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Electron-electron interactions and the Hall insulator

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Using the Kubo formula, we show explicitly that a noninteracting electron system cannot behave like a Hall insulator, i.e., a dc resistivity matrix $\rho_{xx}\rightarrow\infty$ and finite ρ_{xy} in the zero-temperature limit, as has been observed recently in experiment. For a strongly interacting electron system in a magnetic field, we illustrate, by constructing a specific form of correlations between mobile and localized electrons, that the Hall resistivity can approximately equal its classical value. A Hall insulator is realized in this model when the density of mobile electrons becomes vanishingly small. It is shown that in noninteracting electron systems, the zero-temperature frequency-dependent conductance generally does not give the dc conductance.

A two-dimensional electron system (2DES) in a perpendicular magnetic field displays a rich variety of behaviors at low temperatures as a result of a delicate interplay between localization and electron-electron interaction. The most interesting phenomenon for this 2DES is the quantum Hall effect (QHE), which has been a focus of tremendous amount of research effort for more than a decade. Recently, much attention has been directed to the behavior of the Hall resistivity for a 2DES in an insulating phase. Magnetotransport studies of the 2DES have shown,[1] among many other interesting properties, that the Hall resistivity of a 2DES in a perpendicular magnetic field equals approximately the disorder-free value $\rho_{xy}\sim B/nec$ for a wide range of applied magnetic field strength, except near the QHE plateaus, where B is the applied magnetic field, n is the electron density of the 2DES, e and c are the electron charge and speed of light, respectively. The fact that a large portion of the electrons may become localized by disorder scattering does not affect the value of ρ_{xy} substantially. This is in sharp contrast to the behavior of the diagonal resistivity, which is found to change from $\rho_{xx}\rightarrow 0$ in the QHE phase to $\rho_{xx}\rightarrow\infty$ in an insulating phase. This implies, in particular, the existence of an insulating phase with $\rho_{xx}\rightarrow\infty$ and finite ρ_{xy}, the so-called Hall-insulating phase, which has generated much experimental[1–3] and theoretical[4,5] work recently. The goal of this paper is to show that the Hall-insulating behavior is *necessarily* an interaction effect. It cannot be explained with any independent-particle model.

It is well known that there exists a different kind of insulator characterized by $\rho_{xx}\rightarrow\infty$ and $\rho_{xy}\rightarrow\infty$ in a magnetic field, which will be called conventional insulators here for convenience. Examples of the conventional insulators are band insulators in semiconductors and the bulk Anderson insulators. The difference between a conventional insulator and a Hall insulator is determined by the different behaviors in the Hall resistivities in a magnetic field: while ρ_{xy} in a conventional insulator is a measure of density of *mobile* carriers n_c, i.e., $\rho_{xy}\sim B/n_c ec$, in a Hall insulator it is not, but rather $\rho_{xy}\sim B/nec$. A successful description of the Hall-insulating behavior should explain the origin of the difference in ρ_{xy}. So far, much of the theoretical work[4] on the Hall insulator has been based on calculating the zero-temperature frequency-dependent conductivity tensor, where a finite value of ρ_{xy} is achieved by finding $\sigma_{xx}\sim i\omega$ and $\sigma_{xy}\sim\omega^2$ in the low frequency $\omega\rightarrow 0$ limit. We shall see that this is not a complete description. If one naively takes $\sigma(\omega\rightarrow 0)$ to be the dc conductivity, one would conclude that all insulators are Hall insulators. In fact, we will see that one cannot, in general, take the $T\rightarrow 0$ limit before taking $\omega\rightarrow 0$.

