Temperature and electric-field dependence of hopping transport in low-dimensional devices

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(Received 27 January 1994)

A theoretical calculation has been performed for the variable-range-hopping (VRH) conduction mechanism in the presence of temperature and electric field for quasi-two-dimensional (QTD) and quasi-one-dimensional (QOD) systems. In the present calculation, it is assumed that the localized states are randomly distributed both in energy and space coordinates. The states both below and above the Fermi level are included in the calculation of the hopping range and conductivity. The present approach differs significantly from the percolation method and others in the calculation of the mobility and the conductivity. The expressions for the hopping range, the mobility, and the conductivity are obtained for the constant and the energy-dependent density of states. The expression of the conductivity for the constant density of states can be reduced to that of Mott in certain approximations. The effect of electron-electron interaction in the calculation of the conductivity and hopping range has been included through the density of states. After some approximations, the present expression of the conductivity can be reduced to that of Efros and Shklovskii. The logarithm of the conductivity follows the $(1-\beta^2)^{3/8}$ and $\sqrt{1-\beta^2}$ electric-field dependence for QTD and QOD systems, respectively, in the presence of the electron-electron interaction and a weak electric field. Here β is directly proportional to an electric field. The present calculations are applied to explain the recent conductivity experiments on $PrBa_2Cu_3O_{7-y}$ (PBCO) films. A possible crossover from Mott-type VRH to Efros-and-Shklovskii-type VRH has been observed in PBCO.

I. INTRODUCTION

Recently, there has been a considerable interest in the study of the proximity effect in S/N and S/N/S junctions.[1-13] Here S and N stand for a high-temperature superconductor (HTS) and a normal metal or oxide superconductor, respectively. Tarutani *et al.*[4] have fabricated S/N/S junctions where S is a $HoBa_2Cu_3O_{7-y}$ (HBCO) compound and N is either $La_{1.5}Ba_{0.5}CuO_{4-y}$ (LBCO) or $PrBa_2Cu_3O_{7-y}$ (PBCO). They have studied the conductivity of these junctions in the superconducting and normal states. They observed the proximity effect even when the thickness of the N is larger than the coherence length. This effect is called the long range proximity effect. Tarutani *et al.*[4] have also measured the temperature-dependent resistivity of the PBCO junctions and films in the normal state. They observed the variable-range-hopping (VRH) type of conduction in the resistivity measurements when the thickness of the normal metal layer is greater or equal to 0.5 μm. However, for the sample of thickness 0.2 μm, they observed that the resistivity is independent of the temperature for $T<8$ K.

The study of the VRH mechanism in bulk materials has been the subject of number of investigations,[14-27] but not much theoretical work has been done in quasi-two-dimensional (QTD) and quasi-one-dimensional (QOD) systems such as thin films, S/N, and S/N/S junctions. Recently, Singh *et al.*[21] have performed a theoretical calculation for variable-range hopping in QTD systems in an electric field and various temperatures. An analytical expression for the conductivity has been obtained for a constant density of states (DOS). A theoretical calculation has been done to explain the temperature-dependent resistivity experiments of Kabasawa *et al.*[5] on PBCO films. A good agreement between the theory and the experiments is found at high temperatures, but at low temperatures there is a discrepancy between theory and experiments. In the above calculations, the effect of electron-electron interactions was not included. This discrepancy may be due to omission of the electron-electron interactions in the calculation of the VRH conductivity.

In the present work, the hopping range, the mobility, and the conductivity are calculated for QOD and QTD systems by using the method developed in the previous paper.[21] It is assumed that the localized states are distributed randomly in both space and energy coordinates. The states occupied both above and below the Fermi level have been included in the calculations. We assume that states are occupied according to Fermi-Dirac statistics. The mobility is calculated as a function of energy with respect to the Fermi level. Then the conductivity is calculated by integrating over energy with the help of the mobility. The present calculation differs significantly from theories of percolation where the conductivity is calculated for a resistance network at a particular or critical energy. This is sometimes called the critical conductivity. Using a very general energy-dependent DOS [i.e., $D(\omega)=D_1\omega^n$] and a constant DOS, expressions for the hopping range, the mobility, and the conductivity are obtained. Here D_1 and n are constants. Analytical expressions for the above quantities have also been obtained in certain approximations. The effect of the electron-electron interaction has been included through the DOS (Refs. 16,17) in the present calculations.

If we assume that all states above and below the Fermi

level are empty and occupied, respectively, then for constant DOS our expressions for the conductivity give $\sigma \approx \exp(T/T_0^M)^{-1/2}$ and $\sigma \approx \exp(T/T_0^M)^{-1/3}$ for QOD and QTD systems, respectively. Here T_0^M is called the Mott characteristic temperature. These results are consistent with the results of Mott.[14,15] Similarly, our expressions for the conductivity obtained in the presence of electron-electron interactions can be reduced to $\sigma \approx \exp(T/T_0)^{-1/2}$ for both QOD and QTD systems. This expression agrees with that of Efros and Shklovskii (ES).[16,17] In the presence of electron-electron interactions, the logarithm of the conductivity follows the $(1-\beta^2)^{3/8}$ and $\sqrt{1-\beta^2}$ electric-field dependence for QTD and QOD systems, respectively, in a weak electric field. Here β is directly proportional to an electric field. The present theoretical calculations are applied to explain the recent temperature dependence of resistivity experiments of PBCO films.[4] A good agreement between theory and experiments is observed if one includes the effect of electron-electron interactions in the calculation of the resistivity at low temperatures. A possible crossover from Mott-type VRH to Efros-and-Shklovskii-type VRH is predicted in PBCO films. The crossover temperature was found to be about 11 K which is in agreement with the theoretical calculated value.

