Flux flow and flux cutting in type-II superconductors carrying a longitudinal current

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(Received 8 November 1993; revised manuscript received 16 March 1994)

A heuristic model of flux flow and vortex cutting that does not imply a buildup of longitudinal flux is given for type-II superconducting cylinders carrying a current in the presence of an axial magnetic field.

I. INTRODUCTION

The existence of flux-flow resistance in type-II superconducting cylinders carrying a current in the presence of a parallel axial magnetic field (in what has been called longitudinal geometry) indicates that an azimuthal component of flux must be moving inward toward the center of the cylinder in order to generate the observed electric field. Brandt[1] has argued that since the longitudinal flux must remain constant, the moving transverse component must somehow interpenetrate the longitudinal component of magnetic flux, and that flux cutting must therefore occur. Clem[2] has published a proof showing that "so long as the specimen remain in the mixed state, the production of longitudinal voltage by a succession of inward-collapsing helical vortices leads inevitably to an ever-increasing longitudinal flux through the cross section." He maintains that this is true even if vortex cutting occurs. Kogan[3] has argued that, for a stationary current flow in a cylindrical sample of homogeneous material, the basic relation $\mathbf{E}=\mathbf{B}\times\mathbf{v}$ used to derive the electric field is incorrect. He points out that this relation could continue to hold if there were a breakdown in homogeneity as reported by Ezaki and Irie,[4] but that such a breakdown is not confirmed by the experiments of Cave and Evetts.[5]

A. Clem's proof

Clem uses the basic relations

$$\nabla\times\mathbf{E}'=-\frac{\partial\mathbf{B}}{\partial t}\ ,\quad \mathbf{E}'=\mathbf{B}\times\mathbf{v}\ , \tag{1}$$

where $\mathbf{E}'$ is the "effective electric field" defined by Josephson[6] and $\mathbf{E}'$, $\mathbf{B}$, and $\mathbf{v}$ are averaged over a few intervortex spacings. Using Clem's notation, define the helical vortex at the surface of the cylinder by $\mathbf{B}_s=B_{s\phi}\hat{\boldsymbol{\phi}}+B_{sz}\hat{\mathbf{z}}$ and the inward velocity of the vortices as $\mathbf{v}_s=-\hat{\mathbf{r}}v_s$. Then the electric field $\mathbf{E}_s'$ at the surface is given by

$$\mathbf{E}_s'=\mathbf{B}\times\mathbf{v}_s=-B_{sz}v_s\hat{\boldsymbol{\phi}}+B_{s\phi}v_s\hat{\mathbf{z}}=E_{s\phi}'\hat{\boldsymbol{\phi}}+E_{sz}'\hat{\mathbf{z}}\ . \tag{2}$$

Integrating the first of Eqs. (1) over the surface of the cylinder and using Stokes' theorem on the left-hand side gives

$$\int_0^{2\pi}E_{s\phi}'d\phi=-\dot{\Phi}\ . \tag{3}$$

Clem now assumes a steady-state value of $\mathbf{E}_s'$ so that $E_{s\phi}'$ is constant and Φ is therefore a constant times the time. Thus, the longitudinal flux increases without bound.

However, assuming a steady-state value of $\mathbf{E}_s'$ is a very strong condition. It means that the spiral vortex as a whole is moving inward toward the axis. In particular, a nonzero $E_{s\phi}'$ *means* that axial flux is continually crossing the surface into the superconductor thus increasing the axial flux; the "proof" therefore assumes its conclusion.

B. Kogan's proof

Kogan assumptions are

$$\begin{aligned}&\nabla\times\mathbf{E}=0\ ,\\&\mathbf{E}=\{0,E_\phi(r),E_z(r)\}\ .\end{aligned} \tag{4}$$

Carrying out the indicated operation leads to

$$\nabla\times\mathbf{E}=-\frac{dE_z}{dr}\hat{\boldsymbol{\phi}}+\left\{E_\phi+r\frac{dE_\phi}{dr}\right\}\frac{\hat{\mathbf{z}}}{r}=0\ . \tag{5}$$

Thus, E_z is constant and E_ϕ must vanish. But since $B_z\neq 0$, $\mathbf{E}=\mathbf{B}\times\mathbf{v}$ implies that E_ϕ cannot vanish and Kogan concludes that $\mathbf{E}=\mathbf{B}\times\mathbf{v}$ cannot be correct, subject to the condition that there is no breakdown of homogeneity.

