Effects of a Gaussian random field in the Sherrington-Kirkpatrick spin glass

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The infinite-range Ising spin glass in the presence of a Gaussian random field is investigated by means of the replica method. The replica-symmetric solution and its instability are discussed; it is shown that the Almeida-Thouless instability regions are always reduced by the random field. The crossover exponent ϕ associated with the irreversibility line is shown to be nonuniversal in the presence of a random field, varying in a range from 1 to 3. Some of our results are compared with recent experimental measurements performed in the diluted antiferromagnet $Fe_xZn_{1-x}F_2$.

I. INTRODUCTION

Disordered systems represent one of the most investigated areas in physics nowadays, due to their relevance for the understanding of the real world and numerous applications. Two of these systems, namely, spin glasses and the ferromagnet in a random magnetic field, have attracted special attention from many workers. Although a lot of progress has been made on these problems, many questions remain open, some of them being quite fundamental.

The theory of spin glasses[1,2] has been mostly concentrated on the study of infinite-range interaction systems, whose prototype is the Sherrington-Kirkpatrick (SK) model.[3] The solution of the SK model turned out to be highly nontrivial, presenting many unexpected features; among many, one can point out a phase transition in the presence of an external magnetic field, signaled by the Almeida-Thouless (AT) line,[4] and the correct description of the spin-glass phase, in terms of an infinite number of order parameters, i.e., an order-parameter function.[5] Being infinite range, these systems are appropriate in the limit of infinite dimensions, representing mean-field approaches for more realistic (short-range interaction) models.[6] One has no guarantee that such unusual properties will survive in three dimensions, and this has been a point of a lot of controversy lately.[7–13] Apart from this, many experimental investigations[1] claim to have observed the AT line, and so it may be that at least some of the predictions of the SK model could be present in real systems.

The random-field problem, as formulated originally by Imry and Ma,[14] remained purely academic, until the appearance of its physical realization as a diluted antiferromagnet in a uniform magnetic field[15] and the proof that the static critical behavior is identical in these two systems.[16] Since then, a lot of effort has been devoted to this system from both experimental and theoretical points of view.[17–20] In particular, an important question which has been addressed concerns what the random-field problem has in common with spin glasses in a uniform field.[21–24]

The joint study of spin-glass and random-field problems has been proposed as an appropriate system for the description of mixed hydrogen-bonded ferroelectrics and antiferroelectrics, the so-called proton and deuteron glasses;[25–30] these may be considered as the electric counterparts of magnetic spin glasses. Also, experiments on the diluted antiferromagnet $Fe_xZn_{1-x}F_2$ in an external magnetic field have presented a crossover between random-field and spin-glass behaviors;[31–39] in particular, the phase diagram in the field versus temperature plane is affected by the concentration x. This suggests that the joint study of these two problems may also be suitable for a better understanding of some diluted antiferromagnets. In this paper we study the Sherrington-Kirkpatrick model in the presence of a Gaussian-distributed random magnetic field, by means of the replica method. In Sec. II we start by defining the model and considering the replica-symmetric (RS) solution. We analyze the instabilities of the RS solution in Sec. III. Finally, we discuss our main results in view of some experimental measurements carried out for the diluted Ising antiferromagnet $Fe_xZn_{1-x}F_2$.

II. THE MODEL AND ITS REPLICA-SYMMETRIC SOLUTION

The SK model[3] in the presence of an external random magnetic field is defined in terms of the Hamiltonian

$$\mathcal{H} = -\sum_{(ij)} J_{ij} S_i S_j - \sum_i h_i S_i \ , \tag{2.1}$$

where $S_i = \pm 1$, with $i = 1, 2, \ldots, N$, and the sum $\sum_{(i,j)}$ applied to all pairs of spins. The coupling constants $\{J_{ij}\}$

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and the random fields $\{h_i\}$ follow independent probability distributions,

$$P(J_{ij})=\left[\frac{N}{2\pi J^2}\right]^{1/2}\exp\left[-\frac{N}{2J^2}\left[J_{ij}-\frac{J_0}{N}\right]^2\right], \quad (2.2)$$

$$P(h_i)=\left[\frac{1}{2\pi\Delta^2}\right]^{1/2}\exp\left[-\frac{1}{2\Delta^2}(h_i-h_0)^2\right]. \quad (2.3)$$

