Effective action of a magnetic monopole in three-dimensional electrodynamics with massless matter and gauge theories of superconductivity

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We compute the one-loop effective action of a magnetic monopole in three-dimensional electrodynamics of massless bosons and fermions and find that it contains an infrared logarithm. So, when the number of massless matter species is sufficiently large, monopoles are suppressed and in the weak-coupling limit charged particles are unconfined. This result provides some support to the idea of new gauge interactions in planar electronic systems. It also provides a mechanism by which interlayer tunneling of excitations with one unit of the ordinary electric charge can be suppressed while that of a doubly charged object is allowed.

Gauge theories of high-temperature superconductors[1] assume that in certain planar electronic systems spin and charge are separated and the resulting new quasiparticles interact via Abelian gauge forces. This idea is seemingly in contradiction with three-dimensional confinement due to magnetic monopoles.[2] One may try to resolve the contradiction by assuming that some of the charged excitations are gapless. Then it is possible that the quantum effects of those excitations are sufficiently strong in the infrared to modify the interaction between monopoles in such a way that the confinement of charges is lost. To see how this can happen, consider the relativistic version of the problem—three-dimensional quantum electrodynamics of massless bosons or fermions. A simple calculation shows that the one-loop contribution of massless charged particles to the gauge field propagator causes its small-momentum behavior to change from the usual $1/p^2$ to $1/|p|$. The bilinear part of the corresponding effective action for the gauge field calculated on the monopole configuration then produces a logarithm of the total size of the system. Indeed, the field strength of a monopole is $F_{ij} \sim \epsilon_{ijk}x_k/r^3$; therefore

$$\int d^3x F_{ij} \frac{1}{\sqrt{\partial^2}} F_{ij} \sim \ln \frac{R}{a} , \tag{1}$$

where R and a are infrared and ultraviolet cutoffs, respectively. Essentially the same calculation appears in Ref. 3. The coefficient of the logarithm increases proportionally to the number N of massless species. This suggests that in the presence of massless charged particles, at least when N is large enough, monopoles are suppressed via an "infrared catastrophe" and charged particles are unconfined.

The reason why Eq. (1) is not sufficient to determine the fate of magnetic monopoles in the presence of massless charged particles even in the large-N limit is that the interaction of a charged particle with a monopole has no small parameter, so the bilinear part (1) of the effective action is in no way distinguished relative to terms containing more powers of the gauge field. To make a reliable conclusion, we need all those terms.

On the other hand, the gauge propagator in the large-N limit is of order $1/N$, so the large-N limit suppresses higher-loop contributions to the effective action. Because we find logarithmically large contributions to the effective action, such as Eq. (1), one may worry that the $1/N$ suppression of the higher-loop terms can be overcome by additional powers of the infrared logarithm that might come from these higher loops. If that were the case, the $1/N$ expansion would be inapplicable. We can argue, however, that this does not happen in relativistic three-dimensional electrodynamics of massless bosons or fermions. We use the fact that in every order of the $1/N$ expansion the effective action for the gauge field in these theories remains ultraviolet finite in the limit when the dimensionful gauge coupling e^2 is taken to infinity.[4] It follows then that this effective action as a function of momenta of the external gauge field (that is, before the substitution of the monopole field into it) does not contain any logarithms of e^2 or the ultraviolet cutoff a. For example, the bilinear part of the effective action in any loop order, at $e^2 \to \infty$, is a constant times the one-loop expression appearing on the left hand side of Eq. (1), the constant being suppressed by the power of $1/N$ corresponding to the given loop. Then, upon substituting the monopole field, only a single logarithm of R/a is produced—that already encountered at one loop. We thus argue that in the large N limit the problem reduces to the calculation of one-loop determinants of massless bosons and fermions in the monopole background.

This paper presents the results and the main steps of such a calculation. We chose to proceed with relativistic three-dimensional particles mainly because this makes the computations more transparent. The infrared divergence of the bilinear part of the gauge field action is not limited to this particular case.[3] Besides, the precise dispersion laws for quasiparticles that may occur in real electronic systems are unknown at present. Emergence of relativistic fermions in quantum insulators is possible, as discussed, for example, by Ioffe and Larkin.[3] We also found it advantageous to consider the case of massless bosons along with that of massless fermions: the two treatments run in parallel and complement each other.

