Thermodynamics of nucleation in current-carrying conductors

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This work studies the thermodynamics of phase transitions of the first kind in current-carrying conductors when these transitions are accompanied by sharp change of the electric conductivity. The expression for the work of formation of a nucleus of a new phase in a current-carrying conductor as a function of the geometrical parameters of the problem is derived. The dynamics of the evolution of a macroscopic interphase boundary when both phases form the coaxial cylindrical domains is investigated. It is shown that the character of equilibrium of an interphase boundary depends upon the mutual location of phases with lower and higher conductivity. If a low-conductivity phase occupies the external cylinder the position of the interphase boundary is stable. In the opposite case, namely when the low-conductivity phase occupies the internal cylinder, the position of the interphase boundary is unstable. It is shown that the current-carrying conductor melts from the surface at the temperature lower than the melting temperature. However, during propagation of the liquid (low-conductivity) phase from the surface to the axis of a conductor, the nuclei of a solid phase (high conductivity) may form inside it. The possible scenario of melting of the current-carrying conductors is suggested.

I. INTRODUCTION

This investigation is a continuation of previous works[1,2] which analyzed the effect of ponderomotive forces upon the dynamics of phase transitions, when these phase transitions are accompanied by a sharp change of electric conductivity. It was showed that ponderomotive forces prevent formation of nuclei with electric conductivity lower than that of a surrounding medium, and promote nucleation when an electric conductivity of a nucleus is higher than that of a surrounding medium. Various effects arising during phase transitions of the first kind were considered using the latter finding.

The analysis performed in Ref. 1 was restricted to the case in which phase transition begins inside the conductor, and only the initial stage of phase transition, i.e., nucleation, was studied. Several important questions—the dependence of the work of formation of a nucleus upon its shape and the distance from the conductor's surface, the work of nucleus formation in a "layered" conductor where the interphase boundary is already formed, the dynamics of the interphase boundary, etc.—remained unexplored. All these problems are investigated in the present work. We also study a case when phase transition begins at the surface of a conductor. In the latter case it is assumed that phase transition skips the nucleation stage, and the conductor's surface itself acts as a nucleus. It is shown that unlike the case of a phase transition in a current free conductor, the interphase boundary in a current-carrying conductor is not in a state of neutral equilibrium but, depending upon the mutual location of phases, can be in a state of stable or unstable equilibrium. Similar to a previous investigation,[1] it is assumed that a high-temperature phase has lower conductivity, and a low-temperature phase has higher conductivity. It is shown that if the external phase is a high-temperature phase, then the equilibrium position of the interphase boundary is stable. If the surrounding phase has a higher conductivity, then the equilibrium position of the interphase boundary is unstable. The phase diagram describing the rate of phase transition at the interphase boundary $\dot{\beta}$ as a function of the volume fraction β of the external phase is shown in Fig. 1.

It is also shown that during heating of a conductor the liquid (high-temperature) phase remains unstable over a wide temperature range with respect to the formation of solid (low-temperature) nuclei inside it. The results obtained allow us to refine the model developed in the previous investigation,[1] and to relax the assumptions which were previously adopted when the phase transition in the vicinity of the conductor's surface was inhibited for some reason. The qualitative picture of a conductor's fusion as

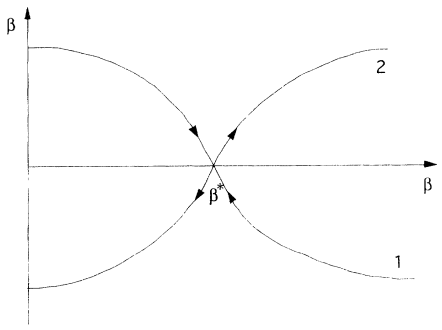

FIG. 1. Scheme of the phase transition on a phase plane. (1) Conductivity of the external phase is less than that of the internal phase. (2) Conductivity of the internal phase is less than that of the external phase. β-volume fraction of an external phase.

predicted by the developed theory is summarized in Sec. V.

The paper is organized as follows. In Sec. II we derive an expression for the work of formation of a nucleus with a conductivity different from the conductivity of a surrounding medium as a function of its shape and the distance from the conductor's surface. An expression for the force applied to the center of mass of a nucleus is obtained. It is shown that in the case when the electric conductivity of a nucleus is higher than that of a surrounding medium, the nucleus is attracted to the center of a conductor. In the opposite case the ponderomotive forces repel a nucleus to the surface.

In Sec. III we investigate the dynamics of the interphase boundary in a two-layer conductor. It is shown that when melting propagates from the surface in current-carrying conductors it can begin at temperatures lower than the melting temperature of the material. In the opposite case, when melting begins inside the conductor, it occurs at temperatures higher than the melting temperature of the material.