II. GENERAL METHOD

Let us briefly discuss the method used in the present calculations. We assume that localized states are randomly distributed in energy and space coordinates and they form a discrete array of sites. This space is called the hopping space. Then the probability of hopping of a charge carrier from an initial state to a final state in this space is given by[14,15]

$$W(R)=\exp(-R) , \tag{1}$$

where R is the distance between two states in the hopping space and is called the range. In the presence of an electric field E, the range is given as[21]

$$R = x(1+\beta\cos\theta)+\omega-\varepsilon \quad \text{for } x\beta\cos\theta+\omega<\varepsilon ,$$
$$R = x \quad \text{for } x\beta\cos\theta+\omega>\varepsilon . \tag{2}$$

Here $\beta=qE/2\alpha k_B T$ and x is the distance between two sites. ε and ω are the energy of the initial and the final sites, respectively. q is the charge of a carrier, α is the inverse of the attenuation length, and θ is the angle between x and the electric field. Note that the space and energy variables are presented in reduced coordinates.[20,21] The reduced coordinate x should be multiplied by $1/2\alpha$ to express it in the distance units. Similarly, ε and ω should be multiplied by $k_B T$ to write them in the energy units. It can be seen from Eq. (1) that as the effective distance between two states decreases the hopping probability increases.

Let us consider a site with energy ε in the hopping space. The most probable hop for a carrier of energy ε on this site will be to its nearest-neighbor empty site. Conduction is the result of many series of hops through this hopping space. The average nearest-neighbor hopping distance R_{NN} in the hopping space is given as[20,21]

$$R_{NN}=\int_0^\infty R P_{NN}(R) dR . \tag{3}$$

In the rest of the paper, R_{NN} will be called the hopping range. Here $P_{NN}(R)$ is the probability distribution of the nearest neighbors in the hopping space. It is defined as[20]

$$P_{NN}=\Delta N(R)\exp[-N(R)] , \tag{4}$$

where $N(R)$ is the total number of localized states within range R for the nearest-neighbor sites and $\Delta N(R)$ is the number of localized states between R and $R+dR$.

In the presence of an electric field, the spatial displacement in the hopping space is more likely to be downfield than upfield. Let X_f be the average spatial distance traveled in the direction of the electric field and $W(R_{NN})$ be the hopping probability. Then $\nu X_f W(R_{NN})/2\alpha$ is the average spatial distance traveled per unit time by a carrier in the direction of the electric field. Here X_f is expressed in the reduced units. We have divided X_f by 2α in the above equation to express it in the units of distance. Here ν is the hopping attachment frequency and it may be taken as a phonon frequency. The mobility is obtained by dividing this quantity by the electric field:

$$\mu=\frac{\nu}{2\alpha E} X_f \exp(-R_{NN}) . \tag{5}$$

Finally, the conductivity of carriers can be obtained from the mobility as[20,21,15]

$$\sigma(T,E)=k_B T q \int_{-\infty}^{\infty} D(\varepsilon) f(\varepsilon) \mu(\varepsilon,T,E) d\varepsilon . \tag{6}$$

Here $D(\varepsilon)$ and $f(\varepsilon)$ are the DOS and the Fermi distribution function, respectively. In the present approach, the mobility is calculated as a function of energy with respect to the Fermi energy. Therefore it is straightforward to include extended states in the DOS distribution for the calculation of the conductivity.

III. VRH IN QOD DEVICES

Here we calculate the VRH hopping conduction in QOD devices. Let us consider a system of length L and width W and assume that the width of the sample is smaller than the hopping distance. If this criterion is satisfied, then the sample behaves as a quasi-one-dimensional system. First, let us calculate the number of unoccupied states within a range R in the hopping space as a function of temperature and electric field for a particular site of energy ε. For a given T,E, ε, the number of vacant sites within range R, is given in the form

$$N=\frac{k_B T W}{2\alpha}\int_0^{x_{max}} dx \int_0^{\omega_{max}} D(\omega)[1-f(\omega)] d\omega , \tag{7}$$

where x_{max} and ω_{max} are the maximum value of integration for x and ω, respectively. For a given contour R, x_{max} is obtained from Eq. (2) by putting $\omega=0$. This gives $x_{max}=(R-\varepsilon)/(1\pm\beta)$. Note that θ has two possible values, $\theta=0$ and $\theta=\pi$, in the QOD. Similarly, for a given x and R, ω varies from 0 to $\omega_{max}=R-\varepsilon-x(\pm\beta)$. Here $D(\omega)$ is the DOS for two-dimensional systems. Us-

ing the above equation, one can calculate numerically the hopping range, the mobility, and the conductivity by using Eqs. (3), (5), and (6), respectively. But we will obtain the analytical expressions of the above quantities by considering the constant and the energy-dependent DOS in the following sections since it is one of the aims of this paper.

A. Constant DOS

Mott used the constant DOS, i.e., $D(\omega)=D_0$ to calculate the VRH conductivity of bulk amorphous solids. Here we will use the constant DOS to calculate the mobility and the conductivity. Putting $D(\omega)=D_0$ and replacing the energy derivative of the Fermi distribution by the δ function in Eq. (7), we get the approximate expression of N for $\varepsilon<0$:

$$N=\frac{\kappa}{1+\beta^2}\{(R+\varepsilon)^2[1+f(R+\varepsilon) -2(R+\varepsilon)\ln(1-f(R+\varepsilon)] +2\varepsilon\ln f(\varepsilon)+\varepsilon^2[1-f(\varepsilon)]-\pi^2/3\} . \quad (8)$$

Here κ is defined as $\kappa=(D_0WTk_B)/2\alpha$. From the above equation and Eq. (4), we can calculate the probability distributions for $\varepsilon<0$ as

$$P_{\rm NN}(R)=\frac{\kappa}{1+\beta^2}\{2(R+\varepsilon)+f(R+\varepsilon)[3(R+\varepsilon)+(R+\varepsilon)^2]+(R+\varepsilon)^2[f(R+\varepsilon)]^2-2\ln[1-f(R+\varepsilon)]\}\exp[-N(R)] . \quad (9)$$