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Our results confirm the presence of an infrared logarithm in a monopole's effective action in both bosonic and fermionic cases. We thus show that single monopoles are suppressed by an "infrared catastrophe" in the presence of a sufficient number of massless-matter fields. We cannot state at present what exactly this "sufficient" number is because the answer to this question lies outside the region of validity of the large-N approximation. Other methods, most likely numerical simulations, are needed to establish whether the qualitative picture of the suppression of monopoles holds down to realistic values of N.

Let us describe the physical effect of the infrared logarithm in the weak-coupling limit of three-dimensional electrodynamics, defined by the condition that the gauge coupling e^2 is much smaller than the ultraviolet scale $M \sim a^{-1}$ at which the internal structure of the monopole becomes essential. In this limit, in the absence of massless matter, monopoles and antimonopoles would form a dilute gas. When the number of massless matter species is sufficiently large, the infrared logarithm causes monopoles and antimonopoles to assemble into "molecules"—pairs of typical size d that is much smaller than the average distance between the pairs. These pairs interact by a short range potential of order $(d^2/r^2)\ln(r/d)$, so it is natural to expect that the charged particles are unconfined. This picture provides some support to the idea of new gauge interactions in planar electronic systems. At the end of this paper we discuss some further applications of our results.

We now proceed to calculating the one-loop contributions to the effective action of a monopole that are due to quantum fluctuations of a single massless charged bosonic field and a single massless charged fermionic field in the monopole background. The quantum Euclidean actions of bosonic and fermionic theories are respectively of the form

$$\mathcal{S}_B = \int \left(\frac{1}{4e^2} F_{ij}^2 + |D_i\phi|^2 + V(\phi^+\phi) \right) d^3x \, ,$$
$$\mathcal{S}_F = \int \left(\frac{1}{4e^2} F_{ij}^2 + \bar{\chi}\sigma D\chi \right) d^3x \, ,$$

where $F_{ij} = \partial_i A_j - \partial_j A_i$ is the field strength, A_i is the vector potential, $D_i = \partial_i - iA_i$ is the covariant derivative, σD is a shorthand for $\sum_i \sigma_i D_i$, σ_i are the Pauli matrices, and $i,j = 1,2,3$. A scalar potential $V(\phi^+\phi)$ does not enter explicitly our one-loop computation but is necessary, with properly tuned parameters, to keep bosons massless in full quantum theory. At the classical (tree) level, the effective action of the monopole is $S^{(0)} = \int d^3x F_{ij}^2/(4e^2)$, where the vector potential is taken to be that of the monopole. This action is of order M/e^2, where M is the ultraviolet scale of order of the inverse monopole core size.

The one-loop contributions of bosons and fermions to the effective action are respectively of the form

$$S_B^{(1)} = \mathrm{Tr}\ln(-D^2) - \mathrm{Tr}\ln(-\partial^2) \, ,$$
$$S_F^{(1)} = -\mathrm{Tr}\ln(\sigma D) + \mathrm{Tr}\ln(\sigma\partial) \, , \tag{2}$$

where A_i inside the covariant derivatives is the monopole vector potential. Equation (2) requires both ultraviolet and infrared regularizations. We compute not expressions (2) directly but rather

$$S_B^{(1)}(R) = \mathrm{Tr}\ln\mathcal{M}_B - \mathrm{Tr}\ln\mathcal{M}_{B0} \, ,$$
$$S_F^{(1)}(R) = -\tfrac{1}{2}[\mathrm{Tr}\ln(-\mathcal{M}_F^2) - \mathrm{Tr}\ln(-\mathcal{M}_{F0}^2)] \, , \tag{3}$$

where

$$\mathcal{M}_B = -\frac{1}{4R^4}(r^2+R^2)D^2(r^2+R^2) \, ,$$
$$-\mathcal{M}_F^2 = -\frac{1}{4R^4}(r^2+R^2)(\sigma D)^2(r^2+R^2) \, , \tag{4}$$

