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Intrinsic inductive characteristics of resonant tunneling

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We have studied the current $I(\omega,t)=I_0(\omega)+I_1(\omega,t)$ through a double-barrier resonant tunneling system (DBRTS) under a dc-ac bias. With a 2D emitter, the *ac shift* of the resonant level can be determined from $I_0(\omega)$, and the intrinsic inductance of a DBRTS is unambiguously defined from $I_1(\omega,t)$. For presently available DBRT diodes under maximum operation frequency, the *photon replica* cannot be observed in the I-V curve. With a 3D emitter, a quantum-equivalent-circuit model can be constructed to calculate the maximum operation frequency of the diode, with good agreement with the measured value.

Quantum transport in nanostructures, especially in a double-barrier resonant tunneling system (DBRTS), has attracted the attention of current research. There has been much work on the I-V characteristics of DBRTS under a dc bias, focusing on the resonant tunneling (RT) through the quasibound state in the well. Since this quasibound state has a finite lifetime $1/\Gamma$, there is a time delay of the current following the switch-on of the bias, resulting in an *intrinsic inductive characteristics of each RT process*. Under a dc bias, such a phenomenon manifests itself in the transient behavior.[1,2] However, when the system is tuned to the RT condition by a dc bias, the intrinsic inductive property of the RT process will respond to an additional ac bias. Such a feature, hardly understood at the present, is of fundamental theoretical interest, as well as of crucial importance to the performance of RT devices at high frequency above 100 GHz.[3–6] When a quantum inductance L is included in the equivalent circuit for a RT diode, calculated results agree better with experimental data.[7,8]

There are some theoretical calculations of RT current under dc-ac bias.[9–12] Assuming a small ac signal, Liu[13] has used the low-order perturbative method to calculate the tunneling current. The linear response of a DBRTS to a dc-ac bias[14] and to a pure ac bias[15] has also been studied recently. The work on pure ac bias has generated some controversial discussions in the literature.[16,17]

The central theme of the problem is to clarify the intrinsic inductive property of a DBRTS, the physical origin of which is the finite lifetime $1/\Gamma$ of the quasibound state. This finite lifetime causes a delay of the tunneling current with respect to the applied bias, and the amount of delay depends also on the electron transmission probability which is a function of the kinetic energy of the tunneling electron. Therefore, the inductive characteristic is intrinsic to each individual tunneling process, specified by the electron kinetic energy, which contributes a *partial tunneling current*. On the other hand, all tunneling electrons contribute to the same measured global current $I(\omega,t)=\mathrm{Re}\ I(\omega,t)+i\ \mathrm{Im}\ I(\omega,t)$. To the best of our knowledge all existing works on the effect of ac bias on resonant tunneling in a DBRTS have focused their attention on the global current $I(\omega,t)$. Consequently, the question arises under what condition the global quantities Re $I(\omega,t)$ and Im $I(\omega,t)$ can accurately describe the intrinsic inductive properties of a RT process.

In this paper, with an independent-particle model, we will not only answer this question, but also solve another important problem described as follows. In the single-electron picture, the DBRTS is illustrated by part (a) of Fig. 1 with the corresponding electronic Hamiltonian

$$H_e=\sum_{p,\nu,\mathbf{k}_\parallel}[\epsilon_{p,\nu}(t)+\epsilon_{\mathbf{k}_\parallel}]a^\dagger_{p,\nu,\mathbf{k}_\parallel}a_{p,\nu,\mathbf{k}_\parallel}+\sum_{\mathbf{k}_\parallel}[\epsilon_c(t)+\epsilon_{\mathbf{k}_\parallel}]a^\dagger_{c,\mathbf{k}_\parallel}a_{c,\mathbf{k}_\parallel}+\sum_{p,\nu,\mathbf{k}_\parallel}[V_{p,\nu}(t)a^\dagger_{c,\mathbf{k}_\parallel}a_{p,\nu,\mathbf{k}_\parallel}+\mathrm{H.c.}]\ , \tag{1}$$

where $\mathbf{k}_\parallel$ and p are, respectively, the parallel and perpendicular component of momentum. $\nu=l$ (or r) refers to the emitter (or collector). The effect of ac bias $V(\omega t)=V_1\sin\omega t$ on the energy levels in H_e depends on the geometry of the DBRTS, which is assumed symmetric for convenience of

