# Nonuniversal electron-electron interaction effects on the fluctuation conductivity above the superconducting transition

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We study the effects of the Coulomb electron-electron interaction on the Aslamazov-Larkin fluctuation correction to the conductivity in impure two-dimensional metals. We show that unlike the correction to the superconducting transition temperature, the above correction to the conductivity is sensitive to the form of the Coulomb interaction. In particular we studied the case of a thin metallic film separated from a massive metallic substrate by an insulator, when the electron-electron interaction is modified by image forces.

The correction to the BCS transition temperature $T_{c0}$ in a two-dimensional impure electron system due to modification of the Coulomb interaction was studied by many authors using different techniques.[1-6] For the transition temperature $T_c$ renormalized by quantum fluctuations from the Coulomb interaction the following result was obtained:

$$\ln\left[\frac{T_c}{T_{c0}}\right]=-\rho\ln^3\left[\frac{1}{2\pi T_c\tau}\right],\quad \rho=\frac{e^2}{6\pi h}R_\square=\frac{1}{4p_F^2ld}, \tag{1}$$

where $e$ is the electron charge, $R_\square=1/\sigma d$, $\sigma$ is the conductivity of a film, $d$ is its thickness, $p_F$ is the Fermi momentum, $l$ is the electron mean free path due to elastic electron-impurity scattering, and $\tau$ is the corresponding momentum relaxation time. We assume in Eq. (1) that $d<l$. This result was obtained for the screened Coulomb potential in Refs. 1, 4–6 and for a local potential in Refs. 2 and 3. It was not clear why the results were the same in these different cases. Nevertheless, the theory which takes into account higher order terms in the Coulomb potential seems to describe experimental data for homogeneous films quite well.[6]

However, there is uncertainty in extracting the transition temperature from analysis of the temperature dependence of the conductivity. It was assumed that all Coulomb corrections were absorbed in the renormalized transition temperature $T_c$, and the fluctuation conductivity depended on $T_c$ as a parameter. Direct effects of the Coulomb interaction on the fluctuation conductivity were not taken into account. The main purpose of the present paper is a complete study of the effect of the Coulomb electron-electron interaction on the fluctuation conductivity. First of all we shall show that the correction of the transition temperature is universal in the sense that it does not depend on the detailes of the screening of the Coulomb potential. Then we shall show that the fluctuation conductivity is not described by the renormalization of the transition temperature alone. There are additional corrections which strongly depend on the form of the Coulomb potential and therefore they are not universal.

Our calculations show that the effect of the Coulomb interaction on the Aslamazov-Larkin[7] correction to the conductivity $\Delta\sigma_{\rm AL}$, the diamagnetic susceptibility $\Delta\chi_d$, and the upper critical magnetic field $H_c$ close to the transition $T-T_c\ll T_c$ may be presented in the form

$$\begin{aligned}\Delta\sigma_{\rm AL}&=\frac{\alpha_\Omega}{\beta}\Delta\tilde{\sigma}_{\rm AL}(T-T_c),\\ \Delta\chi_d&=\frac{\alpha_Q}{\beta}\Delta\tilde{\chi}_d(T-T_c),\\ H_c&=\frac{\beta}{\alpha_Q}\tilde{H}_c(T-T_c).\end{aligned} \tag{2}$$

Here $\Delta\tilde{\sigma}_{\rm AL}(T-T_c)$ is the correction to the conductivity which depends on the renormalized transition temperature $T_c$ as a parameter. The corrections $\Delta\tilde{\chi}_d(T-T_c)$ and $\tilde{H}_c(T-T_c)$ are defined in the same way. The functions $\alpha_\Omega$ and $\alpha_Q$ describe the effects of the Coulomb interaction not included in the renormalized transition temperature. The function $\beta$ is defined by the equation

$$\beta=1-3\rho\ln^2\left[\frac{1}{2\pi T_c\tau}\right], \tag{3}$$

which is obtained by linearizing an expression similar to Eq. (1) close to $T_c$. Therefore $\beta$ is a universal function. However, the functions $\alpha_\Omega$ and $\alpha_Q$, which describe the renormalization of the "kinetic term" and the "gradient term" in the nonstationary Ginsburg-Landau equation depend strongly on details of the screening of the Coulomb interaction.