and $\mathcal{M}_{B0}$ and $-\mathcal{M}_{F0}^2$ are obtained analogously from the operators $-\partial^2$ and $-(\sigma\partial)^2$ of the vacuum sector. This replacement is similar to the one used by 't Hooft in his four-dimensional instanton calculation.[5] If, as it is in the four-dimensional case, the effective action were not infrared sensitive, the additional factors of $(r^2+R^2)/R^2$ would cancel between the vacuum and nonvacuum contributions in (3). In the present case, we intend to show that the effective action *is* infrared sensitive. In this case Eq. (3) provides an infrared regularization of Eq. (2), R being the regulator radius. In what follows we measure all distances in units of R; hence we set $R = 1$.

To calculate the traces in (3), we diagonalize the operators (4) and sum up the logarithms of the corresponding eigenvalues. The eigenvalue equations for the operators (4) are

$$\left(D^2 + \frac{4\lambda}{(1+r^2)^2} \right) \Psi_B = 0 \, , \tag{5}$$

$$\left((\sigma D)^2 + \frac{4\lambda'}{(1+r^2)^2} \right) \Psi_F = 0 \, , \tag{6}$$

where λ and λ' stand for the eigenvalues. In bosonic Eq. (5), radial and angular variables are separated by $\Psi_B = \psi(r) Y_{q,l,m}(\theta,\phi)$, where $Y_{q,l,m}$ are the monopole harmonics of Ref. 6. One gets the radial equation

$$\left[\left(\frac{\partial}{\partial r}\right)^2 + \frac{2}{r}\frac{\partial}{\partial r} - \frac{\alpha(\alpha+1)}{r^2} + \frac{4\lambda}{(1+r^2)^2} \right] \psi = 0 \, , \tag{7}$$

where $\alpha = [(l+1/2)^2 - q^2]^{1/2} - 1/2$, $l = |q|, |q|+1, \ldots$, and the multiplicity of the eigenvalue λ is $(2l+1)$. Parameter q assumes integer and half-integer values as a consequence of the Dirac quantization condition. All our results depend only on $|q|$, so in what follows we take $q \geq 0$. In fermionic Eq. (6), the variables are separated by using any of the three angular dependences $\xi_{jm}^{(1)}$, $\xi_{jm}^{(2)}$, η_m introduced in Ref. 7. We find that in all three cases the resulting radial equations have the same form as Eq. (7) but with different values of α. For the angular dependence $\xi^{(1)}$, $\alpha = \mu - 1$, where $\mu = [(j+1/2)^2 - q^2]^{1/2}$ and $j = q+1/2, q+3/2, \ldots$, while for the angular dependences $\xi^{(2)}$ and η, $\alpha = \mu$ with $j = q+1/2, q+3/2, \ldots$ for $\xi^{(2)}$ and $j = q - 1/2$ for η. In all cases the multiplicity is $(2j+1)$. In three dimensions, fermionic wave functions (or, more precisely, wave sections[6,7]) Ψ_F in (6) are doublets. This doublet structure is carried by the angular dependences, so both for bosons and fermions the radial functions ψ in Eq. (7) are one-component objects. Therefore, unlike the scattering problem in (3+1) dimensions,[7] our calculation does not require any special treatment of $j = q - 1/2$ fermionic modes.

We can treat the bosonic and fermionic cases simultaneously using Eq. (7) if we adopt the following notation:

$$\alpha = [(j+1/2)^2 - q^2]^{1/2} - \kappa - 1/2\,, \qquad j = q+\kappa, q+\kappa+1, \ldots\,. \tag{8}$$

Then, $\kappa = 0$ corresponds to bosons, $\kappa = 1/2$ to fermions with the angular dependence $\xi^{(1)}$, and $\kappa = -1/2$ combines fermions with the angular dependencies $\xi^{(2)}$ and η. The results for the vacuum sector are obtained by substituting $q = 0$. Though such a substitution leads to an unphysical value $j = -1/2$ for $\kappa = -1/2$, the eigenvalues corresponding to this unphysical value do not participate in the fermionic trace in (3) because of vanishing multiplicity factor $(2j+1)$.

