.12) in terms of the meaningful physical parameters given in Eqs. (2.1) and (2.2), we finally obtain that

$$\tan\phi_p=-k_0\mu_e\mu_hE_0/[(\mu_e(D_hk_0^2+q\mu_hp_0/\epsilon+1/2t_p) +\mu_h(q\mu_hp_0/\epsilon-1/2t_n)] , \tag{3.13}$$

and that

$$|\Delta p|=|G_0|/(\mu_hE_0k_0) . \tag{3.14}$$

As one can see immediately, $\tan\phi_p\to\infty$ for $E_0\to\infty$, so that $\phi_p\to\pi/2$, and $|\Delta p|$ decreases as $1/E_0$ as has been predicted already for this case.[8,12,14] Furthermore, it is clear from our general treatment, given in Sec. II, that for this case we will obtain similar results for electrons, except that μ_e replaces μ_h and that $\phi_n\to-\pi/2$.

IV. GENERAL THEOREMS ON THE FUNCTIONAL FORM OF $\Delta p(x)$ AND $\Delta n(x)$

We have seen that under sinusoidal carrier generation, the hole and electron distributions are also sinusoidal. Since we have also seen that in general the carrier distributions do not have the functional form of the carrier-generation function, the sinusoidal grating is a special case. In this section we investigate a more fundamental question, i.e., what is the most general form of the generation function for which $\Delta p(x)$ or $\Delta n(x)$ are functionally similar to this function. This similarity requirement can be expressed in the most general case by the conditions:

$$\Delta p(x)=T_p\Delta G(x-s_p) , \qquad \Delta n(x)=T_n\Delta G(x-s_n) , \tag{4.1}$$

where T_p, T_n, s_p, and s_n are constants. In other words, we allow $\Delta p(x)$ of $\Delta n(x)$ to differ from $\Delta G(x)$ only by a scale factor and a phase shift.

To answer the above question, let us take the Fourier transforms of Eqs. (4.1). We obtain that these conditions are

$$p_k=T_p\exp(-iks_p)G_k\ ,$$
$$n_k=T_n\exp(-iks_n)G_k\ , \tag{4.2}$$

where G_k is the Fourier transform of $\Delta G(x)$. Substitution of (4.2) into the general relations (2.10) and (2.11) yields the following general equations for the above constants:

$$T_p^2=[(D_ek^2+d_p-c_n)^2+k^2b_n^2]/[(Ak^4-Ck^2+E)^2+k^2(Bk^2-D)^2]\ , \tag{4.3a}$$

$$T_n^2=[(D_hk^2+d_n-c_p)^2+k^2b_p^2]/[(Ak^4-Ck^2+E)^2+k^2(Bk^2-D)^2]\ , \tag{4.3b}$$

$$\tan(ks_p)=k[(Bk^2-D)(D_ek^2+d_p-c_n)-b_n(Ak^4-Ck^2+E)]/[(D_ek^2+d_p-c_n)(Ak^4-Ck^2+E)+k^2b_n(Bk^2-D)]\ , \tag{4.3c}$$

and

$$\tan(ks_n)=k[(Bk^2-D)(D_hk^2+d_n-c_p)-b_p(Ak^4-Ck^2+E)]/[(D_hk^2+d_n-c_p)(Ak^4-Ck^2+E)+k^2b_p(Bk^2-D)]\ . \tag{4.3d}$$

The cumbersome expressions for T_p and T_n can be rewritten as polynomial equations for a new variable $z\equiv k^2$:

$$(D_ez+d_p-c_n)^2+zb_n^2=T_p^2[(Az^2-Cz+E)^2+z(Bz-D)^2] \tag{4.4a}$$

and

$$(D_hz+d_n-c_p)^2+zb_p^2=T_n^2[(Az^2-Cz+E)^2+z(Bz-D)^2]\ . \tag{4.4b}$$

These equations depend only on k^2, as they should, since k and $-k$ must enter them symmetrically to ensure that $\Delta p(x)$ and $\Delta n(x)$ are real. Equations (4.4) can always be accommodated for one value of k. This value simply defines the appropriate T_p and T_n. This is essentially the sinusoidal case discussed in Sec. III. In the more general case, once T_p and T_n are fixed, Eqs. (4.4) greatly constrain the possible forms of $\Delta G(x)$. Let us consider, for example, the holes' distribution. For a given T_p, Eq. (4.4a) is quartic in z, which means that it has four solutions at most. As a result, $\Delta p(x)$ can be functionally similar to $\Delta G(x)$ only if the generation grating's Fourier spectrum is discrete and contains *at most a superposition of four sinusoidal gratings.*

