Transmittance fluctuations and nonlinearity in random chains in the presence of applied electric fields

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We have carried out numerical investigations of transmittance fluctuations in disordered chains in the presence of external electric fields. We have obtained an almost constant fluctuation in a length scale smaller than the localization length. However, the value of the fluctuation in the plateau region is dependent on the external electric field and the strength of the disorder. We have also studied the transmittance autocorrelation as a function of external electric field to probe the nonlinearity in the transmittance.

Universality has been predicted in the zero-temperature aperiodic conductance fluctuations of a mesoscopic sample as a function of chemical potential, magnetic field, or impurity configurations, by various authors (ALSF).[1–4] It is now well understood that the phenomenon originates from quantum mechanical interference of multiply scattered electrons. The universal value of the root-mean-square conductance fluctuations Δg, is of the order of $(e^2/\hbar)$ and has been found to be independent of randomness and length of the sample in the weak-disorder limit. The value depends weakly on the dimensionality of the system.

The above theory (ALSF) has been invoked by the pioneering experimental studies of aperiodic magnetoresistance fluctuations in metallic wires,[5] quasi-one-dimensional metal-oxide-semiconductor configurations,[6] and periodic Aharonov-Bohm magnetoresistance oscillations in loops.[7] The universal characteristics have been tested numerically by Giordano,[8] Xie and Das Sarma,[9] Harris and Houari,[10] as well as our group.[11] Most of the studies of universal conductance fluctuations have made use of Anderson tight binding Hamiltonian.[12] Disorder is usually introduced in the diagonal terms. Harris and Houari[10] have recently studied the phenomenon in disordered binary allows with the Hamiltonian characterized by δ-function like potential at each site. Manna and Mookerjee[11] have examined the UCF in systems with continuously and randomly varying potentials using the invariant imbedding method.[13–16]

In the previous theoretical works,[1–4,8–10] quasi-one-dimensional systems (width much less than localization length) have been considered and have direct relevance to the experimental studies in thin wires where evidence for quantum diffusion has been revealed. The striking point is that UCF is independent of width of the system which has been verified by decreasing the width up to even a lattice spacing.

Manna and Mookerjee[17] have pointed out that although the distribution of phase angles for the amplitude transmission or reflection coefficient is approximately uniform in large, weakly disordered chains, for smaller lengths or stronger disorders the distribution of the phase angle is twin peaked as the phases are *pinned*. Recently, Pradhan and Kumar[18] observe the same and point out that its consequence, strong correlation between the phase and the magnitude of the amplitude reflection or transmission coefficient, leads to finite average conductances and finite averaged conductance fluctuations for smaller length scales in weakly disordered chains. This correlation seems to have escaped notice prior to our numerical work on disordered chains[11,19] and Pradhan and Kumar's[18] analytical treatment. It then becomes reasonable to study UCF in disordered chains.

We concentrate first on the behavior of fluctuations in sufficiently weakly disordered one-dimensional systems. We have found that in a length scale within a phase coherence length shorter than the localization length, such systems do exhibit UCF. Then we proceed to study the fluctuation and underlying nonlinearities in transmittance in the phase coherence regime due to applied electric fields. In this regime, the interference pattern is extremely sensitive to the details of impurity potentials. With the presence of an electric field, the overall scattering potential structure is modified. This changes the details of the interference effect. This aspect causes highly nonlinear I-V characteristics.

To date there are not many studies on mesoscopic transport phenomena in the presence of an electric field. Al'tshuler and Khmelnitskii[19] have shown that conductance of a mesoscopic sample is a random function of voltage and this variation is characterized by the correlation function. This is a signature of the fact that when quantum interference, the I-V curve is strongly nonlinear. They have predicted a voltage scale V_c where conductance fluctuations is of the order of $(e^2/\hbar)$. And beyond V_c fluctuation decays as $(e^2/\hbar)\sqrt{(V_c/V)}$. The experimental results[21] in small metallic samples are in qualitative agreement with the AKL (Al'tschuler-Khmelnitskii-Larkin) theory.[19] Recently, Tang and Fu[20] have proposed a theory that claims to explain the strong nonlinearity as observed by Webb, Washburn, and Umbach.[21] According to this theory, nonlinearity appears when the change in electron energy ΔE becomes equal to the order of level spacing ε instead of $\Delta E = g\varepsilon$, as predicted in AKL theory.