By the change of variables

$$\psi(r) = r^{\alpha}(1+r^2)^{-\alpha-1/2}\phi(x)\,, \quad x = (1+r^2)^{-1}\,, \tag{9}$$

Eq. (7) is converted into a hypergeometric equation. The resulting eigenvalues are

$$\lambda_n = (n+\alpha+1/2)(n+\alpha+3/2)\,, \quad n = 0,1,\ldots\,. \tag{10}$$

We still need an ultraviolet regularization for the traces in Eq. (3). A convenient one is provided, again in parallel with 't Hooft's calculation,[5] by two Pauli-Villars regulators with masses M_i and metrics e_i satisfying $\sum_i e_i = -1$, $\sum_i e_i M_i^2 = 0$, $i = 1,2$. Regularized traces that we will need are

$$\mathrm{Tr}\ln\mathcal{M}(M_i;\kappa) = \sum_{j=q+\kappa}^{\infty}(2j+1)\sum_{s=1}^{\infty}\sum_{i=0,1,2} e_i \ln[(s+\alpha)^2+\mu_i^2]\,, \tag{11}$$

where we have defined $e_0 = 1$, $M_0 = 0$, and $\mu_i^2 = M_i^2 - 1/4$, $i = 0,1,2$. The effective action in the fermionic case is obtained from the half sum of the traces (11) with $\kappa = \pm 1/2$. Those two traces are in fact related to each other in a simple way.

Because the one-loop effective actions (2) are dimensionless, and do not depend on the gauge coupling l^2, they can depend only on products of infrared and ultraviolet regulator parameters, $M_i R$. In the system of units where $R = 1$, we are then interested in the dependence of the effective action on M_i in the limit when M_i are large. The nonregulator, $i = 0$, terms in Eq. (11) cannot produce such dependence. Applying the Euler-Maclaurin formula to the regulator terms in Eq. (11), we get

$$\sum_{s=1}^{\Lambda}\ln[(s+\alpha)^2+M^2] = \int_0^{\Lambda} ds\,\ln[(s+\alpha)^2+M^2] + \left(\tfrac{1}{2}\ln[(s+\alpha)^2+M^2] + \frac{1}{6}\frac{s+\alpha}{(s+\alpha)^2+M^2}\right)\Bigg|_0^{\Lambda} + O([\alpha^2+M^2]^{-3/2})\,, \tag{12}$$

where $\Lambda \gg M$. The remainder in Eq. (12) gives a convergent contribution of order $1/M$ when summed over j with the multiplicity factor $(2j+1)$ and hence may be neglected. Terms divergent at $\Lambda \to \infty$ as well as those proportional to M_i^2 get canceled when all the regulator and nonregulator contributions are added together and we obtain

$$\mathrm{Tr}\ln\mathcal{M}(M_i;\kappa) = \sum_{j=q+\kappa}^{\infty}(2j+1)\left\{\sum_{i=1,2} e_i\left[-(\alpha+1/2)\ln(\alpha^2+\mu_i^2) + 2\mu_i\left(\frac{\pi}{2} - \arctan\frac{\alpha}{\mu_i}\right) - \frac{1}{6}\frac{\alpha}{\alpha^2+\mu_i^2}\right] + C(j)\right\}\,, \tag{13}$$

where $C(j)$ denotes terms independent of M_i.

The dependence of Eq. (13) on M_i can be found by the following method. We decompose each sum over j into two—one running from $q+\kappa$ to some value $J-1$ such that $q \ll J \ll M_i$, another running from J to a cutoff $\Lambda' \gg M$. As in the sum over s before, the dependence on the cutoff will disappear when all regulator and nonregulator terms are added together. The number J is integer or half integer when $q+\kappa$ is integer or half integer, respectively. Now, in the region $q+\kappa \le j \le J-1$ we can neglect j compared to M_i, while in the region $J \le j \le \Lambda'$ we can use the Euler-Maclaurin formula. At $J \le j \le \Lambda'$ we can also use the expansion

$$\alpha = \alpha_1 - q^2(2j+1)^{-1} + O(j^{-3})\,, \qquad \alpha_1 = j - \kappa\,. \tag{14}$$