Putting Eq. (9) into Eq. (3), the hopping range $R_{\rm NN}$ can be calculated as

$$R_{\rm NN}=\int_0^\infty {\rm Re}^{[-N(R)]}dR . \quad (10)$$

While it is possible in principle to calculate the value of $R_{\rm NN}$ numerically from Eq. (10), a good estimate of $R_{\rm NN}$ (or the critical value of $R_{\rm NN}$) can be obtained when the total number of the accessible states is equal to 1.[20,21,46] For $\varepsilon<0$, this gives the following approximate value of $R_{\rm NN}$:

$$R_{\rm NN}=B_0R_0-B_1-\frac{B_2}{2R_0} , \quad B_0=1+f(\varepsilon+R_0) ,$$
$$R_0=\kappa^{-1/2}\sqrt{1-\beta^2)/2}=(T_0^M/T)^{1/2}(1-\beta^2)^{1/2} , \quad T_0^M=(2\alpha)/(k_BWD_0) ,$$
$$B_1=\varepsilon[1+f(R+\varepsilon)]-2\ln[1-f(\varepsilon+R_0)] ,$$
$$B_2=\varepsilon^2[2+f(\varepsilon+R_0)-f(\varepsilon)]-2\varepsilon\{\ln[1-f(\varepsilon+R_0)]-\ln[f(\varepsilon)]\}-\frac{\pi^2}{3} . \quad (11)$$

Now let us calculate the average spatial forward hopping distance X_f appearing in Eq. (5). A particle of energy ε at a given site hops a distance $R_{\rm NN}$ in the hopping space, but these hops in the spatial space will be in random directions and the net displacement of a carrier in the spatial space will be zero. However, in the presence of an electric field, a greater spatial distance will be hopped in the downfield direction than in the upfield direction. If one sums over all final sites for an initial energy ε, there will be an average spatial forward distance hopped, X_f. This quantity is obtained by averaging $x\cos\theta=$const. as follows. X_f has been evaluated for QTD systems in Ref. 21. For the QOD system it is given as

$$X_f=\frac{\Sigma_\pm\int_{-\infty}^{\varepsilon+R_{\rm NN}}D(\omega)[1-f(\omega)]xd\omega}{\Sigma_\pm\int_{-\infty}^{\varepsilon+}D(\omega)[1-f(\omega)]d\omega} . \quad (12)$$

Here $x=(R+\varepsilon-\omega)/(1\pm\beta)$ for $\omega\pm x\beta>\varepsilon$ and $x=R$ for $\omega\pm x\beta<\varepsilon$. For a constant DOS and $\varepsilon<0$, the above equation reduces to

$$X_f=\frac{\beta}{(1-\beta^2)}(R+\varepsilon)\eta(R_{NN},\varepsilon) ,$$
$$\eta(R_{\rm NN},\varepsilon)=\left[1-\frac{\varepsilon\ln f(\varepsilon)/(R+\varepsilon)}{\ln f(R_{\rm NN}+\varepsilon)}+\frac{(R+\varepsilon)[1-f(R_{\rm NN}+\varepsilon)]}{2\ln f(R_{\rm NN}+\varepsilon)}-\frac{\varepsilon^2[1-f(\varepsilon)]/(R_{\rm NN}+\varepsilon)}{\ln f(R_{\rm NN}+\varepsilon)}+\frac{\pi^2/(R+\varepsilon)}{6\ln[f(R_{\rm NN}+\varepsilon)]}\right] . \quad (13)$$

Note that, in the absence of an electric field, the above expression becomes zero. Similarly, one can also calculate the expressions for N, $P_{\rm NN}$, $R_{\rm NN}$, and X_f for $\varepsilon>0$. For $\varepsilon=0$, N, $P_{\rm NN}$, $R_{\rm NN}$, and X_f are calculated by putting $\varepsilon=0$ in Eqs. (8), (9), (11), and (13), respectively. The hopping range at the Fermi level is denoted by $R_{\rm NN}^0$. A simpler expression for N, $R_{\rm NN}$, and X_f is obtained if it is assumed that for $\varepsilon<0$ the Fermi distribution function is

close to 1:

$$N=\frac{\kappa(R+\varepsilon)^2}{(1-\beta^2)} ,$$
$$R_{NN}=R_0-\varepsilon , \tag{14}$$
$$X_f=\frac{2\beta}{1-\beta^2}(R_{NN}+\varepsilon) ,$$

where R_0 is given in Eq. (11).

The mobility and the conductivity can be numerically calculated by putting the expressions for X_f and R_{NN} into Eqs. (5) and (6), respectively, but an analytical expression for the conductivity can be found if we make the following approximations. For bulk materials, Apsley and Hughes[20] pointed out that the mobility is relatively independent of X_f so they assumed that X_f at $\varepsilon=0$ is valid for all ε of interest. They also pointed out that, for $\varepsilon>0$, the hopping range is a relatively weak function of ε.[20,21] Therefore it is a reasonable approximation to assume that Eq. (11) is valid for all $\varepsilon>0$. Using these approximations, the expression for the mobility takes the following form:

$$\mu=\frac{\nu\beta}{\alpha E(1-\beta^2)}R^0_{NN}\exp(-R_{NN}) . \tag{15}$$

If we assume further that the most important contribution to the conductivity comes from electrons located near the Fermi level, then we can expand R_{NN} near the Fermi energy in powers of ε by using the Taylor series:

$$R_{NN}=R^0_{NN}+\lambda_1\varepsilon ,$$
$$\lambda_1=-1+(2-R_0)f(R_0)+R_0f^2(R_0)+\frac{\ln\{2[1-f(R_0)]\}}{R_0} . \tag{16}$$

We retained the dominating terms of order ε in the above equation. Using Eqs. (15), (16), and (6) the analytical expression for the conductivity has the form

$$\sigma=\frac{qD_0k_BT\nu\eta R^0_{NN}\beta}{\alpha E(1-\beta^2)}\frac{(1+\lambda_1)}{\lambda_1}R^0_{NN}\exp(-R^0_{NN}) . \tag{17}$$

If the values of R_{NN} and X_f at the Fermi level are used in the calculation of the conductivity then the above expression can be further simplified to

$$\sigma=\frac{2qD_0k_BT\nu\beta}{\alpha E(1-\beta^2)}R_0\exp(-R_0) . \tag{18}$$

The above expression for the conductivity predicts that $\ln\sigma$ has $(T_0^M/T)^{1/2}$ temperature dependence, which agrees with the result of Mott. Note also that the above equation predicts that $\ln\sigma$ has the $(1-\beta^2)^{1/2}$ electric-field dependence.