For this purpose we consider two cases which may be realized experimentally. First is the case of a thin metallic film on an isolating substrate where the Coulomb potential may be considered two dimensional. In this case

 

$$\alpha_\Omega = 1 + 3\rho \ln\left[\frac{1}{2\pi T_c \tau}\right]\ln\left[\frac{E_0}{T_c}\right],\quad E_0 = \frac{(l\kappa_2)^8}{18\pi\tau}, \tag{4}$$

where $D$ is a diffusion coefficient, and $\kappa_2$ is the two-dimensional Debye screening momentum. Second is the case of a metallic film separated from a massive metallic substrate by an insulator where the Coulomb potential is modified by image forces. In this case

$$\alpha_\Omega = 1 - 3\rho \ln\left[\frac{1}{2\pi T_c \tau}\right]\ln\left[\frac{1}{2\pi T_c \tau \mu}\right], \tag{5}$$

where $\mu$ is the dimensionless coupling constant [see Eq. (16)]. In the approximation we use a first order of perturbation theory in the Coulomb potential, $\alpha_\Omega = \alpha_Q$. It is seen from Eqs. (3)–(5) that corrections to the kinetic term $\alpha_\Omega$ and to the transition temperature (the function $\beta$) are equally important close to the transition. We expect them to be equally important when the corrections are not small.

Now we present some details of the calculations. The Aslamazov-Larkin correction to the conductivity[8] close to the transition may be written in the following way:

$$\Delta\sigma_{\rm AL} = \frac{T}{\pi^3 d}\int d^2Q\left[\mathbf{n}\frac{\partial P^R(\mathbf{Q},0)}{\partial \mathbf{Q}}\right]^2 \int\frac{d\Omega}{\Omega}\mathrm{Re}\left[L^A(\mathbf{Q},\Omega)\frac{\partial L^R(\mathbf{Q},\Omega)}{\partial\Omega}\right], \tag{6}$$

where $\mathbf{n}$ is a unit vector and $L^R(\mathbf{Q},\Omega)$ is the retarded fluctuation propagator which effectively describes the electron-electron interaction in the Cooper channel $L^R(\mathbf{Q},\Omega)^{-1} = \lambda^{-1} - P^R(\mathbf{Q},\Omega)$. Here $P^R(\mathbf{Q},\Omega)$ is the retarded Cooper polarization operator and $\lambda$ is the interaction constant. The transition temperature is determined from the pole of the fluctuation propagator for $\Omega=0$ and $Q=0$. When the Coulomb interaction effects are not included, we obtain for the polarization operator $P_0$ the following result:

$$P_0^R(\mathbf{Q},\Omega) = -\frac{\nu}{2}\left[\ln\left[\frac{2\gamma_E\omega_D}{\pi T}\right] - \Phi(-i\Omega + DQ^2)\right],$$
$$\Phi(x) = \Psi\left[\frac{1}{2} + \frac{x}{4\pi T}\right] - \Psi\left[\frac{1}{2}\right], \tag{7}$$

and accordingly

$$L_0^R(\mathbf{Q},\Omega) = -\frac{2}{\nu}\left[\ln\left[\frac{T}{T_{c0}}\right] + \Phi(-i\Omega + DQ^2)\right]^{-1},$$
$$T_{c0} = \frac{2}{\pi}\gamma_E\omega_D \exp\left[-\frac{2}{\nu|\lambda|}\right], \tag{8}$$

where $\nu = mp_F/\pi^2$ is the two-spin electron density of states, $D$ is the diffusion coefficient, $\Psi(x)$ is the logarithmic derivative of the $\gamma$ function, $\omega_D$ is the Debye frequency, and $\gamma_E$ is Euler's constant.