In an insulating phase, the lowest energy extended state is above the Fermi level by a finite energy difference. Low temperature electrical current is carried by electrons thermally activated to the extended states. If the activated carriers are effectively decoupled from the localized electrons in the background, a transport experiment becomes a measurement of the mobile carriers only. As we will show below, the system is then a conventional insulator. Therefore, in order to have a Hall resistivity ρ_{xy} which is not a measure of the mobile carrier density n_c, but a measure of the total electron density n as $\rho_{xy}\sim B/nec$, there must exist a strong correlation between the activated mobile electrons and the localized electrons in the background. This simple argument directly implies that a noninteracting electron system cannot become a Hall insulator. We will exploit this idea and show that the Hall-insulating behavior can be realized for electrical conduction by certain kinds of correlated excitations.

There is another unsettled question related to the Hall insulator. It is about its ground state: is it essentially a pinned Wigner crystal or a new type of insulator? The assumption of a Wigner crystal is found to be qualitatively consistent with some results from the transport study,[6] radio-frequency measurement,[7] and photoluminescence experiment.[8] However, a ground state of disorder-localized electrons rather than a pinned Wigner crystal has been suggested by a recently proposed globe phase diagram[9] and the observation of a similar Hall-insulating phase[3] at the Landau level filling factor $\nu>2$. Our work reported here does not settle the question of which ground state is correct in a given situation, but rather shows that the Hall resistivity does not necessarily distinguish between these possibilities. We will see, instead, that it is the properties of the excited states that determine the Hall resistivity.

In the following, we will first discuss the noninteracting 2DES and show explicitly that it cannot display Hall-insulating behavior. We will then discuss the interacting

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2DES with correlated excitations and explain how the Hall-insulating behavior is realized.

We start with the Kubo formula for conductivity σ_{ij},

$$\sigma_{ij}(T,\omega)=\frac{i}{\omega}\frac{ne^2}{m}\delta_{ij}+\frac{i}{\omega}\Pi_{ij}(T,\omega). \tag{1}$$

For noninteracting electrons with an arbitrary disorder, the correlation function $\Pi_{ij}(T,\omega)$ can be evaluated using the complete set of eigenfunctions of the Hamiltonian. It is straightforward to obtain the result

$$\Pi_{ij}(T,\omega)=\frac{1}{A}\sum_{nm} J^i_{nm}J^j_{mn}\frac{f(\epsilon_n)-f(\epsilon_m)}{\epsilon_n-\epsilon_m+\omega+i0}, \tag{2}$$

where n and m label the eigenstates of the Hamiltonian ($H=\Sigma_i h_i$ and $h_n|n\rangle=\epsilon_n|n\rangle$), J_{nm} is the matrix element of current operator, and $f(\epsilon)$ is the Fermi distribution function. From the above expression, one can prove the following properties of the conductivity matrix: (1) $\mathrm{Im}[\sigma_{ij}(T,\omega)]$ is an odd function of ω,

$$\mathrm{Im}[\sigma_{ij}(T,\omega)]=O(\omega)+O(\omega^3)+O(\omega^5)+\cdots. \tag{3}$$

(2) $\mathrm{Re}[\sigma_{ij}(T,\omega)]$ is an even function of ω,

$$\mathrm{Re}[\sigma_{ij}(T,\omega)]=\mathrm{Re}[\sigma_{ij}(T,0)]+O(\omega^2)+O(\omega^4)+\cdots. \tag{4}$$

In the following, we show that if the system is insulating, it must be a conventional insulator. We will reach this conclusion by calculating the finite-temperature dc conductivity and then taking the low temperature limit.

From Eq. (3) and Eq. (4), we can see that $\mathrm{Im}[\sigma(T)]=0$ for $\omega=0$. We only need to consider the real parts, which are easily obtained from Eq. (2). The diagonal conductance is

$$\sigma_{xx}(T)\sim\sum_{nm}|J^x_{nm}|^2\frac{\partial f(\epsilon_n)}{\partial\epsilon_n}\delta(\epsilon_n-\epsilon_m). \tag{5}$$

For any localized state,[10] we have

$$J_{nm}=\langle n|J|m\rangle\sim\langle n|Hr-rH|m\rangle=r_{nm}(\epsilon_n-\epsilon_m). \tag{6}$$