B. Energy-dependent DOS

Mott[14,15] and others[18,20–42] assumed that the DOS is constant even at the Fermi level. Using this DOS, we have calculated the conductivity in the previous section. In this section, we include the effect of the electron-electron interaction through the DOS in the calculation of the conductivity. Efros and Shklovski[16,17] calculated the DOS in the presence of electron-electron interaction for amorphous solids and found that $D(\omega)=D_1\omega^{(d-1)}$ at low temperatures. Here d is the dimensionality of the material and ω is measured with respect to the Fermi level. Note that this equation gives a zero DOS at the Fermi level. This contradicts the approximation made by Mott and others at low temperatures. Efros and Shklovskii have also predicted that the electron excitation spectrum has an energy gap at the Fermi level. This gap is called the Coulomb gap and denoted by Δ_{CG}. Recently, some authors[43–45] considered a more general energy-dependent DOS, i.e., $D(\omega)=D_1\omega^n$, in the calculation of the conductivity for bulk materials, where n can be any number. This equation reduces to that of Efros and Shklovskii for $n=d-1$. We use this DOS in the calculation of the conductivity.

Using the energy-dependent DOS and following the method for the previous section, we obtain the expressions for N and R_{NN} for $\varepsilon<0$ as

$$N(\varepsilon,R)=A_0\{(R+\varepsilon)^{n+2}[1-f(R+\varepsilon)]-\varepsilon^{n+2}[1-f(\varepsilon)]\} , \tag{19}$$
$$A_0=\frac{(k_BT)^{n+1}WD_1}{\alpha(n+1)(n+2)(1-\beta^2)} ,$$
$$R_{NN}=R_0-\varepsilon+\frac{f(R_0+\varepsilon)}{n+2}+\frac{R_0^{-(n+1)}}{n+2}\left[\frac{\varepsilon}{1+\varepsilon/R_0}\right]^{n+2}[1-f(\varepsilon)] , \tag{20}$$
$$R_0=\left[\frac{T_0^{ES}}{T}\right]^{(n+1)/(n+2)}[1-\beta^2]^{1/(n+1)} ,$$
$$T_0^{ES}=\left[\frac{k_B^{n+1}WD_1}{2\alpha(n+1)(n+2)}\right]^{-1/(n+1)} ,$$
$$X_f=\frac{2\beta}{1-\beta^2}\frac{(R+\varepsilon)}{n+2}\times\left[1-R_{NN}^{-(n+1)}\times\frac{\varepsilon^{n+2}(1-f(\varepsilon)}{(1+\varepsilon/R_{NN})^{n+2}[1-f(R_{NN}+\varepsilon)]}\right] . \tag{21}$$

To get the above analytical expressions for R_{NN}, N, and X_f, we have replaced the energy derivative of the Fermi-distribution function by a δ function. This approximation is valid at low temperatures. Without this approximation, it is difficult to get the analytical expressions of the above quantities. If we further assume that all states above and below the Fermi level are empty and filled, respectively, then Eq. (20) reduces to

$$R_{NN}=R^0_{NN}=\left[\frac{T_0^{ES}}{T}\right]^{(n+1)/(n+2)}[1-\beta^2]^{1/(n+2)} . \tag{22}$$

Note that for $n=0$ (i.e., constant DOS) the above expression gives the same temperature dependence as that of Eq. (14) of the last section. Finally, we can calculate the conductivity as follows:

$$\sigma=\frac{q\nu\beta R_{\rm NN}^{0}(k_BT)^{n+1}(\varepsilon_F)^{n+1}}{(1-\beta^2)(n+1)(n+2)E\alpha}\exp(-R_{\rm NN}^{0}) . \quad (23)$$

Efros and Shklovskii[16,17] obtained $D(\omega)=D_1$ (i.e., $n=0$) for one-dimensional (1D) systems and $D(\omega)=D_1\omega$ (i.e., $n=1$) for 2D systems in the presence of electron-electron interactions. Since our system is quasi-one-dimensional, we can choose either $n=0$ or $n=1$ depending on the thickness of the sample. If we choose $n=0$, the above equation gives $[T_0/T]^{1/2}$ and $[1-\beta^2]^{1/2}$ temperature and electric-field dependence, respectively. The temperature dependence of the conductivity agrees with the analytical calculation of Efros and Shklovskii.[16,17] If we choose $n=1$, we get

$$\sigma=\frac{q\nu\beta(k_BT\varepsilon_F)^2}{6E\alpha}\left[\frac{T_0}{T}\right]^{(2/3)}(1-\beta^2)^{(-2/3)}$$

$$\times\exp\left[-\left[\frac{T_0}{T}\right]^{(2/3)}(1-\beta^2)^{(1/3)}\right] . \quad (24)$$

It is interesting to note from the above expression that $\ln\sigma$ follows $[T_0/T]^{(2/3)}$ temperature dependence instead of $[T_0/T]^{(1/2)}$. Similarly, $\ln\sigma$ follows $(1-\beta^2)^{(1/3)}$ electric-field dependence rather than $(1-\beta^2)^{(1/2)}$.

IV. VRH IN QTD DEVICES

In this section, we calculate the hopping range, the mobility, and the conductivity for QTD systems in the presence of the constant and the energy-dependent DOS. We have already presented preliminary results in a short paper[21] by using the constant density of states but have neglected the effect of electron-electron interactions. Here we present more detailed calculations for the above quantities and will include the effect of electron-electron interaction through the energy-dependent DOS. To get analytical expressions for the above quantities, we will use the same approximations as those of the previous section.