In Sec. IV we consider a problem of nucleus formation in the two-layer conductor. Expressions are derived for the critical radii of such nuclei and for the limit values of temperatures of formation for these nuclei inside the external layer (phase) and the internal phase.

II. NUCLEATION INSIDE HOMOGENEOUS CONDUCTOR

Consider the formation of a nucleus of radius a and conductivity σ_a inside a conductor of radius ρ_0 and conductivity σ_0, with a length $L \gg \rho_0$ carrying an electric current I. Assume also that the distance from a nucleus to the conductor's surface and edges is considerably greater that the size of the nucleus. The change of the free energy of the system after the nucleus formation was studied comprehensively in Ref. 1. In the latter study the range of the applicability of the adiabatic approximation, when the time dependence of the electromagnetic fields is determined in the first approximation by the instant value of the nucleus radius a, was also analyzed. Without repeating this analysis, note only that in this case the change of the free energy of the system $\Delta\Phi$ can be written as follows:

$$\Delta\Phi=\Delta\Phi_0+\Delta W, \quad \Delta W=W_0-W , \tag{1}$$

where $\Delta\Phi_0$ is the free-energy change in a current-free conductor, and ΔW an energy change of the magnetic field:

$$\Delta W=\frac{1}{2c^2}\int\int\frac{d\mathbf{r}\,d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}[\mathbf{j}_0(\mathbf{r})\mathbf{j}_0(\mathbf{r}')-\mathbf{j}(\mathbf{r})\mathbf{j}(\mathbf{r}')] , \tag{2}$$

where $\mathbf{j}_0(\mathbf{r})$ and $\mathbf{j}(\mathbf{r})$-current densities before and after formation of a nucleus, respectively.

Electric current distribution in the adiabatic approximation can be determined from the following equations:

$$\mathrm{rot}\frac{\mathbf{j}}{\sigma}=0, \quad \mathrm{div}\mathbf{j}=0 . \tag{3}$$

The nucleus formation is considered to occur far from the conductor's surface, i.e., $(a/d)\ll 1$, where d is the distance from the center of mass of the nucleus to the conductor's surface. Therefore the electric current-density distribution can be determined for the case of a nucleus in an infinite medium:[1,3]

$$\begin{aligned}\mathbf{j}&=\mathbf{j}_0+\delta\mathbf{j} ,\\ \delta\mathbf{j}&=-\mathbf{j}_0\xi\left[2\eta(a-|\mathbf{r}-\mathbf{r}_a|)\right.\\ &\quad\left.+\frac{a^3}{|\mathbf{r}-\mathbf{r}_a|^3}(3\mu^2-1)\eta(|\mathbf{r}-\mathbf{r}_a|-a)\right]\\ &\quad-3\mathbf{e}_\perp j_0\xi\mu\sqrt{1-\mu^2}\frac{a^3}{|\mathbf{r}-\mathbf{r}_a|^3}\eta(|\mathbf{r}-\mathbf{r}_a|-a) ,\end{aligned} \tag{4}$$

where $\mathbf{j}_0$ is the density of an electric current before the formation of a nucleus which is considered homogeneous over the cross section of a conductor, $\mathbf{r}_a$ is the radius vector of the center of mass of a nucleus, $\mathbf{e}_\perp$, the unit vector in a plane normal to vector $\mathbf{j}_0$ and directed from a center of a nucleus, and

$$\mu=\frac{(\mathbf{r}-\mathbf{r}_a)\mathbf{j}_0}{|\mathbf{r}-\mathbf{r}_a|j_0}, \quad \eta(x)=\begin{cases}1, & x\geq 0\\ 0, & x<0\end{cases}, \quad \xi=\frac{\sigma_0-\sigma_a}{\sigma_a+2\sigma_0} .$$

Employing expressions (2) and (4), and neglecting the small terms proportional to the self-inductivity of a nucleus, we found that

$$\begin{aligned}\Delta W&=\frac{\xi j_0^2}{c^2}\int 2\eta(a-|\mathbf{r}-\mathbf{r}_a|)W_1(\mathbf{r})d\mathbf{r}\\ &\quad+\int a^3\frac{\eta(|\mathbf{r}-\mathbf{r}_a|-a)}{|\mathbf{r}-\mathbf{r}_a|^3}(3\mu^2-1)W_1(\mathbf{r})d\mathbf{r} ,\end{aligned} \tag{5}$$

where

$$W_1(\mathbf{r})=\int\frac{d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} . \tag{6}$$

