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Magnetization curling reversal for an infinite hollow cylinder

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A rigorous analysis of the curling mode for an infinite ferromagnetic hollow cylinder with exchange and uniaxial anisotropy and external field applied along its axis is made using Brown's equation. Curling instability occurs only in a hollow cylinder, but never in a solid one. Moreover, the energy of the curling mode with large-angle deviations in the nonlinear region is always higher than that of the uniform mode. This suggests that at the nucleation field the initial saturated state cannot reverse through the curling mode to a new stable state.

The problem of nucleation in magnetization reversal has been extensively studied.[1–5] Most of the methods used are restricted to a linearization procedure for simplicity, but this allows one to calculate the nucleation field only. This was applied to the magnetization curve of an infinite isotropic ferromagnetic cylinder which then has a rectangular hysteresis loop and the coercive force is given by the nucleation field. Strictly speaking, however, the determination of coercivity actually requires analysis of the nonlinear regime which is lacking till now. The unwarranted identification of coercivity and nucleation field remains to be clarified for decades,[2] but many researchers still successfully use it as a rule, at least phenomenologically.

Micromagnetic analysis of an infinite solid cylinder can be briefly described as follows: For an infinite solid cylinder, the nonlinear Brown's equation admits only the uniform magnetization (trivial solution) at a sufficiently large applied field. When the field reverses, a nucleation field H_n is reached which permits the occurrence of infinitesimal deviations from the initially saturated state along the cylinder axis. The form of these deviations is determined by solving a linearized Brown's equation.[2] Furthermore, it has been shown numerically that the solution of the linearized Brown's equation is not a stable solution to the full nonlinear equation at applied fields larger than the nucleation field, so that the only possible stable states are those of uniform alignment along the axis.[5,6] Thus, magnetization is assumed to reverse completely at the nucleation field and the magnetization curve is rectangular.

Two possible reversal modes have been identified within the linearized model for an infinite solid cylinder at the nucleation field.[2,7] If the cylinder radius is sufficiently large, the above jumplike magnetization reversal starts with a curling mode,[7] in which the deviation of the magnetization angle θ from its initial value increases with increasing radius;[2] it was therefore believed that the magnetization near the cylinder surface reverses first. However, the amplitude of this deviation cannot be determined solely from the linearized Brown's equation[2] and it has recently been shown that the curling solution of the full nonlinear Brown's equation is just a uniform mode at the nucleation field and is unstable at any other applied field.[5] In other words, the undetermined amplitude of the linearized deviations turns out to be zero at the nucleation field. This result indicates that instability occurs at the nucleation field of the linearized Brown's equation but that the curling mode of the full nonlinear equation cannot propagate because of its null amplitude. Thus, magnetization should not jump to the reverse direction and the coercive force is not equal to the nucleation field of the linearized equation. This paradox results from inappropriate boundary condition chosen for the center of a solid cylinder [i.e., $\theta(r=0)=0$],[8] which is a line singularity in micromagnetic calculations with cylindric symmetry. This singularity not only restricts the possible magnetization configurations but also the number of possible magnetization reversal processes.[8] Therefore, the curling mode with nonzero amplitude can never occur during the reversal process in a solid cylinder.

In this paper, we demonstrate that curling instability can only appear in an infinite hollow cylinder. We have calculated the nucleation field for different ratios of the inner and outer radii and found that the nucleation field of a hollow cylinder is always lower than that of a solid cylinder. Moreover, from analysis of the nonlinear behavior it follows that contrary to the solid cylinder the deviations amplitude is nonzero. However, the curling mode has, in the nonlinear region, higher energy than the uniform mode and this suggests that the initial saturated state of an infinite hollow cylinder can not reverse through the curling mode either.