Our approach is based on the invariant imbedding method[13] which yields a nonlinear first order differential equation for the complex coefficient of reflection. We shall follow the approach of Heinrichs[22] which yields a coupled nonlinear differential equation for the complex reflection coefficient. This equation differs from that utilized by Vijayagobindan *et al.*[23] for the work on localization and resistance fluctuations. However, for the zero-field situation, both the equations take the same form.

We consider a disordered chain, stretching from $x=0$ to $x=L$, the ends of which are attached to semi-infinite perfectly conducting leads maintained at a potential difference, FL, where F is the strength of the applied field, directed along negative x axis. The model Hamiltonian of the disordered system is

$$H=(-\hbar^2/2m)(\partial^2/\partial x^2)+V(x) , \quad (1)$$

where $V(x)=-Fx+v(x)$. $v(x)$ at any point x varies uniformly randomly about mean zero between $-w/2$ and $+w/2$. Therefore, $\langle v(x)\rangle=0$ and $\langle V(x)\rangle=-Fx$.

An electron of energy $E=k_0^2/2$ moving from $-\infty$ is incident at $x=L$. Solving the Schrödinger equation in the sample and leads and applying smooth boundary conditions at $x=L$ one obtains a set of coupled differential equations for the real and imaginary parts of the reflection coefficient, $R_1(x)$ and $R_2(x)$:

$$\begin{aligned}\frac{dR_1(x)}{dx}=&-\frac{k^2(x)}{k_1(x)}R_2(x)[R_1(x)+1]\\&+\frac{F}{2k_1^2(x)}\times\cdots\times[1-R_1(x)+R_2^2(x)]\\&+k_1(x)R_2(x)[R_1(x)-1] ,\end{aligned} \quad (2)$$

$$\begin{aligned}\frac{dR_2(x)}{dx}=&\frac{k^2(x)}{2k_1(x)}[1+R_1(x)+R_2(x)]\\&-\frac{F}{k_1^2(x)}R_1(x)R_2(x)+\cdots\\&+\frac{k_1(x)}{2}[1-R_1(x)+R_2(x)]\\&\times[1-R_1(x)-R_2(x)] ,\end{aligned} \quad (3)$$

where

$$k(x)=\sqrt{k_0^2-2[v(x)-Fx]}$$

and

$$k_1(x)=\sqrt{(k_0^2+2Fx)}$$

We have solved these equations numerically for reflectance, $r(x)=R_1^2(x)+R_2^2(x)$ and transmittance $t(x)=1-r(x)$.

We have studied the conductance in the absence of an external electric field by making use of generalized version of Landauer formula as worked out by Fisher and Lee.[24,25] When an electric field is present, we have chosen to emphasize the studies of transmittance.

We present the numerical results for transmittance fluctuation Δt in the mesoscopic regime of one-dimensional disordered systems under the influence of electric field. For the averaging process, we have considered 200 independent samples differing only in the microscopic configurations of the potential $v(x)$. The rms transmittance fluctuations Δt is given by $\Delta t=[\langle t^2\rangle-\langle t\rangle^2]^{1/2}$.

For zero electric field, the generalized Landauer formula provides a simple relation for Δg in terms of $\Delta t, \Delta g=2\Delta t$. To ensure mesoscopic regime in one-dimensional disordered systems, we display in Fig. 1, Δg as a function of sample size L for $w=0.1$, $E=0.125$, and $F=0$. Δg attains a certain value, $0.54e^2/\hbar$, which remains constant in the region $L=300$–700. In an earlier work[11] we have verified that this value does not change by changing the degree of disorder, in the weak disorder limit. Only the extent of the regime, over which Δg remains constant, increases with decreasing disorder. These are features of UCF, as predicted by ALSF.[1–4] Our value of UCF is in reasonable agreement with those observed in quasi-one-dimensional system.[1–4,8–10]

We now proceed to describe the effect of electric field on mesoscopic fluctuations. For this study, we invoke transmittance as the physical quantity of interest. In Fig. 2(a) we find that the transmittance fluctuations are reduced in the presence of external electric fields. We also see that the transmittance fluctuations remain invariant over length scales which decrease as the electric field increases. However, unlike the zero-field results, the finite field Δt in the plateau region is not universal but depends on the strength of disorder. In Fig. 2(b), we have shown the transmittance fluctuations plateaus over a fixed extent from $L=300$ to $L=500$ for three different electric fields: $F=0.0$, 25×10^{-4}, and 50×10^{-4} with $w=0.1$ and $E=0.125$. For $F\geq60\times10^{-4}$, the plateau region disappears due to the presence of nonlinearity and the delocali-