Integration in expressions (5) and (6) is performed over the whole volume of the conductor. Direct integration over $0\leq z'\leq L$ and $0\leq\rho'\leq\rho_0$ in expression (6) yields

$$W_1(\mathbf{r})=\pi\rho_0^2\left[\ln\left[\frac{4z(L-z)}{\rho_0^2}\right]+1-\frac{\rho^2}{\rho_0^2}\right] ,$$
$$z\gg\rho_0, \quad L-z\gg\rho_0 , \tag{7}$$

$$W_1(\mathbf{r})=\pi\rho_0^2\left[\ln\left[\frac{2L}{\rho_0}\right]+\frac{1}{2}\left[1-\frac{\rho^2}{\rho_0^2}\right]\right] ,$$
$$z\ll\rho_0, \quad L-z\ll\rho_0 . \tag{8}$$

Since it was assumed that a nucleus is located far from the edges of the conductor and its surface, in the following it is sufficient to use expression (7). In the Appendix it is shown that the second integral in formula (5) over the domain outside the nucleus is of order ρ_0^2/L^2. Thus relations (5) and (7) allow us to determine the electrodynamic component of the work of nucleus formation ΔW_a:

$$\Delta W_a = \frac{2\xi_a}{c^2}\frac{I^2}{\pi\rho_0^2}V_a\left[\ln\left[\frac{4z_a(L-z_a)}{\rho_0^2}\right]+1-\frac{\rho_a^2}{\rho_0^2}\right], \quad (9)$$

where V_a is a nucleus volume, z_a and ρ_a are nucleus coordinates, and ξ_a is a geometric factor depending upon the shape of a nucleus. In the case when a nucleus has the shape of a spherical ellipsoid with a main axis in the direction of an electric current,

$$\xi_a = \frac{1}{2}\frac{(\sigma_0-\sigma_a)(1-n_z)}{\sigma_a n_z+\sigma_0(1-n_z)}, \quad (9')$$

where n_z is a depolarization factor in the direction of an electric current. Its generalization for the case of arbitrary direction is well known[3] and is not considered here.

When $\xi_a<0$ the optimal shape of a formed nucleus corresponds to a maximum of $|\xi_a|$. If the conductivity of a nucleus σ_a is less than that of the bulk of a material $\xi_a>0$, then the optimal shape of a formed nucleus corresponds to the minimum of a coefficient ξ_a. Expression (9') shows that minimum of a geometric factor ξ_a is achieved for the nuclei flattened along the axis ($n_z\sim 1$). Such nuclei have to be separated from each other by a distance which is considerably higher than their transversal dimensions. In the opposite case, the distribution of the electric current will be completely different and closer to that in a cylinder with low effective conductivity. The longitudinal dimensions of these nuclei during their evolution have to remain considerably smaller than their transversal dimensions. Therefore formation of such nuclei cause stratification of the conductor into separate cylinders. Since at this stage we consider only those nuclei which during their evolution cause the transition of a whole conductor into the new phase, the formation of nuclei flattened in the direction of the axis is not analyzed here. On the other hand, if one considers the feasibility of formation of the long threadlike nuclei with longitudinal dimensions $b\gg\rho_0$, the logarithmic factor does not enter into expression (9) (see below and also Ref. 1). The ponderomotive barrier then becomes considerably lower than that for nuclei with different shape, e.g., for a spherical nucleus. Therefore in the following it is assumed that the long-thread-type nuclei are most favorable for phase transition to the phase with lower conductivity, if this phase transition begins inside the conductor.

Expression (9) determines not only the renormalization of the chemical potential[1] but also ponderomotive forces applied to the center of mass of the nucleus (inhomogeneity):

$$F_{az} = -\frac{\partial(\Delta W)}{\partial z_a}, \quad F_{a\rho} = -\frac{\partial(\Delta W)}{\partial \rho_a}. \quad (10)$$

Expressions (9) and (10) show that if the conductivity of a nucleus is higher than that of the surrounding medium $\xi<0$ then particles are forced to the center of the conductor. In the opposite case $\xi>0$ the nuclei are repelled to the periphery of a conductor. Analysis of systems where the dynamics of inhomogeneities under the action of ponderomotive forces has a significant effect is of interest in itself. For processes occurring in exploding wires, and for the characteristic times involved, the effects of these forces are negligible.

With the logarithmic accuracy expression (9) for $z_i=(L/2)$ (a particle is formed at the center of a conductor) recovers the results derived in Ref. 1. Consider now the stage of the developed phase transition when there exists a macroscopic interphase boundary, and the dynamics of the phase transition is determined by the evolution of this interphase surface.

III. DYNAMICS OF THE INTERPHASE BOUNDARY IN A TWO-LAYER CONDUCTOR

Consider a hollow cylindrical conductor with external radius ρ_0 and conductivity σ_2, with an internal coaxial region of radius ρ_1 and conductivity σ_1. According to (3) the electric currents in these two cylinders are determined by the following expressions:

$$j_1 = \frac{j\sigma_1}{\sigma_1(1-\beta)+\sigma_2\beta}, \quad j_2 = \frac{j\sigma_2}{\sigma_1(1-\beta)+\sigma_2\beta}, \quad (11)$$

where

$$j = \frac{I}{\pi\rho_0^2}, \quad \beta = 1-\frac{\rho_1^2}{\rho_0^2}.$$

The change in the energy of a magnetic field with respect to a homogeneous conductor can be determined similarly to expression (1), $\Delta W = W_0 - W$.