If we require that $\Delta n(x)$ also follows the functional dependence of $\Delta G(x)$ we have further restrictions. From (4.4a) and (4.4b) we obtain that

$$(T_p/T_n)^2=[(D_ez+d_p-c_n)^2+zb_n^2]/[(D_hz+d_n-c_p)^2+zb_p^2]\ . \tag{4.5}$$

For given T_p and T_n this is a *quadratic* equation in z, and it has therefore two solutions at most. Hence *if both Δp and Δn are required to be functionally similar to ΔG*, the carrier-generation grating can be *at most a superposition of two sinusoidal gratings.* There are two trivial exceptions to this rule. In the first case all the electron's parameters are identical to all the hole's parameters, i.e., $D_e=D_h$, $t_p=t_n$, and so on. In this case, $T_p=T_n$ and Eq. (4.5) is verified identically for all values of k, so there is still a possibility of a superposition of four sinusoidal gratings. However, this is an abnormal physical occurrence. The other trivial case is when the carries are not mobile, i.e., $D_e=D_h=\mu_e=\mu_h=0$. In this case the carriers reside where generated, and thus Δn and Δp are trivially proportional to ΔG. Mathematically, in this case, Eqs. (4.4) are independent of z.

In the remainder of this section, we assume, as is always the case in actual materials, that the electrons' parameters differ from the holes' parameters and that the carriers are mobile. Now Eq. (4.5) is not the only constraint. Since $z=k^2$, we must require that z be positive. One can always find one positive z that verifies Eq. (4.5). Let us denote it z_0. As noted above, z_0 simply fixes the value of T_p/T_n. Equation (4.5) can therefore be rewritten as

$$[(D_ez_0+d_p-c_n)^2+z_0b_n^2]/[(D_hz_0+d_n-c_p)^2+z_0b_p^2]=[(D_ez+d_p-c_n)^2+zb_n^2]/[(D_hz+d_n-c_p)^2+zb_p^2]\ . \tag{4.6}$$

Searching for possible solutions for z, we note first that one solution is evidently z_0 itself. In looking for other solutions let us first consider the simplest case of the zero electric field. In this case Eq. (4.6) reduces to

$$(D_ez_0+d_p-c_n)^2/(D_hz_0+d_n-c_p)^2=(D_ez+d_p-c_n)^2/(D_hz+d_n-c_p)^2\ . \tag{4.7}$$

The two solutions x_1 and x_2 of a quadratic equation $ax^2+bx+c=0$ satisfy the relation $x_1x_2=c/a$. In the case of Eq. (4.7), this relation turns out after some algebra and use of the fact that

$$d_p-c_n=d_n-c_p=q(\mu_en_0+\mu_hp_0)/\epsilon>0 \tag{4.8}$$

[which follows our definitions associated with Eq. (2.4)], to be

$$zz_0=-[z_0^2(D_e+D_h)+2z_0(d_p-c_n)](d_p-c_n)/[(D_e+D_h)(d_p-c_n)+2D_eD_hz_0]\ . \qquad (4.9)$$

Hence zz_0 is clearly negative. Since z_0 is positive, this means that the second solution of (4.7) is negative and *cannot* therefore be a proper solution. Hence in the zero-field case we have a single k in our solution, i.e., we obtain that *for Δp and Δn to be both functionally similar to the generation function the latter must be sinusoidal.*