For an infinite hollow cylinder of inner radius a and outer radius b (Fig. 1), the demagnetization energy is zero if the magnetization reverses in the curling mode. The free energy is a sum of the exchange, anisotropy, and Zeeman energy terms:

$$E=\int_a^b(\mathcal{E}_{\rm ex}+\mathcal{E}_K+\mathcal{E}_H)2\pi r\,dr\ . \tag{1}$$

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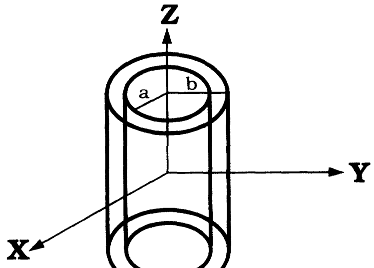

FIG. 1. An infinite hollow cylinder of inner radius a and outer radius b. The easy axis is along the z axis.

The exchange energy density

$$\mathscr{E}_{\rm ex}=A\left[\left(\frac{d\theta}{dr}\right)^2+\frac{1}{r^2}\sin^2\theta\right] , \tag{2}$$

the anisotropy energy

$$\mathscr{E}_K=K\sin^2\theta , \tag{3}$$

and the Zeeman energy

$$\mathscr{E}_H=-M_S H_a\cos\theta , \tag{4}$$

where A is the exchange stiffness constant, M_S is the saturated magnetization, K is the first-order anisotropy constant, H_a the applied field which along the z axis, and θ is the angle between the magnetization and z axis. The Euler differential equation which minimizes the integral (1) is the nonlinear Brown's equation

$$\frac{d^2\theta}{dt^2}+\frac{1}{t}\frac{d\theta}{dt}=\left[\left(\frac{1}{t^2}+Q\rho_0^2\right)\cos\theta+\overline{H}\right]\sin\theta \tag{5}$$

with the boundary conditions

$$\frac{d\theta}{dt}\Big|_{t=\epsilon}=\frac{d\theta}{dt}\Big|_{t=1}=0 . \tag{6}$$

The reduced units are defined as follows: $t=r/b$, $\epsilon=a/b$, $R_0=\sqrt{A}/M_S$, $S=b/R_0$, $\rho_0^2=2\pi S^2$, $\overline{H}=S^2H_a/2M_s$, and $Q=K/2\pi M_S^2$. The nucleation field is defined as the value of applied field at which magnetization oriented along the saturation direction is no longer a local energy minimum state. In order to find the nucleation field, we only need to consider the small-angle approximation

$$\frac{d^2\theta}{dt^2}+\frac{1}{t}\frac{d\theta}{dt}=\left[\left(\frac{1}{t^2}+Q\rho_0^2\right)+\overline{H}\right]\theta , \tag{7}$$

whose general solution is

$$\theta(t)=C_1J_1(\mu t)+C_2N_1(\mu t) . \tag{8}$$

Here $\mu^2=-\overline{H}-Q\rho_0^2, C_1,C_2\ll 1$ are constants, and $J_1(x)$ and $N_1(x)$ are Bessel functions of the first and second kind, respectively. According to the boundary conditions (6), μ satisfies

$$J_1'(\mu)N_1'(\mu\epsilon)-J_1'(\mu\epsilon)N_1'(\mu)=0 , \tag{9}$$

and for the integration constants we obtain the relation

$$C_2=-\frac{J_1'(\mu)}{N_1'(\mu)}C_1 . \tag{10}$$

The nucleation field for an infinite hollow cylinder can thus be derived numerically through μ,

$$H_n=-\frac{2K}{M_S}-\frac{2\mu^2M_S}{S^2} , \tag{11}$$

and it depends sensitively on the radii ratio ϵ.