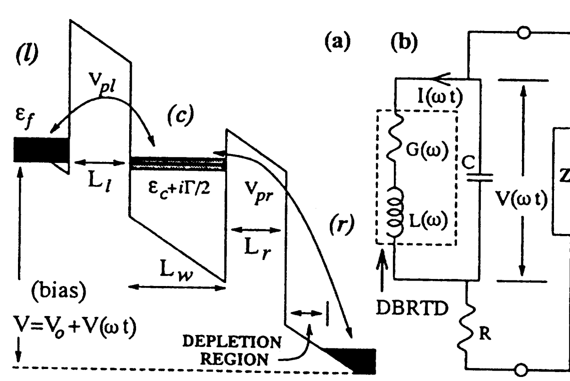

FIG. 1. (a) is the schematic illustration of a double-barrier resonant-tunneling diode under a combined dc-ac bias $V_0+V(\omega t)$. It is represented by the dash-lined box DBRTD in the equivalent circuit (b).

analysis. Because of the uncertainty on the width of the depletion region, the time dependences of the relevant energy levels can only be set as $\epsilon_{p,l}(t)=\epsilon_{p,l}$, $\epsilon_c(t)=\epsilon_c-\alpha eV(\omega t)$, and $\epsilon_{p,r}(t)=\epsilon_{p,r}-eV(\omega t)$. The value of α is crucial to the total power output of a RT diode, but so far cannot be determined theoretically. In this paper we will show that for certain realistic DBRTS, α can be accurately calculated from the characteristic oscillation of the mean tunneling current.

We need the transmission probability $T(t,\epsilon_\perp,\omega,\mathbf{k}_\parallel)$ of an electron from the emitter at initial time $t=0$ with perpendicular energy $\epsilon_\perp$ and parallel momentum $\mathbf{k}_\parallel$, to the collector at a later time $t>0$. To do this we must know the two-particle Green's function

$$\langle a_{c,\mathbf{k}_\parallel}(t_3)a^\dagger_{c,\mathbf{k}_\parallel}(t_2)a_{c,\mathbf{k}_\parallel}(t_2)a^\dagger_{c,\mathbf{k}_\parallel}(t_1)\rangle .$$

Under the practical condition $V_1 \ll V_0$, the *wide band approximation*[18] $\sum_p |V_{p,\nu}|^2\delta(\epsilon_\perp-\epsilon_{p,\nu})\simeq\Gamma_\nu$ for $\nu=l$ and r is valid. Within this approximation, Johansson[19] has calculated this two-particle Green's function exactly. Hence we will neglect all mathematical details and write down the final results. We will assume perfect interfaces and so the transmission probability $T(t,\epsilon_\perp,\omega)\equiv T(t,\epsilon_\perp,\omega,\mathbf{k}_\parallel)$ does not depend on $\mathbf{k}_\parallel$. Let us set $\hbar=1$ and define

$$F_{n,s,m,\pm}(\epsilon_\perp,\omega)=\pm s\; i^{2n+sn+1}\frac{J_m\left(\dfrac{\alpha eV_1}{\omega}\right)J_{m\pm n}\left(\dfrac{\alpha eV_1}{\omega}\right)}{\epsilon_\perp-\epsilon_c+sm\omega\pm si\Gamma/2} ,$$

where $s=\pm 1$, $\Gamma=\Gamma_l+\Gamma_r$, and $J_m(x)$ is the Bessel function of order m. Then, following the derivation of Johansson,[19] we readily obtain

$$T(t,\epsilon_\perp,\omega)=\mathscr{T}(\epsilon_\perp,\omega)+\sum_{n=1}^{\infty}\mathrm{Re}\{2\mathscr{F}_n(\epsilon_\perp,\omega)e^{in\omega t}\} , \quad (2)$$

where

$$\mathscr{T}(\epsilon_\perp,\omega)=\sum_{m=-\infty}^{\infty}\frac{\Gamma_l\Gamma_r|J_m(\alpha eV_1/\omega)|^2}{(\epsilon_\perp-\epsilon_c-m\omega)^2+\left(\dfrac{\Gamma}{2}\right)^2} \quad (3)$$

is the time-independent mean transmission probability, and

$$\mathscr{F}_n(\epsilon_\perp,\omega)=\frac{2\Gamma_l\Gamma_r}{\Gamma+in\omega}\sum_s\left\{\sum_{m=\eta(s,n)}^{\infty}F_{n,s,m,+}(\epsilon_\perp,\omega)+\sum_{m=1-\eta(s,n)}^{\infty}F_{n,s,m,-}(\epsilon_\perp,\omega)\right\} , \quad (4)$$

with $\eta(s,n)=(1+s)(1-n)/2$. Most experiments and device applications are related to the power output, for which the relevant time-dependent part in (2) is the fundamental $n=1$. In this paper we will also consider only this case.