Using expressions (11), ΔW can be represented as follows:

$$\Delta W = \frac{1}{2c^2}\frac{I^2}{\pi^2\rho_0^4}[F_{11}W_{11}+F_{22}W_{22}+2F_{12}W_{12}], \quad (12)$$

where coefficients F_{ik} and W_{ij} are determined by the expressions

$$F_{ik} = \left[1-\frac{\sigma_i\sigma_k}{\bar{\sigma}^2}\right], \quad \bar{\sigma} = \sigma_1(1-\beta)+\sigma_2\beta,$$

$$W_{ik} = \int_{V_i}\int_{V_k}\frac{d\mathbf{r}_i d\mathbf{r}_k}{|\mathbf{r}_i-\mathbf{r}_k|}.$$

With the accuracy of terms of order ρ_0/L, we find that

$$W_{11} = \pi^2\rho_1^4 2L\ln\left[\frac{2L}{\rho_1 e^{3/4}}\right], \quad \ln(e)=1,$$

$$W_{22} = \pi^2\rho_0^4 2L\left[\ln\left[\frac{2L}{\rho_0 e^{3/4}}\right]+\frac{\rho_1^4}{\rho_0^4}\ln\left[\frac{2L}{\rho_1 e^{1/4}}\right]+\frac{\rho_1^2}{\rho_0^2}\ln\left[\frac{e\rho_0^2}{4L^2}\right]\right], \quad (13)$$

$$W_{12} = \pi^2 2L\rho_1^2\rho_0^2\left[\ln\left[\frac{2L}{\rho_0 e^{1/2}}\right]+\frac{\rho_1^2}{\rho_0^2}\ln\left[\frac{\rho_1 e^{1/2}}{2L}\right]\right].$$

Using the identity

$$F_{11}(1-\beta)^2+F_{22}\beta^2+2F_{12}\beta(1-\beta)\equiv 0,$$

we find the following formula for ΔW:

$$\Delta W=\frac{I^2L}{c^2}\left[(1-\beta)^2\ln(1-\beta)\tilde{\xi}^2(\beta)+\tilde{\xi}(\beta)\frac{\sigma_2}{\bar{\sigma}}\beta(1-\beta)\right], \tag{14}$$

where

$$\tilde{\xi}(\beta)=\frac{\sigma_2-\sigma_1}{\bar{\sigma}(\beta)} .$$

For $\beta=0$ and 1, $\Delta W=0$, which corresponds to the extremum change of a magnetic energy as a function of β during the phase transition of a conductor.

Equation (14) was derived under the assumption that the volume of the conductor does not change during the phase transition. In compliance with this assumption, in the following we consider the specific volumes of phases $v_1^0=v_2^0$. Then the dependence of the thermodynamic potential upon the parameter β is given, according to (1), by the following formula:

$$\Phi(\beta)=\bar{\mu}(\beta)N_0, \quad \bar{\mu}(\beta)=\mu_1(1-\beta)+\mu_2\beta+p_m v_0 f(\beta) , \tag{15}$$

where N_0 is the number of particles in the system, μ_1 and μ_2 are chemical potentials of phases 1 and 2, respectively, $p_m=I^2/\pi\rho_0^2c^2$, and

$$f(\beta)=(\tilde{\xi}(\beta))^2(1-\beta)^2\ln(1-\beta)+\tilde{\xi}(\beta)\frac{\sigma_2}{\bar{\sigma}(\beta)}\beta(1-\beta) . \tag{16}$$

Remaining in the framework of the thermodynamic approach, Eq. (16) can be supplemented by the following relation:[4,5]

$$\dot{\beta}\sim-\frac{\partial\bar{\mu}}{\partial\beta}=\mu_1-\mu_2-p_m v_0\frac{\partial f}{\partial\beta} , \tag{17}$$

which describes the phase transition at the interphase boundary in terms of macroscopic kinetics. The equilibrium position of the interphase boundary β^* at fixed temperature and electric current is determined by condition $\dot{\beta}=0$, or

$$\mu_1-\mu_2=p_m v_0\frac{\partial f}{\partial\beta} . \tag{18}$$

Define temperature $\Delta T_1=T_1-T_0$, at which $\beta^*=0$, and temperature $\Delta T_2=T_2-T_0$, at which $\beta^*=1$, where T_0 is phase equilibrium temperature for a current free conductor. Evidently, in the temperature range

$$\Delta T=\Delta T_1(1-\chi)+\Delta T_2\chi, \quad 0\le\chi\le1 \tag{19}$$

the equilibrium position of the interphase boundary β^* occurs inside a conductor. Condition (19) determines the temperature range where there exists stable or unstable layered states of a conductor.