To investigate the reversal behavior after nucleation, one should address the nonlinear dynamic Brown's equation with the same boundary conditions for the curling mode as in the linearized equation. Using fourth-order Runge-Kutta method and a trial and error procedure, we then find the deviation $\theta(r)$. Since the solutions for a fixed field (and radius) are not unique, we choose the lowest energy solution as the reversal mode. Figure 2 shows the deviation of magnetization at the inner surface (i.e., at $r=a$). When $\epsilon\to 0$, $\theta(a)$ approaches zero for any value of the outer radius. Because of symmetry, at $\epsilon=0$, the magnetization at the center can only be oriented along to $\theta=0$ or $\theta=\pi$. Therefore, for an initial state of $\theta=0$, the magnetization at the center will stay in that direction when $\epsilon=0$. However, for $\epsilon=0$, there exists no nonuniform solution of the nonlinear Brown's equation with $\theta=0$ at the center when $H=H_n$. In other words, for a solid cylinder ($\epsilon=0$) at the nucleation field, the curling mode actually is identical to the uniform mode, and the parameters C_1 and C_2 of the solution for linearized Brown's equation are zero. Therefore, during nucleation, the line singularity at the center of a solid cylinder ($\epsilon=0$) not only locally supports the wall structure but also globally resists the reversal. The line singularity in the center thus locks the magnetization and prohibits the reversal process to propagate. At the nucleation field, there exists no nonuniform solution which can both sustain the

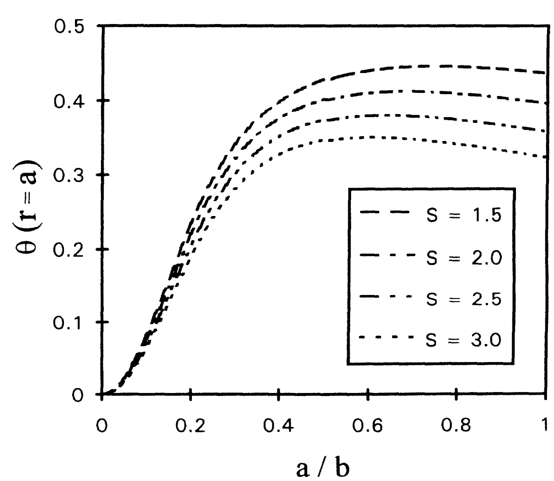

FIG. 2. Relation between the deviation of magnetization $[\theta(r=a)]$ on the inner surface of the hollow cylinder and the ratio a/b for different values of b. When $a/b\to 0$, $\theta(r\to 0)$ approaches zero for any S.

cylindric symmetry and balance the net torque (i.e., satisfy the full Brown's equation) in an infinite solid cylinder; therefore, magnetization stays at its initial state even at applied field higher than the nucleation field when an instability should occur. Without thermal agitation,[9] magnetization can only reverse at a larger value of reverse field until the next solution of nonaxial symmetry appears, e.g., buckling or rotation in unison. From Fig. 2, we see that a nonzero amplitude of the curling mode does exist for $\epsilon \neq 0$; therefore, this micromagnetic defect, line singularity at the center, can be removed legitimately if we consider the magnetization reversal of a hollow cylinder.[10]

Figure 3 shows the dependence of the nucleation field on the ratio a/b with the different values of outer radius, b. Larger values of the ratio a/b result in reduced nucleation fields, but their influence becomes insignificant at very large outer radii. This result also indicates that nanosize particles should be very sensitive to nonmagnetic impurities and that their coercivity is significantly reduced. This is probably the reason for the anomalous behavior of coercivity of granular particles.[11] As the ratio a/b approaches zero the nucleation field approaches the solid cylinder results and this is apparently the reason for the successful application of the curling mode theory to a microsize solid cylinder. We have compared the calculated nucleation fields at different values of a/b with experimental data[12] (Fig. 4). For a large outer radius, the nucleation field is almost independent of a/b; however, for small b the dependence is significant. Most of the experimental data[12] are, indeed, much lower than the theoretical results for a solid cylinder of small radius. Besides the pores or nonmagnetic impurities within the particles, the reduction of coercivity can also be caused by other factors, e.g., by thermal agitation and irregular shape,[9] which are not considered here.