The "proof" fails, however, because the assumption that $\nabla\times\mathbf{E}=0$ *means* that $\partial\mathbf{B}/\partial t=0$ and therefore that no flux is moving into the sample so that $\mathbf{v}\equiv 0$.

II. FLUX-FLOW AND CUTTING IN LONGITUDINAL GEOMETRY

Three fundamental flux regimes will be taken as typical for the purposes of discussion. They are defined as a function of transport current: (A) a nonlinear region of increasing paramagnetic moment; (B) a linear region of increasing paramagnetic moment where flux flow is present; and (C) a second linear region of increasing paramagnetic moment where large-scale flux flow occurs. These regions are shown in Fig. 1 superimposed on data abstracted from Walmsley and Timms.[7]

For the case of longitudinal geometry, it is usually assumed that the vortices form a helical array so as to account for the observed paramagnetic moment. It is often further assumed, particularly before flux-flow voltages are observed, that the array takes a force-free or nearly force-free configuration following the suggestion of Bergeron.[8] In the case of a transverse field, the model

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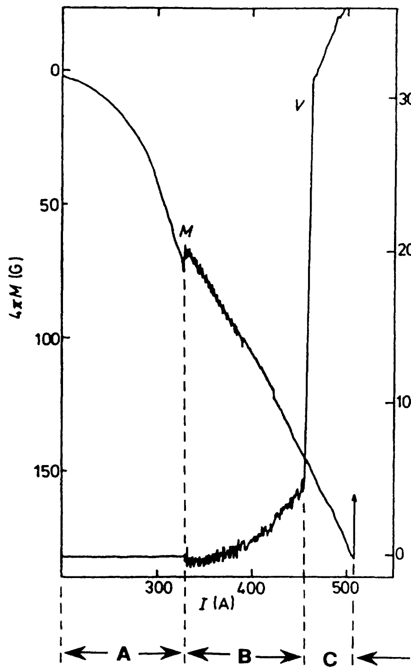

FIG. 1. (A) is a nonlinear region of increasing paramagnetic moment, (B) is a linear region of increasing paramagnetic moment where flux flow is present, and (C) is a second linear region of increasing paramagnetic moment where large-scale flux flow occurs. [Shown superimposed on data taken from D. G. Walmsley and W. E. Timms, J. Phys. F 7, 2373 (1977)].

often used tacitly assumes that current flows *throughout the whole body of the metal* and that this transport current interacts via the Lorentz force with the magnetic flux associated with the normal cores of the vortices.[9] If this model is extended to longitudinal geometry, the Lorentz force on the vortices would only vanish if the vortices were parallel to the symmetry axis, and it would not be possible to account for the observed paramagnetic moment.

If, on the other hand, one assumes that the current flows *along* the vortices, it is possible to argue that the lattice assumes a force-free configuration, and that the transport current could be carried without pinning while producing a paramagnetic moment. However, for this to be true, either the density of vortices, the longitudinal current carried per vortex, or both would have to vary so as to satisfy the force-free relation $\nabla\times\mathbf{B}=\alpha\mathbf{B}$, where α is, in general, a scalar function of position. While some experimental results are not inconsistent with the assumption that the flux-line lattice adopts a force-free configuration,[10] there seems to be no *compelling* experimental or theoretical reason for this to be true. A number of alternative models have been given by Timms and Walmsley.[11]

The term "force free" is also sometimes applied to the following case: Assume the De Gennes-Matricon[12] model of a vortex; if it is further assumed that current flows along the vortex parallel to the field lines, the Lorentz force between the current and field would vanish. Such a configuration is incorrectly called "force free."