A. Constant density of states

For a given ε, R, T, and E the number of unoccupied sites N in the hopping space is given as[20]

$$N=\frac{k_BT}{2\alpha^2}\int_0^{\pi}d\theta\int_0^{x_{\max}}xdx\int_0^{\omega_{\max}}D(\omega)[1-f(\omega)]d\omega , \quad (25)$$

where $x_{\max}$ and $\omega_{\max}$ are the maximum values of x and ω, respectively, and they are obtained from Eq. (2) as $x_{\max}=(R+\varepsilon)/(1+\beta\cos\theta)$ and $\omega_{\max}=R+\varepsilon$. Putting the constant DOS into the above equation, we get

$$N=\frac{\kappa}{(1-\beta^2)^{3/2}}[N_\alpha+N_\beta] ,$$

$$N_\alpha=\{(R+\varepsilon)^2[1+2f(R+\varepsilon)]$$

$$-3(R+\varepsilon)^2\ln[1-f(R+\varepsilon)]\} , \quad (26)$$

$$N_\beta=\{\varepsilon(6R+3)\ln[f(\varepsilon)]$$

$$+\varepsilon^2(3R+4)[1-f(\varepsilon)-(R+\varepsilon)\pi^2]\} ,$$

for $\varepsilon<0$. Here $\kappa=(Dk_BT\pi)/(12\alpha^2)$. The expression for $P_{\rm NN}$ is obtained from Eq. (4) and the above equation as

$$P_{\rm NN}=\frac{\kappa}{(1-\beta^2)^{3/2}}[\Delta N_\alpha+\Delta N_\beta]\exp[-N(R)] ,$$

$$\Delta N_\alpha=([3(R+\varepsilon)^2+3f(R+\varepsilon)][(R+\varepsilon)^3+15(R+\varepsilon)^2]$$

$$-(R+\varepsilon)^2\{f(R+\varepsilon)-6\ln[1-f(R+\varepsilon)]\}) , \quad (27)$$

$$\Delta N_\beta=\{6\ln f(\varepsilon)+3\varepsilon^2[1-f(\varepsilon)]-\pi^2\} .$$

Following the method of the last section, we get the expressions for the hopping range:

$$R_{\rm NN}=A_0R_0-\frac{A_1}{3}-\frac{A_2}{3R_0}-\frac{A_3}{3R_0^2} ,$$

$$R_0=(1-\beta^2)^{1/2}\kappa^{-1/3} , \quad A_0=1+2f(R_0+\varepsilon) ,$$

$$A_1=3\varepsilon+6\varepsilon f(R_0+\varepsilon)-3\ln[1-f(R_0+\varepsilon)] , \quad (28)$$

$$A_2=3\varepsilon^2[2+2f(R_0+\varepsilon)-f(\varepsilon)]$$

$$-6\varepsilon\{\ln f(\varepsilon)-\ln[1-f(R_0+\varepsilon)]-\pi^2\} ,$$

$$A_3=5\varepsilon^2+3\varepsilon^2[f(R_0+\varepsilon)-2f(\varepsilon)]$$

$$-3\varepsilon^2\{\ln f(\varepsilon)-\ln[1-f(R_0+\varepsilon)]\}-\varepsilon\pi^2 .$$

For a QTD system the X_f is given as[20,21]

$$X_f=\frac{X_N}{X_D}=\frac{\int_0^{\pi}\cos\theta d\theta\int_{-\infty}^{(R_{\rm NN}+\varepsilon)}d\omega D(\omega)[1-f(\omega)]x^2}{\int_0^{\pi}d\theta\int_{-\infty}^{(R_{\rm NN}+\varepsilon)}d\omega D(\omega)[1-f(\omega)]x} . \quad (29)$$

Here $x=(R+\varepsilon-\omega)/(1+\beta\cos\theta)$ for $x\cos\theta+\omega>\varepsilon$ and $x=R$ for $x\cos\theta+\omega<\varepsilon$. For the constant DOS, we get

$$X_N=\frac{\beta}{(1-\beta^2)^{3/2}}(R+\varepsilon)^3[1+2f(R_{\rm NN}+\varepsilon)]-3(R_{\rm NN}+\varepsilon)^2\ln\left[\frac{1-f(R_{\rm NN}+\varepsilon)}{f(\varepsilon)}\right]$$

$$-3R_{\rm NN}^2\ln[f(\varepsilon)]+(3R_{\rm NN}\varepsilon^2-2\varepsilon^3)[1-f(\varepsilon)-\pi^2/2] ,$$

$$X_D=\frac{1}{(1-\beta^2)^{1/2}}(R_{\rm NN}+\varepsilon)^2[1+f(R_{\rm NN}+\varepsilon)-2(R+\varepsilon)]\ln[1-f(R_{\rm NN}+\varepsilon)]-2\varepsilon\ln f[\varepsilon+\varepsilon^2(e)+\pi^3/3] . \quad (30)$$

In a similar way one can find the expressions for the above quantities for $\varepsilon>0$ and $\varepsilon=0$. Simpler expressions for N, R_{NN}, and X_f are obtained if the energy derivative of the Fermi-distribution function is replaced by a δ function:

$$N=\kappa(R+\varepsilon)^3\ ,\ R_{NN}=R_0-\varepsilon\ ,$$
$$X_f=\frac{2\beta R_{NN}}{3(1-\beta^2)}\left[1+\frac{\varepsilon}{R_{NN}}\right] \tag{31}$$
$$R_0=\kappa R_{NN}-1/3=\left[\frac{T}{T_0^M}\right]^{-1/3}\ ,\ T_0^M=\frac{12\alpha^2}{\pi D k_B}\ .$$

The above expression for the hopping range at the Fermi level reduces to $R_{NN}=R_{NN}^0=R_0$. Following the method of Sec. III A, we get the following expressions for the mobility and conductivity:

$$\mu=\frac{\beta\eta\nu}{(1-\beta^2)2\alpha E}R_{NN}^0\exp(-R_{NN}^0)\ , \tag{32}$$

$$\sigma=\frac{qD_0k_BT\nu\eta(R_{NN}^0\beta)}{\alpha E(1-\beta^2)}\frac{(1+\lambda_2)}{\lambda_2}R_{NN}^0\exp(-R_{NN}^0)\ , \tag{33}$$

where λ_2 is given as

$$\lambda_2=-\left[1+(3-2R_0)f(R_0)+2R_0[f(R_0)]^2+\frac{\ln[2(1-f(R_0)]+\pi^2}{R_0}-\frac{\pi^2}{3R_0^2}\right]\ . \tag{34}$$

Finally, the above expression for the conductivity can be further simplified if we assume that all the states below and above the Fermi level are filled and empty, respectively. In this approximation, the conductivity takes the form

$$\sigma=\frac{2qD_0k_BT\nu\beta}{3\alpha E(1-\beta^2)}R_0e^{(-R_0)}\ . \tag{35}$$

Note that $\ln\sigma$ has $(T_0^M/T)^{1/3}$ temperature and $(1-\beta^2)^{1/2}$ electric-field dependence. The temperature dependence of the conductivity agrees with the results of Mott.[14,15]