In order to determine ΔT_1, we take into account that since β is considered small, the following condition is satisfied:

$$\beta\ll\min\left\{1,\frac{\sigma_1}{\sigma_2}\right\} . \tag{20}$$

For the calculation of ΔT_2, we used the condition

$$1-\beta\ll\min\left\{1,\frac{\sigma_2}{\sigma_1}\right\} . \tag{21}$$

The difference $1-\beta$ is considered sufficiently large so that the surface energy can be neglected.

When conditions (20) are satisfied, expressions (16) and (18) yield

$$\mu_1-\mu_2=p_m v_0\xi_1, \quad \xi_1=\frac{\sigma_2-\sigma_1}{\sigma_1} . \tag{22}$$

Expanding μ_1 and μ_2 in the vicinity of the equilibrium state of a current free conductor $\mu_2^0=\mu_1^0$, and keeping the linear terms, we find that

$$\Delta T_1=\frac{p_m v_0}{s_2-s_1}\frac{\sigma_2-\sigma_1}{\sigma_1} \tag{23}$$

where s_1 and s_2 are specific entropies of phases. Similarly, using expressions (16), (18), and (21) for the determination of ΔT_2, we find

$$\mu_1-\mu_2=-p_m v_0\xi_2, \quad \xi_2=\frac{\sigma_2-\sigma_1}{\sigma_2}$$

or

$$\Delta T_2=\frac{p_m v_0}{s_1-s_2}\frac{\sigma_2-\sigma_1}{\sigma_2} . \tag{24}$$

Note that when the high-temperature phase has a lower conductivity,

$$\sigma_i>\sigma_k \quad \text{if } s_i<s_k \quad (i,k=1,2) , \tag{25}$$

then the following relations are valid:

$$\Delta T_1<0, \quad \Delta T_2>0 .$$

Consider now the processes when conditions (25) are satisfied, and let us analyze two cases. In the first case the external phase has the lower conductivity and the internal phase the higher conductivity. In the second case the external phase has the higher conductivity and the internal phase the lower conductivity. For convenience, in the following we denote a low-temperature and high-conductivity phase by index S, and a high-temperature and low-conductivity phase by L, referring to solid and liquid phases, respectively.

Expressions (17) and (18) imply that when the external phase is a liquid phase ($\sigma_2=\sigma_L$, $\sigma_1=\sigma_S$) the equilibrium position β^* is stable. Expressions (23) and (24) yield

$$\Delta T_{1L}=-\Delta T_{2L}\frac{\sigma_L}{\sigma_S}, \quad \Delta T_{2L}=T_0\frac{p_m v_0}{\lambda_0}\frac{\sigma_S-\sigma_L}{\sigma_L} , \tag{26}$$

where $\lambda_0=T_0(s_L-s_S)$ is the latent heat of the phase transition.

Expression (26) implies that a liquid phase can be stable at a temperature lower than a melting point if it

occupies the external region of a conductor. It is important to note that here we mean the stability with respect to the phase transition at the interphase boundary, but not the stability with respect to the formation of internal solid nuclei. The latter case is considered separately below.

Consider now the second case, when the liquid phase occupies the internal region of the conductor. Then the equilibrium position of the interphase boundary β^* is unstable. Thus at temperatures $\Delta T_{1S} \leq \Delta T \leq \Delta T_{2S}$ in a conductor with external solid crust with $\beta > \beta^*$, the solid phase will eventually spread over all the conductor. If the position of an interphase boundary $\beta < \beta^*$, the conductor will be completely occupied by the liquid phase.

It can easily be seen that the temperature ΔT_{2S}, which corresponds to the total instability of externally located solid phase, is lower than the temperature ΔT_{2L} corresponding to the stable internally located solid phase. Actually this the same situation which was analyzed in detail in Ref. 1, and which can be summarized as follows. If the phase transition propagates only from the axis of the conductor, then melting begins at temperatures $\Delta T_{2S} > 0$, since at temperatures $\Delta T < \Delta T_{2S}$ internal melting requires the formation of a macroscopic seed liquid volume. The probability of nucleation of such a seed via fluctuations is negligibly small. Until $\Delta T_{2S} < \Delta T_{2L}$, the liquid phase formed inside the conductor and propagating toward its surface is unstable with respect to the formation of internal solid nuclei.

In this investigation primary consideration is given to the case when melting propagates from the surface inside the conductor. The characteristic feature of this process is that the liquid nuclei are not formed inside the solid phase until the temperatures up to $\sim \Delta T_{2S}$. Nevertheless, considering the surface to be a macroscopic "nucleus" we assume that melting can begin directly at the surface skipping the nucleation stage. In order to proceed further, we must determine the work of nucleus formation in a layered conductor.