For a given realization of bonds and site fields $(\{J_{ij}\},\{h_i\})$, one has a corresponding free energy $F(\{J_{ij}\},\{h_i\})$, such that the average over the disorder $[\cdots]_{J,h}$ becomes

$$[F(\{J_{ij}\},\{h_i\})]_{J,h} = \int\prod_{(ij)}[dJ_{ij}P(J_{ij})]\prod_i[dh_iP(h_i)]F(\{J_{ij}\},\{h_i\}). \quad (2.4)$$

One can now make use of the replica method[1] in order to get the free energy per spin as

$$-\beta f=\lim_{N\to\infty}\frac{1}{N}[\ln Z(\{J_{ij}\},\{h_i\})]_{J,h} = \lim_{N\to\infty}\lim_{n\to 0}\frac{1}{Nn}([Z^n]_{J,h}-1), \quad (2.5)$$

where Z^n is the partition function of the replicated system. The usual procedure leads to

$$\beta f=-\frac{(\beta J)^2}{4}-\frac{(\beta\Delta)^2}{2}+\lim_{n\to 0}\frac{1}{n}\min g(m^\alpha,q^{\alpha\beta}), \quad (2.6)$$

where

$$g(m^\alpha,q^{\alpha\beta})=\frac{\beta J_0}{2}\sum_\alpha(m^\alpha)^2+\frac{(\beta J)^2}{2}\sum_{(\alpha\beta)}(q^{\alpha\beta})^2 -\ln \mathrm{Tr}_\alpha\exp(\mathcal{H}_{\mathrm{eff}}), \quad (2.7a)$$

$$\mathcal{H}_{\mathrm{eff}}=\beta J_0\sum_\alpha m^\alpha S^\alpha+(\beta J)^2\sum_{(\alpha\beta)}q^{\alpha\beta}S^\alpha S^\beta +(\beta\Delta)^2\sum_{(\alpha\beta)}S^\alpha S^\beta+\beta h_0\sum_\alpha S^\alpha. \quad (2.7b)$$

Above, α and $\beta(\alpha,\beta=1,2,\ldots,n)$ are replica labels and $\sum_{(\alpha\beta)}$ denote sums over distinct pairs of replicas.

The extrema of the functional $g(m^\alpha,q^{\alpha\beta})$ give us the equilibrium equations

$$m^\alpha=\langle S^\alpha\rangle, \quad (2.8a)$$

$$q^{\alpha\beta}=\langle S^\alpha S^\beta\rangle \quad (\alpha\neq\beta), \quad (2.8b)$$

where $\langle\cdots\rangle$ indicate thermal averages with respect to the "effective Hamiltonian" $\mathcal{H}_{\mathrm{eff}}$ in (2.7b).

The analytic continuation $n\to 0$ in (2.6) may be easily performed if one chooses the "naive" replica-symmetric solution,[3]

$$m^\alpha=m,\ \forall\alpha;\quad q^{\alpha\beta}=q,\ \forall(\alpha\beta). \quad (2.9)$$

With this choice, the free energy per spin [Eq. (2.6)] and the equilibrium equations [Eqs. (2.8)] become

$$\beta f=-\frac{(\beta J)^2}{4}(1-q)^2+\frac{\beta J_0}{2}m^2 -\int_{-\infty}^{\infty}\left[\frac{1}{2\pi}\right]^{1/2}dz\,\exp(-z^2/2)\ln(2\cosh\xi), \quad (2.10)$$

$$m=\int_{-\infty}^{\infty}\left[\frac{1}{2\pi}\right]^{1/2}dz\,\exp(-z^2/2)\tanh\xi, \quad (2.11)$$

$$q=\int_{-\infty}^{\infty}\left[\frac{1}{2\pi}\right]^{1/2}dz\,\exp(-z^2/2)\tanh^2\xi, \quad (2.12)$$

where

$$\xi=\beta J_0 m+\beta J\left[q+\left[\frac{\Delta}{J}\right]^2\right]^{1/2}z+\beta h_0. \quad (2.13)$$