B. Energy-dependent DOS

As we pointed out in the previous section, Efros and Shklovskii obtained $D(\omega)=D_1\omega$ for QTD systems in the presence of electron-electron interactions. Here we use a more general form of the DOS, i.e., $D(\omega)=D_1\omega^n$. Putting this into Eq. (25), we get the analytical expression for N:

$$N(\varepsilon,R)=B_0\{(R+\varepsilon)^{n+3}[1-f(R+\varepsilon)]-[R(n+3)\varepsilon^{n+2}-\varepsilon^{n+3}](1-f(\varepsilon)\}\ ,$$
$$B_0=\frac{\pi(k_BT)^{n+1}D_1}{2\alpha^2(n+1)(n+2)(n+3)(1-\beta^2)^{3/2}}\ . \tag{36}$$

Following the method of Sec. III B, R_{NN} and X_f for $\varepsilon<0$ are calculated as

$$R_{NN}=R_0-\varepsilon+\frac{1+f(R_0+\varepsilon)-f(\varepsilon)}{n+3}\left[\frac{(n+3)R_0^{-(n+1)}}{1+\varepsilon/R_0}\left[\frac{\varepsilon}{1+\varepsilon/R_0}\right]^{-(n+2)}+R_0^{-(n+2)}\left[\frac{\varepsilon}{1+\varepsilon/R_0}\right]^{n+3}\right]\ , \tag{37}$$

$$X_f=\frac{C_N\left[1-\frac{1-f(\varepsilon)}{1-f(R_{NN}+\varepsilon)}\left[\frac{\varepsilon}{(R_{NN}+\varepsilon)}\right]^{n+3}-\frac{2n(2n+3)}{(R_{NN}+\varepsilon)}\left[\frac{\varepsilon}{R_{NN}+\varepsilon}\right]^{n+2}\right]}{C_D\left[1-\frac{1-f(\varepsilon)}{1-f(R_{NN}+\varepsilon)}\left[\frac{\varepsilon}{(R_{NN}+\varepsilon)}\right]^{n+2}\right]}\ , \tag{38}$$

where R_0, T_0^{ES}, N, and C_D are given as

$$R_0=\left[\frac{T_0^{ES}}{T}\right]^{(n+1)/(n+3)}[1-\beta^2]^{3/(2n+6)}\ ,$$
$$T_0^{ES}=\left[\frac{k_B^{n+1}\pi D_1}{2\alpha^2(n+1)(n+2)(n+3)}\right]^{-1/(n+1)}\ ,$$
$$C_N=\frac{(R+\varepsilon)^{n+3}[1-f(R_{NN}+\varepsilon)]}{(n+1)(n+2)(n+3)}\frac{2\beta}{(1-\beta^2)^{3/2}}\ , \tag{39}$$
$$C_D=\frac{(R+\varepsilon)^{n+2}[1-f(R_{NN}+\varepsilon)]}{(n+1)(n+2)}\frac{1}{(1-\beta^2)^{1/2}}\ .$$

Again, if we assume that all states above and below the Fermi level are empty and filled, respectively, then Eqs. (37) and (38) reduce to

$$R_{NN}=R_{NN}^0=\left[\frac{T_0^{ES}}{T}\right]^{(n+1)/(n+3)}[1-\beta^2]^{3/(2n+6)}\ , \tag{40}$$

$$X_f=\frac{2R_{NN}^0\beta}{(n+3)(1-\beta^2)}\ . \tag{41}$$

Note that for $n=0$ (i.e., constant DOS), Eq. (40) gives a similar temperature and electric-field dependence as Eq. (31). For $D(\omega)=D_1\omega$, Eq. (40) is reduced to the form

$$R_{NN}=R_{NN}^0=\left[\frac{T_0^{ES}}{T}\right]^{1/2}[1-\beta^2]^{3/8}\ . \tag{42}$$

Finally, following the method of the previous sections, the expression for the conductivity is calculated as

$$\sigma=\frac{q^2\nu\beta R_{NN}^0(k_BT)^{n+1}}{(1-\beta^2)(n+3)2\alpha^2}\exp(-R_{NN}^0)\ . \tag{43}$$

For $D(\omega)=D_1\omega$, the conductivity takes the following form:

$$\sigma=\frac{q^2\nu\beta k_B T}{4\alpha^2}\left[\frac{T_0^{\mathrm{TS}}}{T}\right]^{(1/2)}(1-\beta^2)^{(-3/8)} \times\exp\left[-\left[\frac{T_0^{\mathrm{ES}}}{T}\right]^{(1/2)}(1-\beta^2)^{(3/8)}\right]. \quad (44)$$

It is interesting to note from the above expression that $\ln\sigma$ follows a $(T_0^{\mathrm{ES}}/T)^{1/2}$ temperature and $(1-\beta^2)^{3/8}$ electric-field dependence. The temperature dependence of the conductivity agrees with the analytical results of Efros and Shklovskii.[16,17]

V. RESULTS AND DISCUSSIONS

In this section, we compare our theoretical calculations with the resistivity experiments on PBCO thin films. Kabasawa *et al.*[5] measured the resistivity of the PBCO single crystal film as a function of temperature. The thickness of the film was 3000 Å and the film orientation was in the [110] direction. This means that the [110] direction and CuO plane were perpendicular to the film surface. The current path at the junction area was parallel to the CuO plane and perpendicular to the c axis. Because of this reason the PBCO films act like QTD systems. The resistivity-temperature characteristics are shown in Fig. 1. These experimental data are obtained under a very weak electric field of about 65 V/m. Since the electric field in the experiments is very weak, we will put zero electric field in our numerical calculations. Recently Singh *et al.*[21] tried to explain these experiments by using the VRH theory based on constant DOS. They found a fairly good agreement between theory and experiments at high temperatures but there was disagreement between theory and experiments at low temperatures, i.e., $T<10$ K. In their calculations, they did not include the effect of electron-electron interaction.