IV. NUCLEATION IN A CONDUCTOR WITH DEVELOPED INTERPHASE BOUNDARY

Denote the magnetic energy of a layered conductor calculated in above by W_0'. The work of nucleus formation is determined as before:

$$\Delta W_{ia} = W_0' - W_{ia} ,$$

where W_{ia} is a magnetic energy after formation of a nucleus a in the ith $(i=1,2)$ cylindrical layer. Assuming that a nucleus is formed far from the surface of a conductor and at a distance $d \gg a$ from the interphase boundary, we can use solution (4) with the corresponding parameter

$$\xi_{ia} = \frac{1}{2} \frac{(\sigma_i - \sigma_a)(1-n_z)}{\sigma_a n_z + \sigma_i (1-n_z)} , \tag{27}$$

where σ_i conductivity of a medium where the nucleus is formed. Taking (11) into account, and as before neglecting the self-inductivity of the nucleus and the small term considered in the Appendix, arrive at

$$\Delta W_{ia} = 2 p_m V_a \xi_{ia} \left[\frac{\sigma_i}{\pi \rho_0^2 \bar{\sigma}^2} \sum_{k=1,2} \sigma_k \int \frac{d\mathbf{r}}{|\mathbf{r} - \mathbf{r}_a|} \right] . \tag{28}$$

Calculating the integral in (28) over the volume of the conductor, we find that if a nucleus is formed inside the internal cylinder ($\Delta W_{ia} = \Delta W_{1a}$),

$$\begin{aligned} \Delta W_{1a} &= 2 p_m V_a \xi_{1a} \frac{\sigma_1}{\bar{\sigma}} \Gamma_1 , \\ \Gamma_1 &= \ln \left[\frac{4 z_a (L - z_a)}{\rho_0^2} \right] \\ &\quad + 1 - \frac{\rho_a^2}{\rho_0^2} + \frac{\sigma_2 - \sigma_1}{\bar{\sigma}} \left[\beta \frac{\rho_a^2}{\rho_0^2} + (1-\beta) \ln (1-\beta) \right] , \end{aligned} \tag{29}$$

where $a^2 \ll \rho_a^2 < \rho_0^2 (1-\beta)$.

For a nucleus formed in the external cylinder,

$$\begin{aligned} \Delta W_{2a} &= 2 p_m V_a \xi_{2a} \frac{\sigma_2}{\bar{\sigma}} \Gamma_2 , \\ \Gamma_2 &= \ln \left[\frac{4 z_a (L - z_a)}{\rho_0^2} \right] + 1 - \frac{\rho_a^2}{\rho_0^2} \\ &\quad + (1-\beta) \frac{\sigma_2 - \sigma_1}{\bar{\sigma}} \left[1 - \frac{\rho_a^2}{\rho_0^2} + \ln \left[\frac{\rho_a^2}{\rho_0^2} \right] \right] , \end{aligned} \tag{30}$$

where $\rho_a^2 \geq \rho_0^2 (1-\beta)$.

When $\beta \to 0$, i.e., only the internal phase exists, expression (29) recovers formula (9). Similarly, when $\beta \to 1$, expression (9) recovers formula (30). Expressions (29) and (30) allow us to determine the size of the critical nucleus for a layered conductor as was done in Ref. 1.

Assume that temperature ΔT satisfies condition (19) where ΔT_1 and ΔT_2 are determined by expressions (23) and (24). The size of a critical nucleus of a liquid phase inside the internal solid phase is

$$a_{1L} = \frac{2 \alpha v_0}{\mu_S - \mu_L - \dfrac{\Delta W_{1a}^L}{N_0}} = \frac{2\alpha}{p_m \theta_{1L}(\chi, \beta)} , \tag{31}$$

where N_0 is the number of particles in a nucleus, α is the coefficient of the surface tension, and

$$\begin{aligned} \theta_{1L} &= M(\chi) - \Gamma_{1L} \frac{2\sigma_S}{\bar{\sigma}} \xi_{1a} , \\ \bar{\sigma} &= \sigma_S (1-\beta) + \sigma_L \beta , \quad \chi = \frac{\Delta T - \Delta T_{1L}}{\Delta T_{2L} - \Delta T_{1L}} , \\ M(\gamma) &= \frac{1-\gamma}{\gamma} [\chi + (\chi - 1)\gamma] , \\ \gamma &= \frac{\sigma_L}{\sigma_S} , \quad \xi_{1a} = \frac{1}{2} \frac{(\sigma_S - \sigma_a)(1-n_z)}{\sigma_a n_z + \sigma_S (1-n_z)} > 0 , \end{aligned}$$

$$\Gamma_{1L}(\beta)=\ln\left[\frac{4z_a(L-z_a)}{\rho_0^2}\right]+\beta_a + \frac{\sigma_L-\sigma_S}{\bar{\sigma}}[\beta(1-\beta_a)+(1-\beta)\ln(1-\beta)] ,$$

$$\beta_a=1-\frac{\rho_a^2}{\rho_0^2} .$$

The minimum temperature $T_L^{\rm ins}$ at which such nuclei may already form is determined from the condition

$$\frac{2\alpha}{p_m\theta_{1L}(\chi,\beta)}\le\rho_0\sqrt{1-\beta} . \quad (32)$$