At this level, the case $h_0\neq 0$ leads to the uninteresting situation of field-induced parameters ($m\neq 0$, $q\neq 0$); we shall therefore consider $h_0=0$, for the moment. Nevertheless, the random-field width (Δ) plays the role of an inducing field which acts on the parameter q only, i.e., $q\neq 0$, if $\Delta>0$. In such case, there is no *spontaneous* spin-glass order like the one found in the case $\Delta/J=0$ (SK model). However, one can still have a phase transition associated with the parameter m. Hence, two phases are possible in this case; namely, the ferromagnetic F ($m\neq 0$, $q\neq 0$) and the independent I ($m=0$, $q\neq 0$), as shown in Fig. 1. The critical frontier separating these two phases is given by the softening to zero of the magnetization parameter m (throughout this paper, we will work in units $k_B=1$, i.e., $\beta=1/T$),

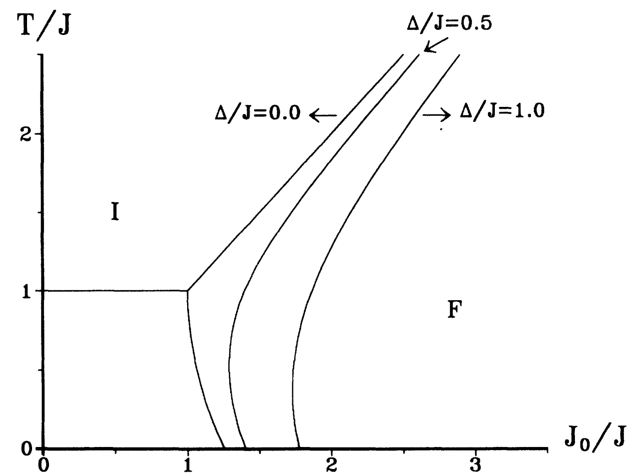

FIG. 1. Phase diagram of the Sherrington-Kirkpatrick model in the presence of a Gaussian random field of width Δ (for typical values of Δ/J), within the replica-symmetric solution. Only in the case $\Delta/J=0$ does one have, besides the ferromagnetic (F) phase, the paramagnetic at high temperatures and spin-glass at low temperatures (which are separated by the horizontal line at $T/J=1$). For any $\Delta/J>0$, only the F and the independent (I) phases are present. The reentrance effect is stronger in the SK case ($\Delta/J=0$) and gets reduced as Δ/J increases.

$$\frac{T}{J}=\frac{J_0}{J}\int_{-\infty}^{\infty}\left[\frac{1}{2\pi}\right]^{1/2}dz\,\exp(-z^2/2)\times\text{sech}^2\beta J\left[q+\left[\frac{\Delta}{J}\right]^2\right]^{1/2}z\ . \qquad (2.14)$$

The critical frontier F-I may be obtained analytically only in the high- and low-temperature regimes. For high temperatures, one gets, in first approximation,

$$\frac{T}{J}=\frac{J_0}{J}\left[1-\left[\frac{J}{J_0}\right]^2\left[\frac{\Delta}{J}\right]^2\right]. \qquad (2.15)$$

For low temperatures,

$$\frac{T}{J}=\pi\left[1+\left[\frac{\Delta}{J}\right]^2\right]^2\times\left[\frac{J}{J_0}-\left[\frac{2}{\pi}\right]^{1/2}\frac{1}{[1+(\Delta/J)^2]^{1/2}}\right], \qquad (2.16)$$

such that at $T=0$ one has

$$\frac{J_0}{J}=\left[\frac{\pi}{2}\right]^{1/2}\left[1+\left[\frac{\Delta}{J}\right]^2\right]^{1/2}. \qquad (2.17)$$

In the intermediate-temperature region the F-I critical frontier is obtained by solving Eqs. (2.14) and (2.12) numerically. In Fig. 1 this line is represented for typical values of Δ/J. One notices that the random field favors the independent phase and reduces the reentrance already observed in the replica-symmetric phase diagram of the SK model. Alos, all curves for $\Delta/J>0$ are smooth, due to the fact that the parameter q is always finite along these lines; only in the SK limit ($\Delta/J=0$) does q go from zero to a nonzero value, at the multicritical point ($T/J=1$, $J_0/J=1$), leading to a cusp in the phase-transition line.[40]