Let us turn now to the more complicated case given by Eq. (4.6), i.e., when the electric field is not zero. We can still apply the same arguments, if we assume that the relation

$$\mu_e/D_e=\mu_h/D_h=\alpha \qquad (4.10)$$

is valid. This is a somewhat "weaker" condition than the Einstein relation, since we only require the same α for both carriers but we do not specify its value . Such a case can occur even if a degenerate statistic takes place.[8] We then obtain, from Eq. (4.6) after some algebra and after using Eqs. (4.8) and (4.10), the following condition for a second solution for z;

$$z_0z=-(d_p-c_n)W/V\ , \qquad (4.11)$$

where

$$W=[z_0^2+z_0(b_n/D_e)^2](D_e+D_h)+2z_0(d_p-c_n)$$

and

$$V=2z_0D_eD_h+(d_p-c_n)(D_e+D_h)\ .$$

Again, it is clear from the fact that $W,V>0$ that z is negative and cannot be a proper solution. We have therefore shown that if the electric field is zero or *if the "Einstein relation" (4.10) holds, the only case where $\Delta p(x)$ and $\Delta n(x)$ both follow the functional form of $\Delta G(x)$ is the pure sinusoidal case.*

We can apply the same arguments to the general case which we have discussed above where only $\Delta p(x)$ or $\Delta n(x)$ are required to follow the functional form of $\Delta G(x)$. This case is more complex, and the analysis correspondingly more difficult. Concentrating on the dependence of $\Delta p(x)$ for definiteness, we begin by introducing z_0, which defines the value of T_p in Eq. (4.4a). We can then rewrite Eq. (4.4a) in the form

$$[(D_ez+d_p-c_n)^2+zb_n^2][(Az_0^2-Cz_0+E)^2+z_0(Bz_0-D)^2]$$
$$=[(D_ez_0+d_p-c_n)^2+z_0b_n^2][(Az^2-Cz+E)^2+z(Bz-D)^2]\ . \qquad (4.12)$$

Since z_0 is a solution of (4.12), we can divide this equation by z_0-z and obtain a third-order equation for the remaining values of z. After some tedious algebra, we obtain an equation of the form

$$a_3z^3+a_2z^2+a_1z+a_0=0\ , \qquad (4.13)$$

where

$$a_3=A^2\{[D_ez_0+(d_p-c_n)]^2+z_0b_n^2\}\ ,$$
$$a_2=(A^2z_0+B^2-2AC)\{[D_ez_0+(d_p-c_n)]^2+z_0b_n^2\}\ ,$$
$$a_1=[(Az_0-C)^2+B^2z_0-2BD+2EA]$$
$$\times[z_0b_n^2+(d_p-c_n)^2+D_ez_0(d_p-c_n)]$$
$$-D_e^2[E^2+D^2-2ECz_0]$$

and

$$a_0=(d_p-c_n)^2[z_0(Az_0-C)^2+(Bz_0-D)^2$$
$$+2E(Az_0-C)]$$
$$-E^2[D_e^2z_0+2D_e(d_p-c_n)+b_n^2]\ .$$

Recalling now that from Eq. (2.4) we have that b_pb_n, c_pD_e, and c_nD_h are all negative, we find that [according to Eq. (2.11)] $C<0$. As a result $Az_0-2C>0$, and therefore

$$A^2z_0-2AC+B^2=(Az_0-2C)A+B^2>0\ . \qquad (4.14)$$

From Eq. (4.14) and the definitions of a_3 and a_2 [Eq. (4.13)] we see immediately that

$$a_2>0 \ \text{ and } \ a_3>0\ . \qquad (4.15)$$

It is well known[18] that the three solutions z_1, z_2, and z_3 of a cubic equation such as Eq. (4.13) must satisfy the relation

$$z_1+z_2+z_3=-a_2/a_3<0\ . \qquad (4.16)$$

Hence *at least* one solution *must* be negative, and therefore improper. This reduces the number of possible solutions of (4.3a) to only three. We have thus reached the conclusion that in this case the *most general solution* will consist at most of a superposition of *three sinusoidal* contributions. Hence we have narrowed down the number of possible sinusoidal contributions which have the proportionality property for one carrier, from four (see above) to three. Further analysis is too complicated to yield any more general insight.