Here we have examined the role of electron-electron interactions to explain the low-temperature data of the resistivity. We have used Eqs. (35) and (44) to explain the high- and low-temperature experimental data of Kabasawa *et al.*, respectively. We used T_0^M and T_0^{ES} as fitting parameters since the physical parameters such as α, D_0, and ε_r are not known for PBCO. The best fit between theory and experiments is obtained when $T_0^M=2.7\times10^4$ K and $T_0^{\mathrm{ES}}=1.3\times10^3$ K. Our calculations show that electron-electron interaction plays an important role in explaining the low-temperature data of the resistivity. This finding is not surprising, since in oxide compounds the electron-electron interactions play a very important role in explaining the electronic properties of both normal and superconducing states.[46–50] The ratio of these two parameters is about 21. The entire temperature-dependent resistivity data cannot be fitted by considering either Eq. (35) or Eq. (44).

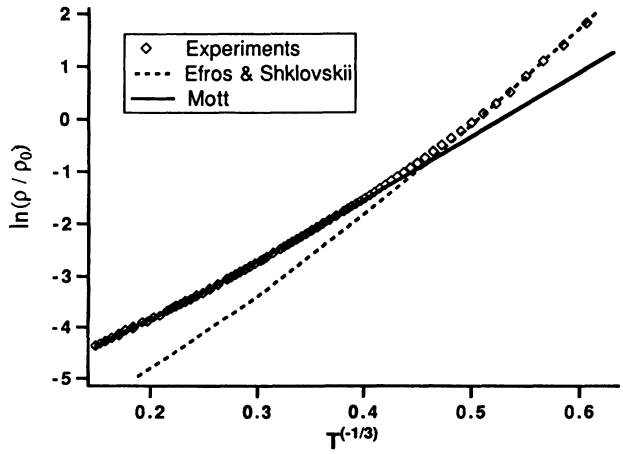

FIG. 1. Plot of ln of resistivity versus temperature for PBCO film.

The constant DOS D_0 can be obtained from the fitted value of T_0^M. We found that its value is about 4.5×10^{35} $\mathrm{J^{-1}\,m^{-2}}$. This value is of the same order of magnitude as that of Refs. 5 and 10. Singh *et al.*[21] obtained a value of about 3.5×10^{35} $\mathrm{J^{-1}\,m^{-2}}$ by fitting the entire range of the temperature dependence experiments. This value of the DOS was not able to explain the experimental data at low temperatures. $1/\alpha=8.5$ nm is taken from Refs. 5 and 21 to calculate the constant DOS.

Let us find the temperature or energy range where Eq. (35) and Eq. (44) play an important role. Efros and Shklovskii showed that, when $\Delta_{\mathrm{hop}}^M>\Delta_{\mathrm{cg}}$, $D(\omega)=D_0$, where Δ_{hop}^m is the mean hopping energy. According to Mott, Δ_{hop}^m for 2D systems is given by $\Delta_{\mathrm{hop}}^m=(\pi R_{\mathrm{NN}}^2 D_0)^{-1}$, where R_{NN} is the hopping range given by Eq. (31) for a simple case. Putting this value into the above equation, we get $\Delta_{\mathrm{hop}}^M=(k_BT/3)(T_0^M/T)^{1/3}$. This equation and the condition $\Delta_{\mathrm{hop}}^M>\Delta_{\mathrm{CG}}$ give the criterion $T>T_M$, where $T_M=[3\Delta_{\mathrm{CG}}/k_B(T_0^M)^{1/3}]^{3/2}$. According to this criterion, Eq. (35) plays an important role for $T>T_M$.

Efros and Shklovskii also showed that, when $\Delta_{\mathrm{hop}}^{\mathrm{ES}}<\Delta_{\mathrm{CG}}$, $D(\omega)=D_1\omega$, where $\Delta_{\mathrm{hop}}^{\mathrm{ES}}$ is the hopping energy in the presence of electron-electron interactions. They obtained $\Delta_{\mathrm{hop}}^{\mathrm{ES}}=(4q^{-2}\varepsilon_0\varepsilon_r R_{\mathrm{NN}})^{-1}$ where ε_r is the dielectric constant of the material. Putting R_{NN} from Eq. (42) into the above equation, we get $\Delta_{\mathrm{hop}}^{\mathrm{ES}}=(k_BT)(T_0^{\mathrm{ES}}/6T)^{1/2}$. From this equation and the condition $\Delta_{\mathrm{hop}}^{\mathrm{ES}}<\Delta_{\mathrm{CG}}$, we get the temperature criterion $T<T_{\mathrm{ES}}$ where $T_{\mathrm{ES}}=[6\Delta_{\mathrm{CG}}/k_B(T_0^{\mathrm{ES}})^2]^2$. According to this criterion, Eq. (44) plays a very important role in the temperature range $T<T_{\mathrm{ES}}$. In other words, to observe the effect of electron-electron interaction, the measurement temperature T should satisfy the criterion $T<T_{\mathrm{ES}}$. The numerical values of T_M and T_{ES} are calculated and found to be about 8.5 and 8 K, respectively. In the calculation of these values, $\Delta_{\mathrm{CG}}=3.5$ meV is taken from the work of Kabasawa *et al.* They obtained this value by measuring the voltage dependence of the conductance of PBCO-based S/N/S junctions.[5,4] Let us compare the above theoretical values of T_M and T_{ES} with those obtained from Fig. 1. One can see from Fig. 1 that a good agreement between Eq. (44) and experiments is found for $T<T_{\mathrm{ES}}$ (=7 K) whereas Eq. (35) is able to explain experiments after $T>T_M$ (=15 K). Hence these values ob-

tained from Fig. 1 agree with the above theoretical values of T_M and T_{ES}.

Let us call Eq. (35) and Eq. (44) Mott-type VRH conduction (MVRH) and Efros-and-Shklovskii-type VRH conduction (ESVRH), respectively. This is because in the derivation of Eq. (35) a constant DOS is used and Mott was the first person who used the constant DOS in the calculation of the conductivity in amorphous materials. Similarity, in the derivation of Eq. (44) the energy-dependent DOS of Efros and Shklovskii is used. We note that the crossover from MVRH to ESVRH occurs when $\Delta_{hop}^{M}=\Delta_{hop}^{ES}$. This condition gives the crossover temperature $T_{cross}=\sqrt{1.5}(T_0^{ES})^3/(T_0^{M})^2$. The nice feature of this equation is that it is expressed in terms of T_0^M and T_0^{ES}, which are obtained by comparing theory and experiments. Using the value of these two parameters, we get $T_{cross}=10$ K. We can also obtain the crossover temperature from Fig. 1. It is found to be 10.5 K, which agrees with the above theoretical value.