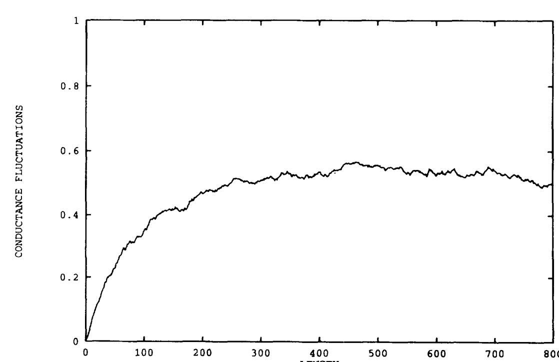

FIG. 1. Zero field Δg as a function of length, for $w=0.1$, and $E=0.125$.

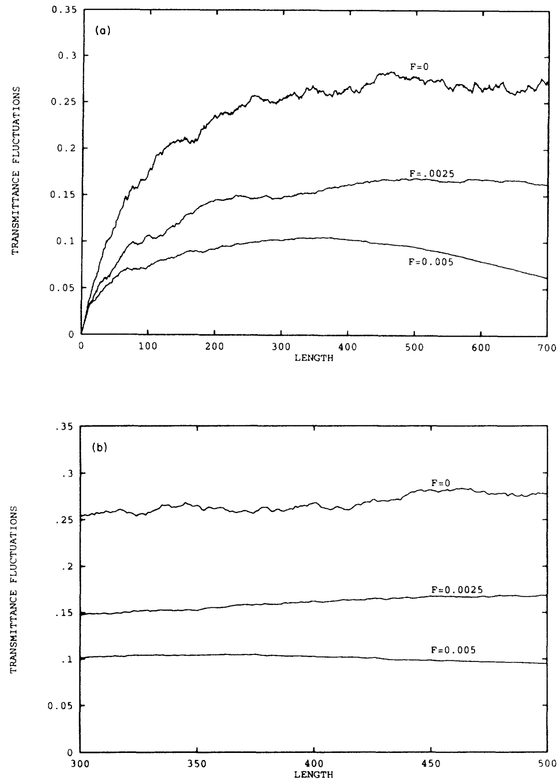

FIG. 2. δt plateaus for $F=0.0,0.0025,0.005$; $w=0.1$, $E=0.125$.

zation effect of the electric field.

In Fig. 3 we show the behavior of transmittance fluctuations as a function of the electric field, for a fixed length $L=400$. We have displayed this in Fig. 3. At field$=0.0$, Δt is universal and of the order of 0.27 $(e^2/\hbar)$. As field is increased in small steps, 5×10^{-6}, the typical pattern evolves gradually. For $F\simeq2\times10^{-4}$, δt starts to change randomly by a considerable amount, about 20% of the zero field value. Further increase of electric field causes the width of the fluctuations to diminish. However, on the overall view, Δt decreases with increasing electric field with a slower variation beyond $F=0.003$. This result is compatible to the theoretical as well as experimental observations.[19,21] We should mention that field-dependent transmittance fluctuations for any other choice of length, within the lengths $L=300-500$, is of the same characteristics as shown for $L=400$.

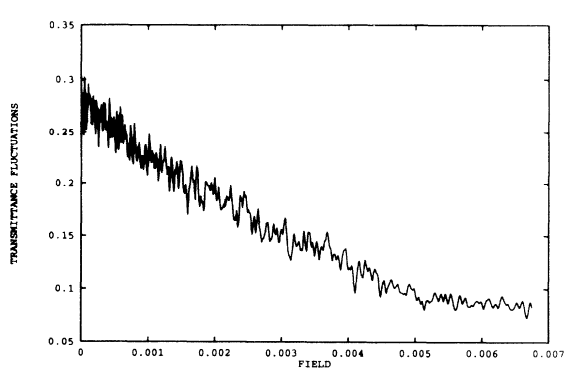

FIG. 3. Δt versus F for the fixed length$=400$; $w=0.1$, $E=0.125$.