The characteristic value of the parameter $2\alpha/p_m$ for exploding wires $(2\alpha/p_m)\sim10^{-6}$ cm.[6] Therefore the condition (32) can be rewritten as $\theta_{1L}=0$ with a good accuracy. For the minimum temperature of internal melting $\Delta T_L^{\rm ins}$, the latter condition yields

$$\Delta T_L^{\rm ins}=\frac{\Delta T_{2S}}{1-\gamma}\frac{2\sigma_S}{\bar{\sigma}}\xi_{1a}\Gamma_{1L}=T_0\frac{p_m v_0}{\lambda_0}\frac{2\sigma_S}{\bar{\sigma}}\xi_{1a}\Gamma_{1L} . \quad (33)$$

Relations (32) and (33) imply that except for the special case which was considered in Sec. I, $\Delta T_L^{\rm ins}>\Delta T_{2S}$ and therefore the minimum temperature for the onset of melting of a conductor from inside is attained for long nuclei with length $b\gg\rho_0$ aligned in the direction of electric current. Then (with obvious stipulations) ΔT_{2S} can be considered the temperature for the internal melting of a conductor. This is exactly what was assumed in the previous investigation,[1] where this temperature was denoted by κ_{21}. From expressions (31) and (32) it follows that the formation of the interphase boundary increases the ponderomotive barrier and causes a positive shift of the melting temperature $\Delta T_L^{\rm ins}$.

Similarly, one can determine the temperature of formation of a liquid nucleus in an external solid phase $\Delta T_L^{\rm out}$, and also the temperatures of formation of solid nuclei $\Delta T_S^{\rm ins}$ and $\Delta T_S^{\rm out}$ in internal and external liquid phases, respectively.

To advance the investigation of melting from the surface, which is our main goal, we now determine the temperature of formation of a solid nucleus in an external liquid phase $\Delta T_S^{\rm out}$. From the formula for the critical size of a nucleus (30), we find that

$$a_{2S}=\frac{2\alpha}{p_m\theta_{2S}(\chi,\beta_a,\beta)} , \quad (34)$$

where

$$\theta_{2S}(\chi,\beta_a,\beta)=-M(\chi)-\frac{2\sigma_L}{\bar{\sigma}}\xi_{2a}\Gamma_{2S}(\beta_a,\beta) ,$$

$$\xi_{2a}=\frac{1}{2}\frac{(\sigma_L-\sigma_a)(1-n_z)}{\sigma_a n_z+\sigma_L(1-n_z)}<0 ,$$

$$\Gamma_{2S}=\ln\left[\frac{4z_a(L-z_a)}{\rho_0^2}\right]+\beta_a + \frac{\sigma_L-\sigma_S}{\bar{\sigma}}(1-\beta)[\ln(1-\beta_a)+\beta_a] .$$

The maximum temperature at which solid nuclei still may appear in a liquid phase can be determined from the condition $\theta_{2S}=0$, or

$$\Delta T_S^{\rm out}=-T_0\frac{p_m v_0}{\lambda_0}\frac{2\sigma_L}{\bar{\sigma}}\xi_{2a}\Gamma_{2S} . \quad (35)$$

Expressions (34) and (35) show that, at a given value of β_a, temperature $\Delta T_S^{\rm out}$ is higher when an interphase boundary appears. At a given value of β the temperature $\Delta T_S^{\rm out}$ is at a maximum for $\beta_a\to\beta$. The maximum temperature when formation of solid nuclei inside an external liquid phase is still possible can be determined from the following formula:

$$\Delta T^*=-2T_0\frac{p_m v_0}{\lambda_0}\frac{\sigma_L-\sigma_a}{\sigma_a+2\sigma_L}\ln\left[\frac{L^2}{\rho_0^2}e\right] . \quad (36)$$

Due to the logarithmic coefficient this temperature can be quite high $\sim T_0$. Thus using data from the experiments with exploding wires[6] for W, Al, and Cu, the values of a parameter $\tau=(p_m v_0/\lambda_0)$ for $p_m=2.85$ Kbar are $\tau_W=0.08$, $\tau_{\rm Al}=0.27$, and $\tau_{\rm Cu}=0.15$, respectively. For a conductor similar to that used in Ref. 6 with a cross section area $S=0.14$ mm^2 and length $L=4$ cm, the logarithmic factor $\ln[(L^2/\rho_0^2)e]\approx11.5$ is quite large, and compensates for the relatively small values of the parameter τ.