The dielectric constant ε_r and Δ_{CG} are not known for PBCO compounds. We can calculate these quantities as follows. Efros and Shklovskii found that $D_1=\varepsilon_r^2/q^4$. With the help of this equation and the equation for T_0^{ES}, we get $\varepsilon_r=(\sqrt{24}q^2\alpha)/(4\pi\varepsilon_0 k_B T_0^{ES})$. Putting the fitted value of T_0^{ES} and α, we get $\varepsilon_r=7.5$. Using this value of ε_r, we get $\Delta_{CG}=3.0$ meV, which is very close to the value 3.5 meV reported by Kabasawa *et al.*[5] All parameters obtained from theory and experiments are summarized in Table I.

Note that the conductivity of the QOD device with constant DOS [Eq. (18)] and the conductivity of the QTD device with electron-electron interactions [Eq. (44)] have the same temperature dependence. Therefore, to explain the present experiment, one can use either of the two equations. If one uses Eq. (18) to explain the low-temperature experiments, then one gets the transition from QTD to QOD. If one uses Eq. (44), one gets the transition from MVRH to ESVRH. But we used Eq. (44) instead of Eq. (18) to compare the experiments and the theory. As we know, if the hopping range is greater than or equal to the thickness of the sample, then there is a crossover from 2D to 1D. The PBCO sample of Kabasawa *et al.* is 3000 Å thick and the film orientation is in the [110] direction. This means that the [110] direction and the CuO plane are perpendicular to the film surface. The current flows parallel to the CuO plane and perpendicular to the c axis. Therefore, if the hopping range is greater than or equal to 3000 Å, the PBCO thin film can act as a QTD system. We have calculated the hopping range and found that the ratio of the hopping range to the sample thickness varies from about 0.12 to 0.26 depending on the temperature. These results are presented in Fig. 2. Hence these results eliminate the possibility of the crossover from 2D to 1D since the hopping range is smaller than the thickness of the sample. It is interesting to note that Eq. (18) and Eq. (44) have different electric-field dependence of the conductivity. Therefore, by measuring the electric field dependence of conductivity, one can remove the above ambiguity. Finally, from the above analysis, we conclude that there is a crossover from MVRH to ESVRH in PBCO samples.

TABLE I. Parameters obtained from experiments and theory.

Parameters	Experiments	Theory
$1/\alpha$ (nm)	8.5	
D_0 (J/m^2)	4.5×10^{35}	
T_0^M/T_0^{ES}	21.0	
T_{cross} (K)	11.0	10.5
Δ_{CG} (meV)	3.5	3.0
ε_r		8.0

There are some other cases where the crossover from MVRH to ESVRH in the temperature dependence of the conductivity is observed in bulk amorphous solids. For example, Glukhov *et al.*[51] have observed this crossover from the hopping exponent 1/4 to 1/2 in granulated Sn-Ge and Ag-Ge. Recently, Rosenbaum[52] has also reported the crossover from MVRH to ESVRH in amorphous In_xO_y. He has found that he can explain his experiments at low temperature by taking the hopping exponent 0.56 rather than 0.5. He has calculated the crossover temperature and found that the crossover temperature is sample dependent and varies from 9.6 to 52 K. Similarly, he also found that the ratio T_0^M/T_0^{ES} is sample dependent and varies from 30 to 80. In Sn-Ge and Ag-Ge samples, the ratio T_0^M/T_0^{ES} is also sample dependent and varies from 3 to 5. In PBCO thin films, we found that this ratio is about 21.

In conclusion, a theoretical calculation has been performed for the hopping conduction mechanism in low-dimensional devices in the presence of temperature and electric field. The hopping range, the mobility, and the conductivity are calculated for constant and frequency-dependent DOS. In the present calculation, the states both below and above the Fermi level are included. The analytical expressions for the hopping range and the conductivity are presented for both types of DOS. The effect of the electron-electron interaction is included in the present calculations through the density of states. In the

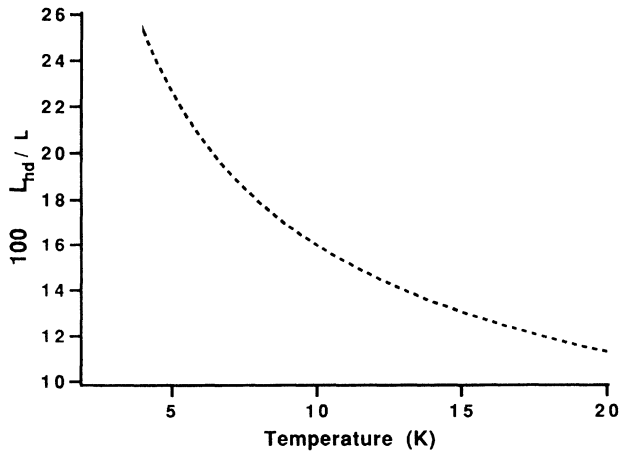

FIG. 2. Plot of the percentage of the ratio of the hopping distance to the thickness of the PBCO film as a function of temperature. Here L_{hd} is the hopping distance.

presence of electron-electron interaction and in a weak electric field, ln σ follows the $(1-\beta^2)^{3/8}$ and $(1-\beta^2)^{1/2}$ electric-field dependence for the QTD and QOD systems, respectively. The present theoretical calculations have been applied to explain the recent temperature-dependent resistivity experiments on PBCO films. Our theoretical calculation predicts a possible crossover from MVRH to ESVRH in PBCO films. The crossover temperature is about 11 K. More experiments are needed to confirm the above findings.

ACKNOWLEDGMENTS

The authors are thankful to Dr. S. Saitoh, M. Suga, and Dr. M. Hatano for their help in preparing the manuscript and numerical calculations. This work was performed under the management of FED (the R&D Association for Future Electron Devices) as a part of the R&D of Basic Technology for Future Industries supported by NEDO (New Energy and Industrial Technology Development Organization).

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