This behavior is very similar to that found by Holweg *et al.* in magnetoconductance of a Ag ballistic point contact, where quantum interference arises from the scattering by sample geometry. Although the systems are very different, both are characterized by quantum interference and the behavior of transmittance fluctuations in the plateau region as a function of electric field are very similar. The main difference with the work of Larkin and Khmelnitskii and Al'tshuler and Khmelnitskii[19] or the experimental situation as pointed by Webb *et al.*[21] is that decay in the conductance fluctuation as shown by Holweg *et al.*[26] is due to the presence of inelastic scattering which is mainly responsible. On the other hand since our treatment is based on elastic scattering from the effective potential landscape, so the numerical results for the decay of transmittance fluctuation of ours is relevant to the work of Larkin and Khmelnitskii and Al'tshuler and Khmelnitskii.[19]

Since the decay of transmittance fluctuation in a disordered mesoscopic sample in the presence of an electric field is understood to be due to the interplay of nonlinearity and quantum interference and at high enough electric fields nonlinearity dominates over the scattering from disorder only, the same effect can be understood at the qualitative level at least for the differential conductance also. This is true whether we try to relate our numerical results with the experimental results as done by Webb *et al.*[21] or the theoretical work of Larkin and Khmelnitskii and

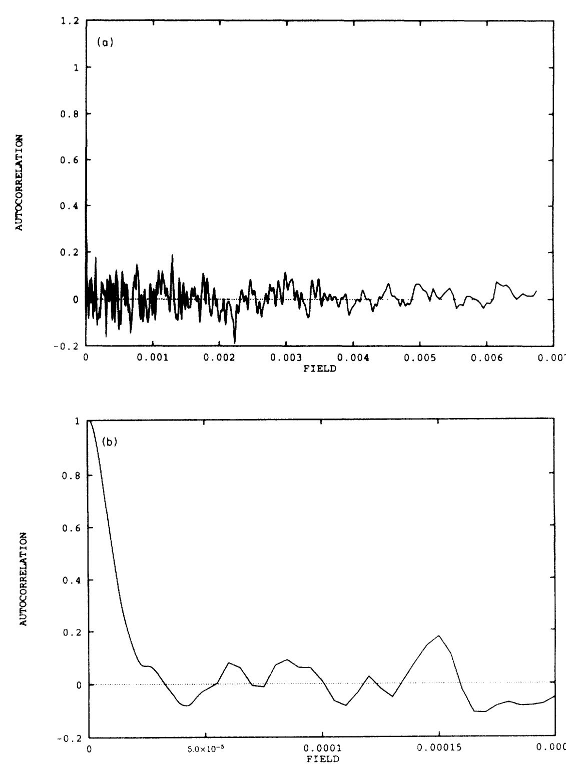

FIG. 4. $C(F)$ for length$=400$, $w=0.1$, $E=0.125$.

Al'tshuler and Khmelnitskii.[19] We think that the results for transmittance for different disorder configurations may be used as input to calculate the nonlinear differential conductance and for the fluctuation studies of the same. This is a formidable task to do in order to compare with experiments.

Transmittance fluctuations for different electric fields are not independent of one another. This essentially provides a long range correlation for transmittance. We have examined this phenomena by evaluating the normalized autocorrelation function defined as

$$C(F)=\frac{\langle t(F_0)t(F_0+F)\rangle-\langle t(F_0)\rangle\langle t(F_0+F)\rangle}{\langle t^2(F_0)\rangle-\langle t(F_0)\rangle^2}$$

as a function of field. The result is demonstrated in Fig. 4. It is shown in Fig. 4(b) that $C(F)$ drops very rapidly with characteristic field scale, of the order of 0.000 02. As the field increases further, we observe random fluctuation of $C(F)$ between the peak amplitudes $+0.2$ and -0.2. This behavior persists up to the field, of the order of 0.003. More increase of the field shows much slower variation of $C(F)$. The behavior of autocorrelation function in different field regimes in our numerical studies is indicative of typical one-dimensional characteristics of the nonlinearity effects in the mesoscopic fluctuations.

The initial drop in the autocorrelation function is very similar to that found in two dimensions by Tang and Fu.[20]

Our numerical results for the effect of vanishingly small and moderate electric field in the mesoscopic regime of one-dimensional disordered systems exhibit the nonlinearity of intrinsic origin. This investigation shows, in a qualitative agreement with the previous studies, that transmittance is a complicated function of electric field. The other striking observation is that the nonzero field transmittance fluctuations in the mesoscopic regime appear as a plateau, so long as it is not destroyed by the delocalizing effect of the electric field and also by the nonlinear nature of the transmittance.

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