As is clear from Eq. (6), to study the Coulomb correction to the fluctuation conductivity we need to calculate the corrections to the polarization operator shown in Fig. 1. We perform calculations with Matsubara technique and make the analytic continuation only at the final step. The result of calculations of the diagrams in Fig. 1 is

$$P_1(\mathbf{Q},\Omega_M) = \frac{2\pi\nu T^2}{d}\sum_m\sum_n\int d^2q\frac{1}{(2\pi)^2}\frac{\Theta(\varepsilon_n(\varepsilon_n-\Omega_M)\Theta(\varepsilon_n+\omega_m)(\varepsilon+\Omega_M-\omega_m))}{(DQ^2+|2\varepsilon_n-\Omega_M|)(DQ^2+|2\varepsilon_n+2\omega_m-\Omega_M|)}$$
$$\times\frac{\Theta(-\varepsilon(\varepsilon_n+\Omega))V(\mathbf{q},\omega)(DQ^2+Dq^2+|\omega_m|)}{(Dq^2+|\omega_m|)^2}, \tag{9}$$

$$P_2(\mathbf{Q},\Omega_M) = \frac{2\pi\nu T^2}{d}\sum_m\sum_n\int d^2q\frac{1}{(2\pi)^2}\frac{\Theta(\varepsilon_n(\varepsilon_n-\Omega_M))}{(DQ^2+|2\varepsilon_n-\Omega_M|)(DQ^2+|2\varepsilon_n+2\omega_m-\Omega_M|)}$$
$$\times\frac{V(\mathbf{q},\omega_m)\Theta((\varepsilon-\Omega_M)(\varepsilon_n+\omega_m))\Theta((\varepsilon+\omega_m-\Omega_M)(\varepsilon_n+\omega_m))}{D(\mathbf{q}+\mathbf{Q})^2+|2\varepsilon_n+\omega_m-\Omega_M|}, \tag{10}$$

$$P_3(\mathbf{Q},\Omega_M) = \frac{2\pi\nu T^2}{d}\sum_m\sum_n\int d^2q\frac{1}{(2\pi)^2}\frac{\Theta(\varepsilon_n(\varepsilon_n-\Omega_M))}{(DQ^2+|2\varepsilon_n-\Omega_M|)^2}$$
$$\times\frac{V(\mathbf{q},\omega_m)\Theta(-\varepsilon_n(\varepsilon_n+\omega_m))}{(Dq^2+|\omega_m|)^2}[Dq^2+DQ^2+|2\varepsilon_n|+|\omega_m|-\Omega_M\mathrm{sgn}(\varepsilon_n)], \tag{11}$$

$$P_4(\mathbf{Q},\Omega_M) = -\frac{2\pi\nu T^2}{d}\sum_m\sum_n\int d^2q\frac{1}{(2\pi)^2}\frac{\Theta(\varepsilon_n(\varepsilon_n-\Omega_M))}{(DQ^2+|2\varepsilon_n-\Omega_M|)^2}\frac{V(\mathbf{q},\omega_m)\Theta(\varepsilon_n(\varepsilon_n+\omega_m))}{D(\mathbf{Q}+\mathbf{q})^2+|2\varepsilon_n+\omega_n-\Omega_M|}, \tag{12}$$

where $\Omega_M = 2\pi MT$, $\omega_m = 2\pi mT$, and $\varepsilon_n = \pi T(2n+1)$, where $M$, $m$, and $n$ are integers; $V(\mathbf{q},\omega_m)$ is the Coulomb electron-electron interaction. We do not write the expression for $P_5$ and present contributions from all terms after summation over $n$:

$$
\begin{aligned}
P(\mathbf{Q},\Omega_M) &= \sum_{i=1}^{5} P_i(\mathbf{Q},\Omega_M) \\
&= -\frac{2T\nu}{d} \sum_{\omega_m>0} \int d^2q \frac{1}{(2\pi)^2} \frac{V(\mathbf{q},\omega_m)[\ln(m) - \Phi(DQ^2+\Omega_M)]}{(Dq^2+\omega_m)} \\
&\quad \times \left[ \frac{1}{\omega_m} + \frac{1}{Dq^2+\omega_m} - 2\frac{\ln(m)-\Phi(DQ^2+\Omega_M)}{\ln(m)} \frac{1}{Dq^2+\omega_m} \right] . \qquad (13)
\end{aligned}
$$

In deriving Eq. (13) we assume that $DQ^2 + \Omega_M \ll \omega_m$, $Dq^2$. The essential region of the summation over $n$ is $0 < \varepsilon_n < \omega_m$. The first term in square brackets originates from $P_1 + P_2$ and the second from $P_3 + P_4$. The last term in the square brackets corresponding to $P_5$ was calculated with logarithmic accuracy which means that the fluctuation propagator was chosen in the form $L(\mathbf{q},\omega_m) \approx -(2/\nu)[\ln(m)]^{-1}$ for $\omega_m \approx Dq^2 \gg 4\pi T$ [see Eq. (4)].

Now we specify a form of the Coulomb potential. The Coulomb potential in an impure metal in three and two dimensions is

$$V^{(3)}(\mathbf{q},\omega_m) = -\frac{1}{P_{d0}(\mathbf{q},\omega_m)} = \frac{1}{\nu} \frac{|\omega_m| + Dq^2}{Dq^2} , \qquad (14)$$

$$V^{(2)}(\mathbf{q},\omega_m) = \frac{1}{\nu} \frac{\kappa_2}{|q|} \left[ 1 + \frac{Dq\kappa_2}{|\omega_m| + Dq^2} \right]^{-1} , \qquad (15)$$

where $P_{d0}$ is the diffusion polarization operator, $\kappa_2 = 2\pi e^2 \nu_2$, and $\nu_2$ is the two-dimensional density of states $\nu_2 = \nu d$. When a metallic film of thickness $d$ is separated from a massive clean metal by a dielectric film of thickness $b$ and permeability $\varepsilon$, the electron-electron interaction is modified by the image forces and has a dipole-dipole character $V_{dd}$. At distances larger than $b$ the screened electron-electron interaction is[8]

$$
\begin{aligned}
V_{dd}(\mathbf{q},\omega_m) &= \frac{V_0(\mathbf{q})}{1 - P_{d0}(\mathbf{q},\omega_m) V_0(\mathbf{q})} , \\
V_0(\mathbf{q}) &= \frac{4\pi e^2 d}{q(1+\varepsilon/bq)} \approx \frac{4\pi e^2 db}{\varepsilon} \equiv \frac{\mu}{\nu} ,
\end{aligned} \qquad (16)
$$

where $V_0$ is a nonscreened interaction and $\mu$ is the interaction constant for the dipole-dipole interaction.

The equation for the renormalized transition temperature is $P_0(0,0) + P(0,0) = \lambda^{-1}$. It is seen from Eq. (13) that for $P(0,0)$ the expression in square brackets is proportional to $Dq^2$. This is a reason why the singular $q$ dependence of the Coulomb potential $V^{(3)}$ cancels out and we obtain Eq. (1). The same result may also be obtained for the potential $V_{dd}$ in the case of strong screening $P_{d0}V_0 \gg 1$. Therefore we conclude that the correction to the transition temperature is not sensitive to the form of the Coulomb potential, i.e., it is universal.