Inserting the above expression into Eq. (5), we see that only extended states can contribute to σ_{xx}, because the δ function requires $\epsilon_n=\epsilon_m$. Now for an insulator, suppose that the lowest extended state has an energy Δ away from the Fermi level, Eq. (5) gives

$$\sigma_{xx}(T)\sim e^{-\beta\Delta}\quad\text{for } T\to 0. \tag{7}$$

The off-diagonal conductance has the form

$$\sigma_{xy}(T)\sim\sum_n f(\epsilon_n)\sum_m\frac{J^x_{nm}J^y_{mn}-J^y_{nm}J^x_{mn}}{(\epsilon_n-\epsilon_m)^2}. \tag{8}$$

Any localized state cannot contribute to σ_{xy}. For example, if $\langle n|$ is localized, then we are allowed to use Eq. (6), and the contribution from the state $\langle n|$ is

$$\sum_m\frac{J^x_{nm}J^y_{mn}-J^y_{nm}J^x_{mn}}{(\epsilon_n-\epsilon_m)^2}=\sum_m[x_{nm}y_{mn}-y_{nm}x_{mn}]$$
$$=\langle n|xy-yx|n\rangle=0. \tag{9}$$

The summation Σ_n in Eq. (8) may then be restricted to extended states only. If the lowest energy extended state is Δ above the Fermi energy, Eq. (8) gives

$$\sigma_{xy}(T)\sim e^{-\beta\Delta}\quad\text{for } T\to 0. \tag{10}$$

Putting together Eq. (7) and Eq. (10), one finds

$$\rho_{xy}(T)=\begin{cases}\dfrac{\sigma_{xy}(T)}{\sigma^2_{xx}(T)+\sigma^2_{xy}(T)}\sim\dfrac{e^{-\beta\Delta}}{e^{-2\beta\Delta}}\\ \infty\quad\text{for } T\to 0.\end{cases} \tag{11}$$

Similarly, one has $\rho_{xx}\to\infty$.

The above result shows that the noninteracting electron insulator is a conventional insulator. We have not made any approximation in our derivation, except the restriction that we consider only noninteracting electrons with a gap between the Fermi level and the lowest extended state. We therefore conclude that noninteracting electrons cannot behave like Hall insulators.

Several authors have described an ac form of the Hall insulator, by calculating the zero-temperature ac conductivity and then considering the low frequency limit; i.e., they took the limit $T\to 0$ first and the limit $\omega\to 0$ second. This method will give a finite value for ρ_{xy} when $\rho_{xx}\to\infty$. It is easy to see that this does not describe the dc conductivity and could mislead one to conclude that every insulator is a Hall insulator.

For an insulator at zero temperature, $\mathrm{Re}[\sigma_{ij}(T=0,\omega=0)]=0$. From Eq. (3) and Eq. (4), we have

$$\sigma^2_{xx}(0,\omega)+\sigma^2_{xy}(0,\omega)=O(\omega^2)+\text{higher powers in }\omega,$$

$$\mathrm{Re}[\sigma_{xy}(T,\omega)]=O(\omega^2)+\text{higher powers in }\omega.$$

Then we have

$$\varrho_{xy}=\frac{\mathrm{Re}[\sigma_{xy}]}{\sigma^2_{xx}+\sigma^2_{xy}}\bigg|_{\omega\to 0}\sim\frac{\omega^2}{\omega^2}=\text{const.} \tag{12}$$

If one associates ϱ_{xy} with the dc Hall resistivity of the system, then every insulator would be a Hall insulator. However, this is not consistent with experiment, suggesting that interchanging the order of $T\to 0$ and $\omega\to 0$ is in general not valid.