The RS solution may give a qualitatively good approximation of the true phase diagram, but it leads to serious problems at low temperatures; a sign of this is a negative entropy at $T=0$. In our case, the entropy per spin in the I phase, at $T=0$, is given by

$$s(0)=-\frac{1}{2\pi}\frac{1}{1+(\Delta/J)^2}\ , \qquad (2.18)$$

which goes to zero as $\Delta/J\to\infty$. As already noticed for the SK model,[1] we believe that the reentrant effect (independent-ferromagnetic-independent), observed in Fig. 1, should be another artifact of the RS solution. Indeed, such an effect is related to the negative entropy at low temperatures; one may show that the derivative of the F-I line at $T=0$ is given by

$$\frac{d(T/J)}{d(J_0/J)}=-2\left[1+\left[\frac{\Delta}{J}\right]^2\right]=\frac{1}{\pi s(0)}\ . \qquad (2.19)$$

The negative entropy is often related to the instability of the RS solution at low temperatures;[4] Eq. (2.18) suggests that, at least in the I phase, the RS solution should be unstable at low temperatures (except in the limit $\Delta/J\to\infty$). This analysis will be considered next.

III. INSTABILITIES OF THE REPLICA-SYMMETRIC SOLUTION

The stability of a given solution is ensured by a positive-definite Hessian matrix associated with the functional $g(m^\alpha,q^{\alpha\beta})$ appearing in Eqs. (2.6) and (2.7). Analogous to what was done for the SK model,[4] it can be shown that, within the RS solution, one of the eigenvalues of this Hessian matrix becomes negative below a temperature defined by

$$\left[\frac{T}{J}\right]^2=\int_{-\infty}^{\infty}\left[\frac{1}{2\pi}\right]^{1/2}dz\,\exp(-z^2/2)\text{sech}^2\xi\ , \qquad (3.1)$$

where ξ was already defined in the previous section.

Usually in the literature,[1,2] two criteria have been used for the identification of a spin-glass (SG) phase, depending on whether one is in a replica-symmetric space (a single order parameter q is employed), or within replica-symmetry breaking (many order parameters are needed; actually, an infinite number of them).

(i) In the first case, the SG phase is associated with the onset of the single parameter q; this can be found, e.g., as one crosses the horizontal line at $T/J=1$ for the case $\Delta/J=0$ (see Fig. 1). Such a line does not exist for the cases $\Delta/J>0$ since q is trivially induced by a finite random-field width Δ.

(ii) In the second case, the SG phase may be associated with the onset of replica-symmetry breaking, i.e., the single-parameter solution (either induced or spontaneous) becomes unstable in a region of the phase diagram delimited by Eq. (3.1). To overcome this instability, an infinite number of order parameters, i.e., an order-parameter function, is required.[5] If, in addition to this, such region contains also a *spontaneous* ferromagnetic order, it is usually called a mixed ferromagnetic phase (like the one found in the T/J-J_0/J phase diagram of the SK model); if no spontaneous ferromagnetism is present, it is classified as a SG phase (like the one found in the h_0/J-T/J phase diagram of the SK model).

In the case $\Delta/J=0$, both criteria described above lead, in the T/J-J_0/J phase diagram, to a horizontal line at $T/J=1$, whereas for $\Delta/J>0$ a horizontal line may be found *only* within criterion (ii). In the discussion which follows, a SG phase will be defined, for $\Delta/J>0$, taking into account criterion (ii).

Let us first consider the stability analysis for the phase diagram in Fig. 1, i.e., $h_0=0$. Throughout the independent phase ($m=0$), Eq. (3.1) defines a line (for a fixed value of Δ/J) which splits it in two regions (see Fig. 2). At high temperatures one gets a region in which replica symmetry is stable, presenting the characteristics of the usual paramagnetic solution (except for the field-induced parameter q); from now on, we shall refer to this as the paramagnetic (P) phase. At low temperatures the RS solution is not valid, characterizing then a SG phase. One should observe that the SG phase is reduced by the random field; actually, as $\Delta/J\to\infty$ the SG phase disappears, supporting the argument that the "negative-entropy catastrophe," the reentrance (paramagnetic–ferromagnetic–spin glass), and the instability of the RS