For example,

$$\sum_{j=q+\kappa}^{\Lambda'}(2j+1)(\alpha+1/2)\ln(\alpha^2+M^2) = \sum_{j=q+\kappa}^{J-1}(2j+1)(\alpha+1/2)\ln M^2 + \sum_{j=J}^{\Lambda'}(2j+1)(\alpha_1+1/2)\ln(\alpha_1^2+M^2) - q^2\sum_{j=J}^{\Lambda'}\left(\ln(\alpha_1^2+M^2) + \frac{\alpha_1(2\alpha_1+1)}{\alpha_1^2+M^2}\right) + O(M^{-2}) + O(J^{-1})\,. \tag{15}$$

It turns out that other terms in Eq. (13) do not produce contributions in either bosonic or fermionic trace that distinguish between the monopole and vacuum sectors. Proceeding with Eq. (15), we finally obtain the one-loop effective action of a monopole with monopole number q in the presence of massless bosons and fermions, up to terms independent of R:

$$S^{(1)}_{B,F}(R) = K_{B,F}(q)\ln M^2R^2 + O(R^0)\ ,$$

$$K_B(q) = \lim_{J\to\infty}\left[\sum_{j=q}^{J-1}(2j+1)[(j+1/2)^2-q^2]^{1/2} -\tfrac{2}{3}J^3+\tfrac{1}{6}J+q^2J\right]\ , \tag{16}$$

$$K_F(q) = -\lim_{J\to\infty}\left[\sum_{j=q+1/2}^{J-1}(2j+1)[(j+1/2)^2-q^2]^{1/2} -\tfrac{2}{3}J^3+\tfrac{1}{6}J+q^2J\right] - \frac{q}{2}\ . \tag{17}$$

We assumed that both regulator masses are of the same order, $M_i \sim M$. The limits in Eqs. (16) and (17) were done by computer. The results for a few values of q are presented in Table I.

When there are N massless species of particles of given type, the corresponding numbers from Table I should be multiplied by N. For sufficiently large N, the logarithms of R coming from the effective action will overpower $3\ln MR$ that comes with the opposite sign from the volume factor in the monopole amplitude. Thus, when there is a large number of massless matter species, monopoles are suppressed. For small N, we cannot draw any conclusions from the present work because the one-loop calculation is not a reliable guide in this case. Possibly, numerical simulations can help to establish whether the suppression of monopoles holds for small N.

Let us now state some applications of our results. They show that nonconfining Abelian gauge interactions are possible in (2+1) dimensions when gapless excitations are present, thus providing some support to the idea of new gauge interactions in planar electronic systems. In a somewhat different interpretation, our results provide a mechanism by which interlayer tunneling of excitations with one unit of the ordinary electric charge is suppressed while that of a doubly charged object is allowed. Recent work[8] shows that a viable theory of high-temperature superconductors can be constructed if such a mechanism is assumed to exist. Let us postulate that a planar system describing one layer in a layered material supports quasiparticles that carry magnetic flux with respect to the new gauge field. There are two varieties of such quasiparticles corresponding to positive and negative fluxes, respectively. Assume further that these quasiparticles carry ordinary electric charge ("holons") which has the same sign for both positive and negative fluxes. Events of interlayer tunneling of these quasiparticles are described by monopoles and antimonopoles in three dimensions. If there are also excitations of another sort ("spinons") that carry charge, rather than flux, with respect to the new gauge field and are gapless, the tunneling of a *single* flux can be suppressed by their infrared effects as discussed above, while a *pair* of positive and negative fluxes can tunnel freely.

TABLE I. Coefficients of $\ln R^2$ in the one-loop effective action of a monopole with monopole number q in the presence of massless bosons and fermions.

q	$K_B(q)$	$K_F(q)$
1/2	0.0968	0.0151
1	0.2266	0.1730
3/2	0.3850	0.4358
2	0.5682	0.7852

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