We will now argue that if correlations, in the Laughlin-Jastrow sense,[11] are important between conduction electrons and the localized electrons, then the Hall-insulating behavior may be obtained. We consider low temperature activated conduction of a 2D electron gas in an insulating phase. Let $N=N_c+N_L$ and $n=N/A$, $n_c=N_c/A$, $n_L=N_L/A$, where A is the system size and N, N_c, and N_L are, respectively, the number of electrons in the system, the number of electrons in extended states, and the number of electrons in localized states, with n, n_c, n_L the corresponding densities. N_c is activated, so $N_c\propto e^{-\Delta/kT}$ as $T\to 0$. We consider only the mo-

tion of conduction electrons and treat the remaining localized electrons as a scattering source. Although the electrons below E_F are well localized, they are still dynamic and responsive to the motion of the conduction electrons. This dynamic correlation is a very complicated problem. We would like to find an effective Hamiltonian to represent the coupling between the conduction electrons and the localized electrons, which is much simpler and yet it still gives the same effect on the motion of the conducting electrons. This may be achieved using a Chern-Simons approach.[12]

The Hamiltonian for the conduction electrons is

$$H_c=\sum_{i=1}^{N_c}\left[-\frac{i\hbar}{2m}\nabla_i-\frac{e}{c}\mathbf{A}^{\rm ext}(\mathbf{r}_i)\right]^2+\frac{1}{2}\sum_{ij=1}^{N_c}V(\mathbf{r}_i-\mathbf{r}_j)+\sum_{i=1}^{N_c}U(\mathbf{r}_i)+H_{cL}\,, \tag{13}$$

where $\mathbf{A}^{\rm ext}$ is the vector potential for applied magnetic field $B^{\rm ext}$, $V(\mathbf{r})$ is the electron-electron interaction between a pair of the conducting electrons, $U(\mathbf{r})$ is the disorder scattering potential. H_{cL} is the interaction between the conduction electrons and the remaining localized electrons, which is important in strongly correlated systems and presumably is the term responsible for Hall-insulator behavior. Our next step is to determine a mean-field form of H_c which characterizes correctly the influence of the localized electrons on the motion of the conduction electrons. It is enlightening to recall an earlier study[13] on correlated interstitials in a weakly disordered Wigner crystal, where one finds that energetically favored excitations are described by the wave function

$$\Psi_{\rm corrl}(z_0)=\Psi_{\rm uncorr}\prod_{i=1}^{N_L}(z_i-z_0)^{m_i}, \tag{14}$$

where $z_i=x_i-iy_i$ are the lattice electrons in complex notation, z_0 is the interstitial coordinate, and ν is the Landau level filling fraction with $B^{\rm ext}=n\phi_0/\nu$. The values of m_i may be chosen to minimize the energy of the excitation, and we argued in Ref. 13 that this may be accomplished if $\langle m_i\rangle=1/\nu$, where $\langle\ \rangle$ denotes an average over lattice sites. $\Psi_{\rm uncorr}$ describes an uncorrelated interstitial, which in a Hartree-Fock approximation would simply be given by a Gaussian orbital at some favorable interstitial site in the lattice. The addition of the Jastrow factor introduces correlations, and it may be shown[13] that its introduction converts the excitation into a delocalized state. Antisymmetrization corrections between the interstitial and the lattice electrons have been shown to be small.[13]

One can clearly see that the physics of such a wave function is more general than the Wigner crystal context in which it was derived. In particular, one can choose $\Psi_{\rm uncorr}$ to be any insulating state of an excited electron, and introducing the Jastrow factor creates an excited electron in an extended state, provided $\langle m_i\rangle=1/\nu$.