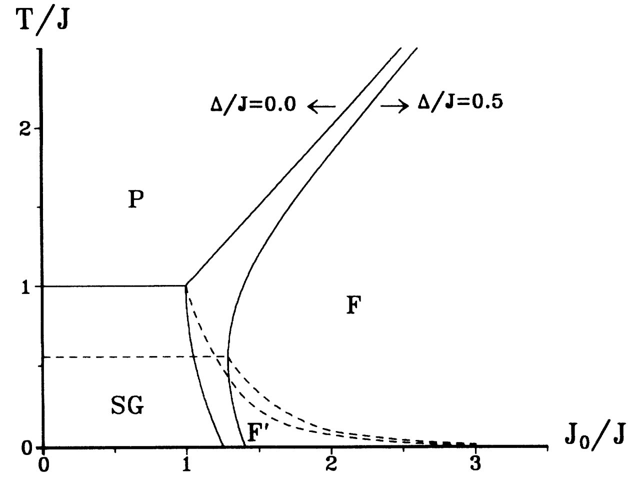

FIG. 2. Phase diagram of the Sherrington-Kirkpatrick model in the presence of a Gaussian random field of width $\Delta(\Delta/J=0.0,0.5)$. The dashed lines define the frontiers of stability of the replica-symmetric solution. In analogy to the $\Delta/J=0.0$ case, four phases are also present for $0<\Delta/J<\infty$: the paramagnetic (P), the spin-glass (SG), the ferromagnetic (F) and the mixed ferromagnetic (F') phases.

solution are strongly correlated. Similar to the SK model, Eq. (3.1) provides a division of the ferromagnetic phase into two regions, namely, the conventional ferromagnetic (F) for which replica symmetry is stable, and the mixed ferromagnetic (F') where the RS solution is unstable (see Fig. 2). Analogous to the SG phase, the F' region is reduced as Δ/J increases.

Let us now consider the instabilities for the case $J_0=0$ and $h_0\neq 0$. For a given value of Δ/J, Eq. (3.1) defines a line in the plane h_0/J versus T/J, below which the RS solution is unstable. This Almeida-Thouless line, which was at first introduced for the SK model ($\Delta/J=0$),[4] is nowadays associated with strong irreversibility effects in real systems, and many experimental workers claim to have observed it.[1] We now analyze how this line is affected by the presence of a random magnetic field. For a strong mean ($h_0/J\gg 1$) one gets

$$\frac{T}{J}\cong\frac{4}{3}\frac{1}{\sqrt{2\pi}}\frac{1}{[1+(\Delta/J)^2]^{1/2}}\exp\left\{-\frac{1}{2}\frac{(h_0/J)^2}{1+(\Delta/J)^2}\right\}, \tag{3.2}$$

i.e., the presence of a finite width ($\Delta/J>0$) gives a slower tendency towards $T=0$.

For both mean and width small ($h_0/J,\Delta/J\ll 1$) one can show that

$$\tau=1-\frac{T}{T_0}\cong\left[\frac{3}{4}\right]^{1/3}\left[\left[\frac{h_0}{J}\right]^2+\left[\frac{\Delta}{J}\right]^2\right]^{1/3}-\left[\frac{3}{4}\right]^{1/3}\left[\frac{\Delta}{J}\right]^{2/3}, \tag{3.3}$$

where T_0 is the temperature obtained from Eqs. (3.1) and (2.12) for $J_0=h_0=0$, i.e.,

$$\frac{T_0}{J}\cong\left[1-2\left[\frac{3}{4}\right]^{1/3}\left[\frac{\Delta}{J}\right]^{2/3}\right]^{1/2} \quad (\Delta/J\text{ small}). \tag{3.4}$$

Equation (3.3) presents, to lowest order, two distinct regimes, namely, the "random-field" one,

$$\tau=\frac{1}{3}\left[\frac{3}{4}\right]^{1/3}\left[\frac{\Delta}{J}\right]^{-4/3}\left[\frac{h_0}{J}\right]^2 \quad (\Delta\gg h_0), \tag{3.5a}$$

and the "spin-glass" one,

$$\tau=\left[\frac{3}{4}\right]^{1/3}\left[\frac{h_0}{J}\right]^{2/3} \quad (\Delta\ll h_0). \tag{3.5b}$$

These two regimes are interpolated by a reversal in the curvature, and the crossover between them may be defined as occurring at the inflection point. This is clearly observed in the AT line for $\Delta/J=0.2$, shown in Fig. 3.