We now address the evolution after instability occurs. For the curling mode we expand the total energy into a series in θ,

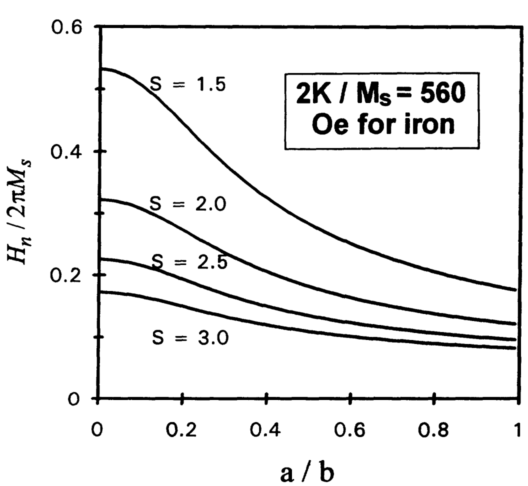

FIG. 3. Relation between nucleation field and the ratio a/b for different values of S. The larger ratio a/b results in the smaller nucleation field. The dependence on a/b becomes insignificant at the large outer radii.

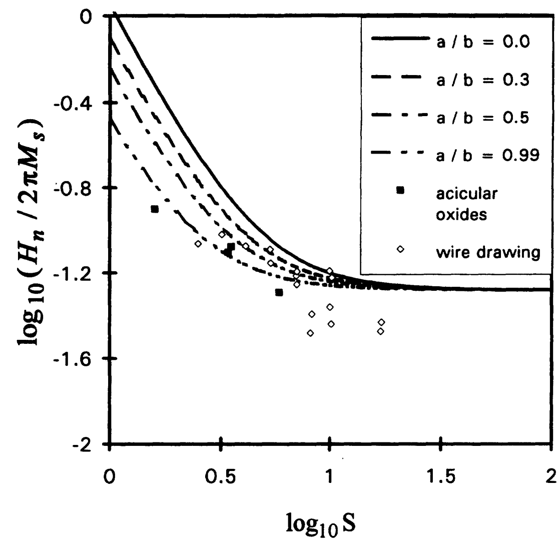

FIG. 4. Comparison between experimental data and calculated nucleation fields for different values of a/b. The symbols ◇ and ■ stand for two different samples (Ref. 12). The values of a/b for the lines from top to bottom are 0.0, 0.3, 0.5, and 0.99, respectively. Large value of a/b yields a better agreement.

$$\frac{E}{2\pi A} = \int_\epsilon^1 -2\bar{H}t\,dt + \int_\epsilon^1 \left\{ \left[\frac{d\theta}{dt} \right]^2 + \left[\left[\frac{1}{t^2} + Q\rho_0^2 \right] + \bar{H} \right] \theta^2 \right\} t\,dt + \Delta \,. \tag{12}$$

For the curling solution of the linearized Brown's equation the second term equals identically zero and the energy difference between the uniform and the curling modes is simply Δ. Neglecting the contributions higher than the sixth order of θ, Δ can be reduced to

$$\Delta^{(4)} = -C_1^4 \int_\epsilon^1 \left[1 + \frac{t^2}{4}(3Q\rho_0^2 - \mu^2) \right] \frac{\xi^4}{3t} dt \,, \tag{13}$$

where

$$\xi(t) = J_1(\mu t) - \frac{J_1'(\mu)}{N_1'(\mu)} N_1(\mu t) \,. \tag{14}$$

From Eqs. (8) and (10), we know that $\theta(t) = C_1 \xi(t)$ and it has been found numerically that the larger the C_1, the lower the energy of the curling mode. Therefore, the magnetization of a hollow cylinder indeed initially reverses by infinitesimal curling. However, because of the growth of the deviation amplitude, the linearized Brown's equation is inappropriate to describe the further reversal behavior at large deviations. Figure 5 shows the numerical solution of a curling mode for nonlinear Brown's equation with different b at $a/b = 0.1$. We also calculate the energy of these solutions and of the uniform mode[13] for $H_a \leq H_n$ (Fig. 6). There exists an energy barrier separating the uniform and the curling modes, and the latter are always at higher energy in the nonlinear region.