The Jastrow factor in Eq. (14) may be thought of as attaching a flux tube of strength $m_i\phi_0$ to the ith localized electron.[11] If we consider the long-wavelength response of the conduction electron, then an appropriate mean-field Hamiltonian will account for interaction with the localized electron by an additional field[12,14] $B^{cL}=n_L\phi_0/\nu$, we then have

$$H_c^{\rm MF}=\sum_{i=1}^{N_c}\left[-\frac{i\hbar}{2m}\nabla_i-\frac{e}{c}\mathbf{A}^{\rm net}(\mathbf{r}_i)\right]^2+\frac{1}{2}\sum_{ij=1}^{N_c}V(\mathbf{r}_i-\mathbf{r}_j)+\sum_{i=1}^{N_c}U(\mathbf{r}_i), \tag{15}$$

where $\mathbf{A}^{\rm net}$ is the vector potential for the net magnetic field $\nabla\times\mathbf{A}^{\rm net}=B^{\rm net}=B^{\rm ext}-B^{cL}=n_c\phi_0/\nu$. dc transport properties of the conduction electrons described by $H_c^{\rm MF}$ of Eq. (15) are easily obtained from the Kubo formula in the memory-function formalism[15]

$$\rho_{ij}=i\Omega_{ij}+\frac{m}{n_ce^2}\Gamma_{ij}\,, \tag{16}$$

where $i\Omega_{xx}=i\Omega_{yy}=0$ and $i\Omega_{xy}=-i\Omega_{yx}=B^{\rm net}/(n_cec)=B^{\rm ext}/(nec)$. Since the electrons are in extended states, the disorder scattering can be treated perturbatively. To the lowest order in the disorder potential, $\Gamma_{xy}=\Gamma_{yx}=0$. Denoting $\Gamma_{xx}=\Gamma_{yy}=1/\tau$, we have

$$\rho_{xx}=\frac{m}{n_ce^2\tau}\,,\quad \rho_{xy}=\frac{B^{\rm ext}}{nec}\,. \tag{17}$$

The above result shows clearly that ρ_{xy} depends only on the external magnetic field and the number of total electrons in 2D system, independent of how many of the electrons are localized.

The idea can be recast in the Drude picture. Suppose there is a current $j_x=n_cev$ with $\rho_{xx}=1/(n_ce\mu)$. A Hall voltage is generated to balance the Lorentz force $\rho_{xy}=E_y/j_x=(vB/c)/(n_cev)=B/(n_cec)$. For uncorrelated conduction electrons, $B=B^{\rm ext}$ and ρ_{xy} depends on n_c, not the total electron number n. One would get $\rho_{xx}\to\infty$ and $\rho_{xy}\to\infty$ when $n_c\to 0$. For strongly correlated systems, we have shown that $B=B^{\rm net}=(n_c/n)B^{\rm ext}$ so that $\rho_{xy}=B/(nec)$, regardless of the number of electrons which are localized.

We have demonstrated this model of correlation between the activated conduction electrons and the remaining localized electrons does make the system a Hall insulator. The key is that we characterize the interaction between the activated conduction electrons and remaining localized electrons as flux-tube-like, and describe them using a Chern-Simons statistical field. However, we would like to emphasize that the introduction of a Jastrow factor in the trial wave function in the early work[13] on correlated interstitials yields extremely low energies with a microscopically realistic Hamiltonian. We note also that previous work[9] has shown that a nondivergent ρ_{xy} may be obtained using the Chern-Simons approach. However, in that case the correlation was introduced in the ground state rather than in the excited states, and while $\rho_{xy}<\infty$, it was not necessarily equal to $B^{\rm ext}/(nec)$. By introducing the correlation in the excited states, it is possible to have localized electrons for which the flux tube strength is site dependent, leading to the classical Hall resistivity.

To summarize, we have shown rigorously with the Kubo formula that a noninteracting electron system cannot display the Hall-insulating behavior and treatments based on zero-temperature frequency-dependent conductivity are insufficient to explain this behavior. Instead, the Hall-insulating behavior should be considered as an interaction effect. We have constructed an explicit form for the strong correlations between the temperature-activated mobile electrons and the localized electrons in the background and demonstrated that this kind of correlation does lead to the Hall-insulating behavior.

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