One may define a crossover exponent ϕ, associated with the AT line [Eqs. (3.5)], through,

$$\tau\sim(h_0/J)^{2/\phi} \quad (h_0/J\text{ small}), \tag{3.6}$$

which may vary, in principle, both by changing Δ/J or along a given line (i.e., changing T/J for Δ/J fixed). For the two regimes discussed above [Eqs. (3.5)], this exponent goes from $\phi=1$ (random-field regime) to $\phi=3$ (spin-glass regime). Such crossover behavior has already been observed experimentally along the irreversibility line of the Ising antiferromagnet $Fe_xZn_{1-x}F_2$, for $x=0.31$.[35–38] As Δ/J increases, T_0 is shifted to the left and the range over which the AT line is dominated by the random-field regime increases, i.e., the inflection point occurs for higher values of h_0/J, as shown in Fig. 4. When Δ/J is large, the AT line becomes nearly a vertical straight line, meeting the h_0/J axis when $\Delta/J=\infty$. Through the whole Δ/J range, the exponent

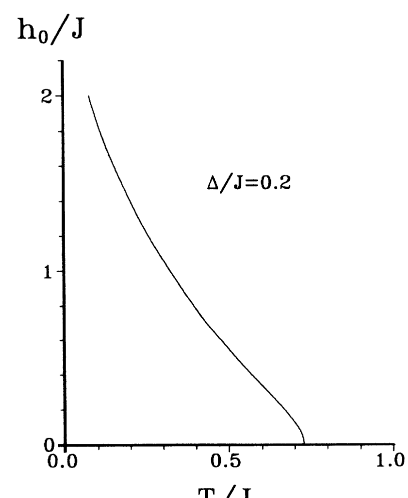

FIG. 3. Almeida-Thouless line for the Sherrington-Kirkpatrick model in the presence of a Gaussian random field of width $\Delta/J=0.2$. An inflection point separates two distinct regimes, the random-field ($\phi=1$) from the spin-glass one ($\phi=3$).

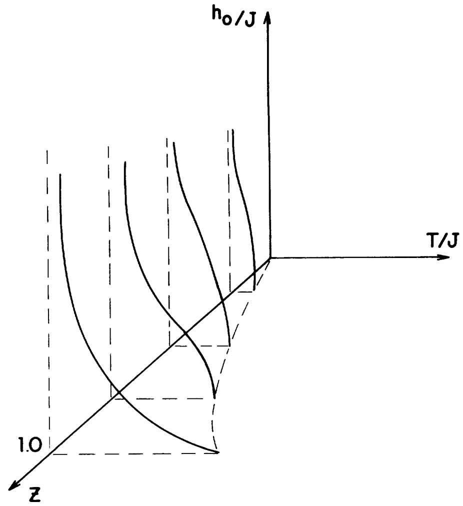

FIG. 4. Evolution of the Almeida-Thouless lines as Δ/J varies $[z=1/(1+\Delta/J)]$. The instability region is reduced as Δ/J increases similarly to what happens in Fig. 2.

$\phi=1$ (h_0 small; $h_0 \ll \Delta$) is found, but as Δ/J gets large the above-mentioned crossover disappears. Analogous to Fig. 2, the region of replica-symmetry instability is reduced by the random field (i.e., as Δ/J increases).

In what follows, we present our main conclusions and discuss our results, comparing them with some previous experimental investigations.

IV. DISCUSSION

We have studied the Sherrington-Kirkpatrick model in the presence of a Gaussian random field of mean h_0 and width Δ. Within replica symmetry and $h_0=0$, the main effects of a finite width ($\Delta/J>0$) are to favor the independent ordering and reduce the reentrance of the independent phase, which is shown to be correlated to the negative entropy at $T=0$. The stability analysis of the RS solution shows that, in the $h_0=0$ phase diagram, the regions of instability (i.e., the spin-glass and mixed ferromagnetic phases) are reduced by the random field. It is well known that the reentrant effect for $\Delta/J=0$ (which is stronger than that of any $\Delta/J>0$) is completely removed by Parisi's replica-symmetry-breaking hypothesis;[1] we believe the same will happen for the cases $\Delta/J>0$.

We have also investigated how the h_0-T phase diagram varies with Δ. Similarly to what happens in the case $h_0=0$, the region of replica-symmetry breaking is reduced by the presence of the random field. Essentially, two distinct types of AT lines were found, both scaling as $\tau \sim (h_0/J)^{2/\phi}$, for h_0/J small, where ϕ is in principle a nonuniversal exponent, as we mention below.