A. Nonlinear region of increasing paramagnetic moment

Assuming here, and in what follows, that the transport current flows along the vortices comprising the flux-line lattice, the nonlinear increase of the paramagnetic moment with increasing current, as shown in region A of Fig. 1, implies that the angle the helical flux-line lattice makes with the symmetry axis must, on the average, be increasing. That is, the lattice must alter its geometric configuration so as to give rise to a greater current-induced magnetization than would result from only an incremental increase in current. This must be true regardless of the model used for the flux-line lattice. It is, of course, clear that since the applied longitudinal field is constant, and the azimuthal field at the surface of the sample increases with the current, the angle nucleating vortices make with the axis must also increase with the current. But because the flux is not necessarily pinned at the ends of the sample after nucleation, this does not constrain the model used for region A, since one cannot argue that helicity[13] is conserved over the length of the sample.

B. Linear region of increasing paramagnetic moment

This region begins where the onset of flux-flow resistance is first observed. The linear increase in paramagnetic moment with increasing current, as shown in region B of Fig. 1, implies that the angle the helical flux-line lattice makes with the symmetry axis must, on the average, be constant. This does not mean that the lattice cannot change its configuration, but only that any change must average out over the lattice so there is no net additional geometrical contribution to the paramagnetic moment. As the transport current increases over region B, vortices continue to nucleate at the surface making an increasingly greater angle with the axis. Although helicity need not be conserved over the length of the sample, the model proposed here to maintain the geometric stability of the flux lattice as a whole over region B relies on restraints due to the elastic properties of the lattice,[14] and on the supposition that vortices nucleating on the surface of the sample are susceptible to a kink instability followed by flux cutting.

A highly idealized representation[15] of the sequence is shown for a single vortex in Fig. 2. The axial flux associ-

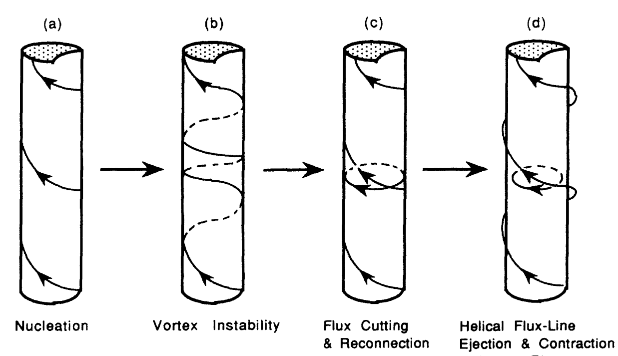

FIG. 2. Idealized representation of the proposed model for vortex nucleation, instability, flux cutting, and flux ejection. The energy of the vortex per unit length is conserved.

ated with the vortex is quantized and is consequently conserved during this process. The energy per unit length of the vortex is therefore also conserved.

Assume [Fig. 2(a)] that it is energetically favorable for additional flux to nucleate on the surface of the sample.[16] To prevent the increase in axial flux with continued nucleation, the following mechanism is envisioned: Because the energy of the helical flux line (per unit length) is unchanged during the vortex instability [Fig. 2(b)] and flux-cutting phases [Fig. 2(c)], it is not energetically favorable for the flux line *and* the vortex ring to remain in the sample; *the helical line is ejected* [Fig. 2(d)], thus precluding a build up of axial flux, and the ring vortex contracts.[17] If the assumption is made that the axial vortex current is conserved, the contracting ring vortex will itself have a circulating current and therefore make a time-dependent contribution to the paramagnetic moment. Such time-dependent behavior was, in fact, observed by Walmsley and Timms. After the ring vortex contracts, it is again energetically favorable for additional flux to nucleate on the surface of the sample, and the cycle is repeated.

It has been suggested[18] that "nascent vortices" of zero order parameter are formed at the surface as part of the nucleation process. If the concept of instability is applicable to such vortices, it is possible to argue that only the ring vortex itself nucleates. Consequently, there is again no build up of axial flux. On the other hand, if the magnetization oscillations observed by Walmsley and Timms are indeed related to the formation of a ring vortex carrying a circulating current as a result of vortex instability and subsequent flux-cutting processes, nucleation of only the vortex ring would seem to be ruled out since there would then appear to be no necessity for a circulating current.