The total current can be calculated from

$$I(\omega,t)=\frac{e}{\pi}\int d\mathbf{k}_\parallel\int d\epsilon_\perp\;\rho(\epsilon_\perp)v_\perp(\epsilon_\perp)T(t,\epsilon_\perp,\omega)\times\{f_l(\epsilon_\perp,\mathbf{k}_\parallel)-f_r(\epsilon_\perp,\mathbf{k}_\parallel)\} , \quad (5)$$

where $v_\perp(\epsilon_\perp)$ is the perpendicular velocity of the tunneling electron, $\rho(\epsilon_\perp)$ the associated partial density of states, and $f_\nu(\epsilon_\perp,\mathbf{k}_\parallel)$ the Fermi distribution function. We are interested in the negative differential resistance (NDR) regime, in which the contributing tunneling processes in (5) have $f_r(\epsilon_\perp,\mathbf{k}_\parallel)\simeq 0$. If we define $g(\epsilon_\perp)\equiv\int d\mathbf{k}_\parallel f_l(\epsilon_\perp,\mathbf{k}_\parallel)$, then (5) reduces to

$$I(\omega,t)=\frac{e}{\pi}\int d\epsilon_\perp\;\rho(\epsilon_\perp)v_\perp(\epsilon_\perp)g(\epsilon_\perp)T(t,\epsilon_\perp,\omega) . \quad (6)$$

In reality there are two types of sample structure. If electrons tunnel directly from the 3D Fermi sea into the quantum well, a case shown in part (a) in Fig. 1, it is called a three-dimensional (3D) emitter. If electrons tunnel into the quantum well from the discrete levels in the triangle-well next to the emitter barrier, such structure is called a 2D emitter. The essential features of a 2D emitter sample have been discussed in our recent work on phonon-assisted resonant magnetotunneling.[20] The main difference between the 3D emitter and the 2D emitter lies in the partial density of states $\rho(\epsilon_\perp)$. Let us consider first the case of a 2D emitter, and for simplicity assume only one discrete level E in the triangle well. Then $\rho(\epsilon_\perp)=\delta(\epsilon_\perp-E)$, and (6) can be expressed as $I(\omega,t)=I_0(E,\omega)+I_1(E,\omega,t)$, with the mean current $I_0(E,\omega)=\theta(E)\mathscr{T}(E,\omega)$ and the fundamental oscillating current $I_1(E,\omega,t)=2\theta(E)\mathrm{Re}[\mathscr{F}_1(E,\omega)e^{i\omega t}]$, where $\theta(E)=v_\perp(E)g(E)$ is a material parameter which depends weakly on the bias through the discrete level E. Since the resonant tunneling current is sharply peaked in the energy range $|E-\epsilon_c|\simeq\Gamma$ around $E=\epsilon_c$, within the corresponding narrow bias region of resonant tunneling, the material parameter can be approximated as $\theta(E)\simeq\theta(\epsilon_c)$ (in the same spirit as the wide band approximation). Consequently, for a DBRTS with a 2D emitter, it is sufficient to study $\mathscr{T}(E,\omega)$ and $\mathrm{Re}[\mathscr{F}_1(E,\omega)e^{i\omega t}]$ in order to understand the measured tunneling current.

In realistic situation the frequency is higher than 100 GHz and eV_1 is no more than a few meV. Hence, both ω/Γ and eV_1/Γ are of the order one. Consequently, with a given $\alpha eV_1/\omega$ in the NDR region, from (3) we can show that the mean current $\mathscr{T}(E,\omega)$ oscillates with ω in such a way that in the very near vicinity of certain frequencies $\omega=\omega_j$ ($j=1,2,\ldots$) which satisfy $J_0(\alpha eV_1/\omega_j)=0$, the mean current $\mathscr{T}(E,\omega_j)$ is a local minimum. To check the quantitative accuracy of this theoretical prediction, we have calculated the normalized mean current $T_0(\epsilon_c,\omega)\equiv(\Gamma^2/\Gamma_l\Gamma_r)\mathscr{T}(\epsilon_c,\omega)$ as a function of ω/Γ, and in Fig. 2 we show two typical results for $\alpha eV_1/\Gamma=2$ and 3. The positions of ω_j are indicated by vertical bars for $j=1,2,3,\ldots$. The inset shows $\alpha eV_1/\Gamma$ as a function of ω_j/Γ for $j=1$, 2, and 3. Within the accuracy of plotting, they form three straight lines with slopes $\mathscr{S}_j\equiv\alpha eV_1/\omega_j$ satisfying $J_0(\mathscr{S}_j)\simeq 0$. Since $\mathscr{S}_j$ are well-defined values, the material parameter $\alpha=(\mathscr{S}_j\omega_j)/(eV_1)$ can be determined with rather high accuracy by measuring the mean tunneling current as a function of frequency for various ac bias amplitude V_1. The accuracy can be improved by taking an average over several so-obtained α's corresponding to different j's.