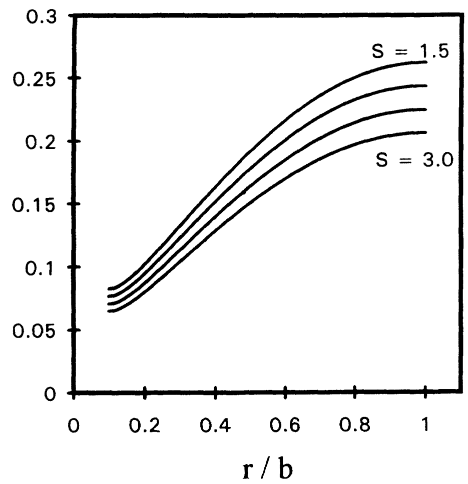

FIG. 5. Dependence of the deviation of magnetization on r for $a/b=0.1$ at different values of S. The curling mode exists in the nonlinear region of the hollow cylinder. The values of S for the lines from top to bottom are 1.5, 2.0, 2.5, and 3.0, respectively.

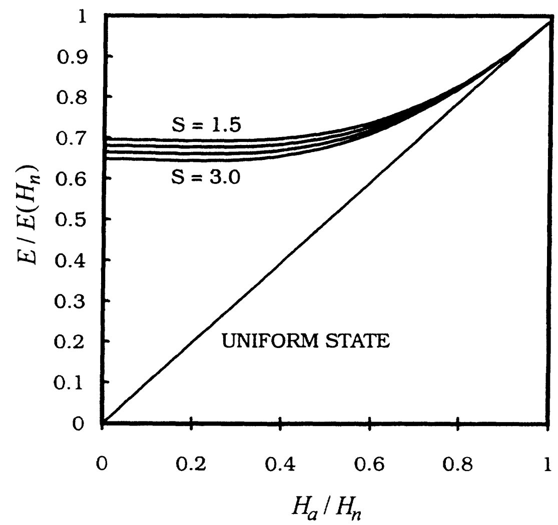

FIG. 6. The energy of the curling mode and the uniform mode for $|H_a| \le |H_n|$. Here $a/b=0.1$. The energy of the curling mode in the nonlinear region is always higher than that of the uniform mode. The values of S for the lines from top to bottom are 1.5, 2.0, 2.5, and 3.0, respectively.

Therefore, magnetization of a hollow cylinder is nucleated by infinitesimal curling, but it is impossible to say what will really happen afterwards because of the higher energy of curling mode in the nonlinear region.

In summary, we have shown that a nonuniform curling mode does not exist in an infinite solid cylinder, even at the nucleation field. Considering a hollow cylinder, we have further shown that magnetization really curls, at nucleation field, at an infinitesimally small angle which depends on the radii of the hollow cylinder, and that the nucleation field is significantly influenced by the ratio of a/b. However, the energy of the curling mode is always higher than that of the uniform mode in the nonlinear region and this implies that magnetization cannot reverse through the curling mode to the reverse stable direction. It should be noted that the above argument does not rigorously prove that the curling reversal does not exist, it only shows that there is no proof, within the micromagnetic analysis, for its existence even within the nonlinear region of an infinite hollow cylinder. However, because of the nonuniform demagnetization field in a finite cylinder,[14] the ferromagnetic samples are never completely saturated.[15] Thus, for a real fine-particle system, the curling mode can still possibly occur due to the lack of initial uniform saturation. Furthermore, thermal fluctuations lead to a finite reversal probability over the energy barrier between the curling mode and the uniform mode.[9] Those are probably the reasons why linear theory, within the framework of the micromagnetic assumptions, agrees quite well with experiments.[16] However, the detailed behavior after nucleation still requires a fully dynamic, nonlinear investigation.

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