The situation is different for the coefficients $\alpha_\Omega$ and $\alpha_Q$ which are defined by the equations

$$
\begin{aligned}
\alpha_\Omega &= 1 - \frac{16T}{\pi\nu} \frac{\partial P(\mathbf{Q},\Omega_M)}{\partial \Omega_M} , \\
\alpha_Q &= 1 - \frac{16T}{\pi\nu} \frac{\partial P(\mathbf{Q},\Omega)}{\partial (DQ^2)} .
\end{aligned} \qquad (17)
$$

We get from Eq. (13)

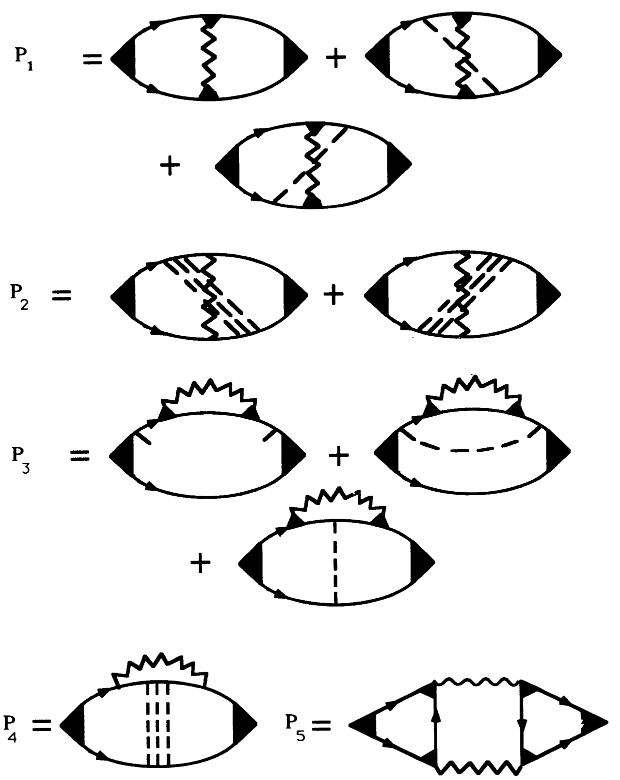


FIG. 1. Coulomb interaction corrections to the Cooper polarization operator. For $P_3$ and $P_4$ only diagrams with interaction on the upper electron Green's function are shown. The triangle vertices correspond to interaction vertices in the diffusion and Cooper channels renormalized by impurities. The zigzag line is the Coulomb potential and the wavy line is the fluctuation propagator.

$$\alpha_\Omega=\alpha_Q=1-\frac{8\pi T}{d}\sum_{\omega_m>0}\int d^2q\frac{1}{(2\pi)^2}\frac{V(\mathbf{q},\omega_m)}{Dq^2+\omega_m}\times\left[\frac{1}{\omega_m}-\frac{3}{Dq^2+\omega_m}\right]. \tag{18}$$

Here there is no cancellation in the expression in square brackets and the right-hand side of Eq. (18) essentially depends on the form of the Coulomb interaction. The right-hand side of Eq. (18) is divergent for $V^{(3)}$; therefore the correct form of the two-dimensional potential from Eqs. (15) and (16) must be used, and as a result we get Eqs. (4) and (5). Note that equality $\alpha_\Omega=\alpha_Q$ holds only in the first order of the Coulomb potential.

In conclusion, we showed that the Coulomb corrections to the fluctuation conductivity, susceptibility, and the upper critical magnetic field are not universal and strongly depend on the screening of the Coulomb potential. The particular interest is to study the effect of image forces in a film separated from a metallic substrate by an insulator. Such a situation may be realized in multilayer systems.[9–11] Equations (2)–(5) show that the effect of image forces is measurable and may be obtained, e.g., from studying the temperature dependence of the conductivity. It is worth mentioning that the screening effect of image forces changes the sign of the logarithmic correction in the function $\alpha_\Omega$. Note also that in granular films under the same conditions the charging effects are also diminished due to weakening of the Coulomb interaction by image forces.

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