The process whereby the ring vortex contracts by cutting through sequential annular layers of decreasing radii has been discussed by Campbell and Evetts.[19] They note (attributing the observation to Frank) that the absorption of a perpendicular vortex implies that there is a critical angle between vortices at which the interaction becomes attractive. This angle must be less than $\pi/2$ radians. The reason for this can be seen in Fig. 2(b) where it is apparent that the angle between nucleating vortices in the layer exhibiting instability and neighboring layers of smaller radii is always less than $\pi/2$ radians. Frank's observation addresses the argument by Josephson[20] that while the free energy can be lowered by untwisting the field lines, a large "activation energy" prevents the cutting of flux lines.

The magnetic field and current associated with a new model or an isolated vortex carrying a longitudinal current has been given elsewhere.[21] They are

$$\mathbf{B}=\{0,cK_1(r),-K_0(r)\}\ ,\qquad \mathbf{J}=\{0,-K_1(r),-cK_0(r)\}\ , \tag{6}$$

where the K_n are modified Bessel functions of the second kind and c is an arbitrary constant proportional to the longitudinal current. When c vanishes, this solution to the Ginzburg-Landau equations reduces to that given by Abrikosov.[22] This vortex model becomes unstable[23] with respect to a kink instability for very small amounts of twisting, although there may be stable intermediate states having a slight corkscrew form. For values of r large compared to unity, **J** and **B** are perpendicular. Since the vortex is subject to a kink instability for very small values of c, the azimuthal flux will be small compared to the quantized axial flux. Note also that the azimuthal flux is not quantized.[24]

The interaction between two Abrikosov vortices, whose axial current vanishes, is repulsive if the vortices are aligned so that their flux is in the same direction, and attractive if they are aligned so that their flux is opposed. The critical angle is $\pi/2$ radians. If the vortices are those represented by Eqs. (6), the interaction becomes attractive at an angle $\gamma=\pi/2-2\beta$, where $\tan\beta=J_z/J_\phi=cK_0(r)/K_1(r)\rightarrow c$ for $r\gg 1$. This model therefore satisfies the condition that the interaction be attractive for an angle between vortices of less than $\pi/2$ radians, although c, and therefore β, might be quite small, consistent with the analysis given by Campbell and Evetts.

C. Second linear region of increasing paramagnetic moment

This region is characterized by the large step increase in the voltage associated with flux flow. Such a large increase indicates that there must be a new flux-flow mode for this region. The linear increase in paramagnetic moment again implies that, on the average, the angle the flux-line lattice makes with the symmetry axis must remain constant.

In region B, it was suggested that nucleating vortices were unstable and through subsequent flux-cutting generated the observed voltages. Because the elastic properties of the flux lattice within the body of the lattice itself help maintain stability with increasing current, vortex instability was restricted to those nucleating at the surface of the sample. Here, at least a portion of the flux lattice itself must become unstable. Since the outermost cylindrical layer of the flux lattice is constrained by intervortex forces only on the "inside" of the layer, it is suggested here that this layer becomes unstable resulting in the large step increase in flux-flow voltage. Such a collapse also eases restraints on the next adjacent layer which may also become unstable with increasing current. The basic sequence of Fig. 2 is still assumed to be valid.

ACKNOWLEDGMENT

This work was supported in part by the U.S. Department of Energy under Contract No. W-31-109-ENG-38.

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current of 200 A and an external field of $H=427$ Oe (3.4×10^4 At/m) is essentially zero, it is assumed that (1) the field produced by the vector sum of the field due to the transport current and the external field approximate the value needed to overcome the surface barrier in type-II superconductors [C. P. Bean and J. D. Livingston, Phys. Rev. Lett. **12**, 14 (1964)]; and (2) the integrating method of measuring the paramagnetic moment has been used.

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[16]Note there is no surface barrier in the sense of Bean and Livingston [C. P. Bean and J. D. Livingston, Phys. Rev. Lett. **12**, 14 (1964)], who considered only the case of a single flux vortex. Here, there is a balance between the magnitude of the flux where it is energetically favorable for a flux line to penetrate the sample and the interaction between the existing flux lattice and the potentially nucleating vortex [see C. G. Kuper, *An Introduction to the Theory of Superconductivity* (Clarendon, Oxford, 1968), Sec. 7.2].

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