Finally, we can address another general question: under what conditions will the hole and electron distributions be functionally similar. In other words, what will be the constraints on the generation function $\Delta G(x)$ if there are to be constants T and s, such that

$$\Delta p(x)=T\Delta n(x-s)\ . \qquad (4.17)$$

To consider this problem, let us recall that the Fourier transform of Eq. (4.17) is

$$p_k = T n_k \exp(-iks) \ . \qquad (4.18)$$

Substituting the expressions for p_k and n_k [Eqs. (2.10) and (2.11)] into Eq. (4.18) yields

$$D_e k^2 - ikb_n + d_p - c_n = T(D_h k^2 - ikb_p + d_n - c_p)\exp(-iks) \ . \qquad (4.19)$$

Taking the norm squared of this equation, we obtain that

$$(D_e k^2 + d_p - c_n)^2 + k^2 b_n^2 = T^2[(D_h k^2 + d_p - c_n)^2 + k^2 b_p^2] \ . \qquad (4.20)$$

This, however, is identical with Eq. (4.5) if we set $T_p/T_n \equiv T$. The analysis of Eq. (4.5) is therefore immediately applicable. Hence for zero electric field, the only possible solution of Eq. (4.19) is a sinusoidal distribution. The same conclusion holds if the electric field is not zero but the "Einstein relation" [Eq. (4.10)] holds. We can conclude then in general that the requirement of similarity of the two carrier-distribution functions also yields their similarity to the generation function if the "Einstein relation" holds, and that this function must be sinusoidal.

V. DISCUSSION AND CONCLUSIONS

In this paper we have used a Fourier-transform analysis to solve the steady-state spatial concentration distributions of the photogenerated carriers due to the application of a small-signal nonuniform carrier generation. We have obtained a *general formula* for the derivation of the distributions for *any* given carrier-generation function using a Fourier-transform analysis of the carrier-generation function. Therefore, we have

$$\Delta p(x) = \int_0^\infty A_{p,k} \cos(kx + \delta_{p,k}) dk$$

and

$$\Delta n(x) = \int_0^\infty A_{n,k} \cos(kx + \delta_{n,k}) dk \ ,$$

where the coefficients $A_{p,k}$ and $A_{n,k}$ and phases $\delta_{p,k}$ and $\delta_{n,k}$ can explicitly be calculated according to the simple algebraic formulas (2.14)–(2.17). In particular, in contrast to previous works,[8,12–14] we have obtained, without any *a priori* assumptions, the *exact analytic* solution for the carrier distributions under the sinusoidal photocarrier generation and in the presence of an electric field. The first and foremost conclusion from the point of view of the photocarrier grating (PCG) technique[9] is that under the application of a sinusoidal generation function the two carrier gratings will also be sinusoidal. This conclusion justifies the procedure and the analysis of the application of the technique for the derivation of phototransport parameters.[9]

We have shown that our Fourier analysis lends itself particularly well to a general discussions of the functional dependence of the two carrier distributions. Following this conclusion we have proved the following theorems (valid under the "very weak" assumption that $D_e \neq D_h$ or $\mu_e \neq \mu_h$) which determine the specific distribution and generation functions when only similarities between pairs of them are given.

(1) If no electric field is applied, the only case in which the excess (above the uniform background) hole and excess electron distributions are functionally the same is the case when they are both forming a sinusoidal grating. This trivially implies that the generation grating is also sinusoidal.

(2) If an electric field is applied, but the Einstein relation between the diffusion constant and the mobility holds, the excess concentrations of both carries are functionally similar if and only if the generation grating is sinusoidal. In this case the distributions of both types of excess carriers are also sinusoidal. We conclude then that since the essence of the Einstein relation holds[8] for most cases relevant to photoconductors (even when some form of degenerate statistics has to be used), this theorem applies quite generally in practice.

(3). Any one of the excess carrier distributions (either that of the holes or the electrons) can be functionally similar to that of the generation signal *only if* the latter's Fourier spectrum is discrete and made, at most, of *three* wave-vectors component. In other words, to have this similarity $\Delta G(x)$ can be a linear combination of no more than three sinusoidal gratings. This has important consequences in regard to carrier-generation functions which are not sinusoidal. For such functions, if they are not well approximated by a superposition of three sinusoidal functions, a full (and probably numerical) Fourier-transform analysis is required.