V. CONCLUSIONS

Summarizing our obtained results, in the following we present a possible picture of the dynamics of nucleation in current-carrying conductors. Assuming that melting from the surface can begin without formation of nuclei of a liquid phase, we can consider that melting begins at temperature $\Delta T_{1L}\approx-T_0(p_m v_0/\lambda_0)(1-\gamma)$. Up to this moment the electric current attains its maximum value and the characteristic temperature ΔT_{1L} which can be attained in the experiments with exploding wires similar to[6] $\Delta T_{1L}\sim-0.1T_0$. The liquid phase is separated from the solid phase by an interphase boundary which relaxes to its equilibrium value $\beta^*(T)$. However, in this process solid nuclei may form inside the liquid phase. The characteristic size of the critical nucleus a_{2S} is determined by expression (34), where one may consider $M(\chi)=0$ $(\Delta T\approx0)$. For the parameters of the experiments,[6] the size of the critical nucleus $a_{2S}\sim10^{-7}$ cm and the nucleation rate at this stage can be quite high.[5] During nucleation the released latent heat of the phase transition compensates for the cooling of the interphase boundary during melting and thus promotes melting. At the same time it smooths the temperature gradients in the vicinity of the interphase boundary, which delays formation of shock waves.

As melting continues and the temperature approaches ΔT_{2S}, formation of long liquid threads at the axis of the conductor theoretically becomes possible, and phase transition inside the conductor also begins. If threads with a

length much longer than the radius of a conductor are formed, they grow toward the surface of a conductor. However, these liquid domains are unstable due to formation of internal solid nuclei inside them. This is the situation in which none of the phases is stable, that was analyzed in the previous investigation.[1] In the case in which there is no mechanism which can support such coherent nucleation of long liquid threads, the temperature for the formation of liquid nuclei is given by formula (33), and ΔT_L^{ins} can be higher or lower than ΔT_{2L}. When $\Delta T_L^{\text{ins}} > \Delta T_{2L}$, the solid phase melts without the formation of internal liquid nuclei. In the opposite case, the solid phase will be dispersed into the liquid nuclei.

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APPENDIX

Estimate the second integral in formula (6):

$$S=a^3\int \eta(|\mathbf{r}-\mathbf{r}_1|-a)\frac{3\mu^2-1}{|\mathbf{r}-\mathbf{r}_i|^3}W_1(\mathbf{r})d\mathbf{r}\ . \tag{A1}$$

Since $\eta(|\mathbf{r}-\mathbf{r}_i|-a)=1-\eta(a-|\mathbf{r}-\mathbf{r}_i|)$, and taking into account that due to the factor $3\mu^2-1$ the integral over the spherical nucleus with the accuracy of terms of order a/ρ_0 vanishes, integration in (A1) can be extended over the whole volume of a conductor. We rewrite (A1) as follows:

$$S=\frac{I^2a^3}{c^2\pi\rho_0^2}\frac{\rho_0^2}{L^2}\int_{-u_i}^{1-u_i}du\int_0^1 dt\int_0^{2\pi}d\phi\,Q(\tilde{u},t)\ ,$$
$$Q(u,t)=\left\{\frac{3u^2}{u^2+A_i^2}-1\right\}\frac{1}{(u^2+A_i^2)^{3/2}} \times\left[\ln\left[\frac{4L^2}{\rho_0^2}\right]+\ln(u+u_i)(1-u-u_i)+1-t^2\right], \tag{A2}$$

where

$$u_i=\frac{z_i}{L},u=\frac{z}{L}-u_i\ ,$$
$$A_i^2=\frac{\rho_0^2}{L^2}[t^2-2tt_i\cos(\phi)+t_i^2],t_i=\frac{\rho_i}{\rho_0}\ .$$

Use the following indefinite integral:

$$\int du\left[\frac{3u^2}{u^2+A_i^2}-1\right]\frac{1}{(u^2+A_i^2)^{3/2}} =\frac{\mathrm{sgn}(u)}{A_i^2}\left[\left[1+\frac{A_i^2}{u^2}\right]^{-3/2}-\left[1+\frac{A_i^2}{u^2}\right]^{-1/2}\right]\ . \tag{A3}$$

Inspection of the expansion the right-hand side of (A3) into a power series of a parameter A_i/u_i shows that the zero-order terms vanish. The logarithmic singularities at the ends of the integration interval associated with the term $\ln(u+u_i)(1-u-u_i)$ are only virtual, since at the end of the integration interval one has to use expression (8) instead of (7), and in any case their contribution cannot compensate for the order of magnitude ρ_0^2/L^2. Therefore the ratio of integral (A1) to the first integral in (5) is of the order of ρ_0^2/L^2.

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