(a) $\Delta/J \ll 1$. A crossover between a random-field regime ($\Delta \gg h_0$), with $\phi=1$ (concave range), to a spin-glass regime ($\Delta \ll h_0$), where $\phi=3$ (convex range) was found. Other nonuniversal behaviors for spin glasses have already been reported in the literature.[41]

(b) $\Delta/J \gg 1$. The AT instability is signaled by a nearly vertical straight line which approaches the h_0/J axis as $\Delta/J \to \infty$; one still has $\phi=1$, but no crossover to $\phi=3$ is observed for h_0/J small.

The above results may be compared with recent measurements performed on site-diluted uniaxial antiferromagnets in the presence of a uniform magnetic field of which one of the most investigated (and puzzling) is $Fe_xZn_{1-x}F_2$.[31–39] Its percolation threshold is $x_p=0.24$, and even not having competing interactions it may present spin-glass behavior. For $x \geq 0.40$ it is well known as a prototype of an Ising random-field system, whereas for $x \approx x_p$ it behaves as a conventional spin glass.[31–33] For intermediate concentrations, both behaviors may be found, depending on the value of the external field. In particular, the case $x=0.31$ has been investigated in detail;[35–38] a line corresponding to the appearance of irreversibility effects, which is usually associated with the Almeida-Thouless instability, has been observed in which (i) for a very small external field (less than 1 T) this line presents $\phi=1.42$; (ii) as the field is increased (above 1.5 T), a reversal in the curvature of such line is observed, which scales then as the AT line for spin glasses, with $\phi=3.4$.

Within the usual precautions in what concerns the results of mean-field approximations, we may say that the breakdown of reversibility for $Fe_{0.31}Zn_{0.69}F_2$ in the presence of an external magnetic field may be satisfactorily identified with the Almeida-Thouless instability presented here (for small Δ/J), as shown in Fig. 3. It is amazing how an infinite-range model can reproduce such a subtle effect.

In Fig. 4 we present the evolution of the AT line with Δ/J, which should be compared with the evolution of the experimental equilibrium boundary $[T_{eq}(h)]$ for $Fe_xZn_{1-x}F_2$ in the range $0.25 \leq x \leq 0.40$ presented in the literature (either Fig. 4 in Ref. 36 or Fig. 9 in Ref. 37). Two important points should be stressed when comparing these figures, as we discuss next.

(1) The instability region delimited by the AT line in our Fig. 4 shrinks as Δ/J increases, whereas in the experimental phase diagram the region below $T_{eq}(h)$ (which starts, for zero external field, at the Néel temperature T_N) is shifted to the right, forced by the increase of T_N as x varies from 0.25 to 0.40. The region in the experimental phase diagram to be associated with the AT instability is the *glassy state* which is defined for temperatures $T_c(h)<T<T_{eq}(h)$, where $T_c(h)$ signals the appearance of long-range antiferromagnetic ordering. Although nonequilibrium effects are present for $T<T_c(h)$, due to critical slowing down associated with activated dynamics, their characteristics are very distinct from those of a spin-glass state. In the experimental phase diagram such a glassy state also gets reduced as x increases. For small magnetic fields, one has $\phi=1.42$ ($x \approx 0.40$) in the experimental case,[36,37] whereas we find $\phi=1.0$ ($\Delta/J \to \infty$); no crossover is observed in either case.

(2) For high magnetic fields our instability region is always dominated by the exponential decay shown in Eq. (3.2), whereas the experimental curve for $T_{eq}(h)$ approaches $T=0$ with zero slope for $x \approx 0.40$. One should remember that the identification of a diluted antiferromagnet in the presence of a uniform external field with

the ferromagnet in a random field is valid only in the limit of small magnetic fields.[15,16]

To conclude, we have presented an infinite-range Ising spin-glass model in the presence of a Gaussian random field, which is able to reproduce several recent experimental results obtained for the diluted antiferromagnet $Fe_xZn_{1-x}F_2$ ($0.25 \leq x \leq 0.40$). Surprisingly, our model is based on competing interactions, whereas the compound $Fe_xZn_{1-x}F_2$ is believed not to have them; this may indicate that spin-glass properties should not be specific only to canonical spin glasses, but should rather be quite generic, common to systems with a large number of low-lying, almost degenerate states, which represent multiple minima in the free-energy surface, as pointed out in Ref. 36

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