Further limitations may follow for Eqs. (4.3b), and (4.3c), and from further analysis of Eq. (4.13). We have not pursued such an investigation because it does not seem to yield any further general or simple results. For any given parameter of the material under investigation (the diffusion coefficients, mobilities, etc.) such an investigation can be pursed numerically.

As a practical matter it is obvious that for any periodic carrier generation (carrier-generation grating) the present Fourier analysis is quite efficient and even for the "square-wave" generation it will converge quite quickly. Moreover, we have found[19] by numerical analysis of that problem that in the zero-field case even for nonsinusoidal gratings, depending on the material parameters (e.g., the relation between the diffusion length and the grating period) a carrier grating can approximate the sinusoidal case (i.e., be written in terms of a small number of k terms). From that stage on it is only the required accuracy which will determine how many k's are needed. On the other hand, for any single "pulse" illumination, such as a single illuminated strip (i.e., a single "square" or a single "window"), the present approach is much less efficient, since it requires a linear combination of many sinusoidal contributions, i.e., a large number of k values. For those cases a much more efficient approach is the approach which we have suggested previously (Appendix A of Ref. 14). That approach, which is essentially the Green's-function approach, i.e., the superposition of the contributions from many illuminated "slits," will be discussed in its general form elsewhere.[19] In the simple case of ambipolar conditions with no applied electric field, both approaches, the one given here and that of the Green's function,[14] yield immediate proof that the sinusoidal generation yields a sinusoidal grating. Following the fact that for other periodic carrier-generation functions the Green's-function approach[14] may not be efficient,[19] while periodic functions are naturally treated by Fourier analysis,[16] it appears that the present approach is the one to be applied for any periodic illumination, i.e., any carrier-generation grating. We also note here that the Fourier-transform analysis has another advantage since one is able to find a *direct* rather than a *convoluted* relation[14] between $\Delta p(x)$ or $\Delta n(x)$ and $\Delta G(x)$. This result is also of practical importance for actually finding the carrier-distribution functions. In many respects, then, the calculations which can be carried out by the present approach are complementary to that of the Green's-function approach, which is more practical, in general, for nonperiodic carrier-generation configurations.[19] However, we see by comparison of the present analysis with that of Ref. 14 that, for the widely used sinusoidal photocarrier grating (PCG) technique and under zero field, the two approaches are equally appropriate. Apart from these conditions the advantages of each approach as discussed above should be considered.

We should point out in passing one aspect of the PCG technique which has not been discussed before. In order to obtain the "true" (one k value) sinusoidal carrier grating, one needs an "infinite" sinusoidal generation grating. In practice about 200 periods are used in a typical PCG measurement,[9] and thus the approximation of an infinite sinusoidal generation grating, i.e, a true sinusoidal carrier grating, is justified to a very high accuracy. In fact, the expression for the dc conductivity is given by an integral over a single period:[8,14]

$$\sigma=\{(N/q)\int_0^\Lambda dx/[\mu_n(n_0+\Delta n)+\mu_p(p_0+\Delta p)]\}^{-1} ,$$

where N is the number of periods per unit length, Λ is the single period length, and the integration is over one period.

In view of the above discussion we can now derive a very general conclusion, i.e., that the Fourier spectra of the carrier distribution are never more complex than the Fourier spectrum of the carrier generation. They can, however, be simpler, i.e., contain fewer components. Physically this occurs because the transport of the carriers causes the carrier distributions to "smear out" the details of the generation function, thus yielding "smoother" distribution functions. In particular this general conclusion explains well the uniqueness of the sinusoidal generation function. The uniqueness is a result of the fact that the sinusoidal generation grating has the simplest possible Fourier spectrum. Since a carrier distribution's Fourier spectrum cannot be more complex, i.e., it cannot have more than a single-k component, it must also be sinusoidal. This is essentially why a sinusoidal generation grating will always yield sinusoidal carrier distributions.

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