Since we choose $V(\omega t)=V_1\sin\omega t$, in terms of the admittance $Y(E,\omega)$, the oscillating current can be expressed as

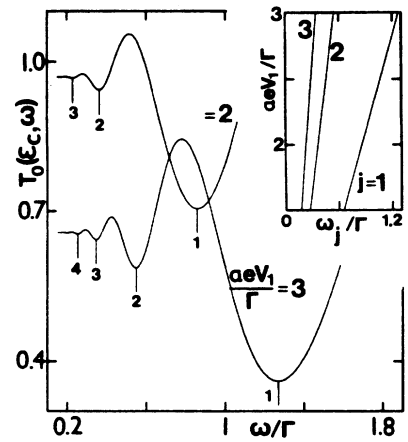

FIG. 2. The normalized mean current $T_0(\epsilon_c,\omega)$ as a function of ω/Γ for $\alpha e V_1/\Gamma=2$ and 3. The inset shows the linear relation of $\alpha e V_1/\Gamma$ and ω_j/Γ for $j=1$, 2, and 3.

$$I_1(E,\omega,t)=(\text{Re }Y)\ V_1\sin\omega t+(\text{Im }Y)\ V_1\cos\omega t\ . \quad (7)$$

The ac conductance $G(E,\omega)\equiv[\text{Re }Y^{-1}(E,\omega)]^{-1}$ and the intrinsic inductance $L(E,\omega)\equiv(1/\omega)\text{Im }Y^{-1}(E,\omega)$ can then be derived. It is convenient to introduce $\Delta\epsilon=E-\epsilon_c$ and define a normalized admittance

$$y(\Delta\epsilon,\omega)\equiv y_r(\Delta\epsilon,\omega)+iy_i(\Delta\epsilon,\omega)$$
$$\equiv[(\Gamma^2/4)/10\Gamma_l\Gamma_r]Y(E,\omega),$$

since then both the real part $y_r(\Delta\epsilon,\omega)$ and the imaginary part $y_i(\Delta\epsilon,\omega)$ are odd functions of $\Delta\epsilon$, and so we only need to consider $\Delta\epsilon\geq 0$. As shown in our earlier work,[21] all matrix elements in H_e can be computed, and hence the current and the admittance can be derived. With a realistic parameter $\alpha eV_1/\Gamma=0.4$, we have calculated $y(\Delta\epsilon,\omega)$ for $\omega/\Gamma=0.3$, 0.5, 0.6, 0.74, and 0.9. The results are shown in Fig. 3 as functions of $\Delta\epsilon/\Gamma\geq 0$, part (a) for $y_r(\Delta\epsilon,\omega)$ and part (b) for $y_i(\Delta\epsilon,\omega)$. We see that in all parameter ranges, $y_i(\Delta\epsilon,\omega)$ is positive and well behaved. Consequently, for a *2D emitter* the DBRTS has a well-defined intrinsic quantum inductance $L(E,\omega)$.

When an electron tunnels through a DBRTS, it can emit or absorb photons with microwave energy ω (we set $\hbar=1$) via all allowed processes. If $\omega\gg\Gamma$, then this phenomenon is similar to the *phonon-assisted resonant tunneling*,[22,18,23] where $\omega_{\text{phonon}}\gg\Gamma$ and so if the electron recoil is neglected[23] the phonon replica in the I-V curve of a pure dc biased RT is well separated from the main tunneling peak. However, even for the maximum operation frequency of a realistic DBRT diode under a dc-ac bias, ω can hardly be larger than Γ. Hence, the interesting RT processes are those with $\epsilon_c-\Gamma/2<E\pm\omega<\epsilon_c+\Gamma/2$, which will not produce an equivalent *photon-replica* separated from the main RT peak in the I-V curve. For the case $\Delta\epsilon\geq 0$ considered here, since $|(E-\omega)-\epsilon_c|<(E+\omega)-\epsilon_c$, the photon emission process contributes more to the RT current than the photon absorption process. The well-approximated condition for the absence of such contribution by a single-photon emission process is $E-\omega\leq\epsilon_c-\Gamma/2$, or $\omega-\Gamma/2\geq E-\epsilon_c=\Delta\epsilon\geq 0$.

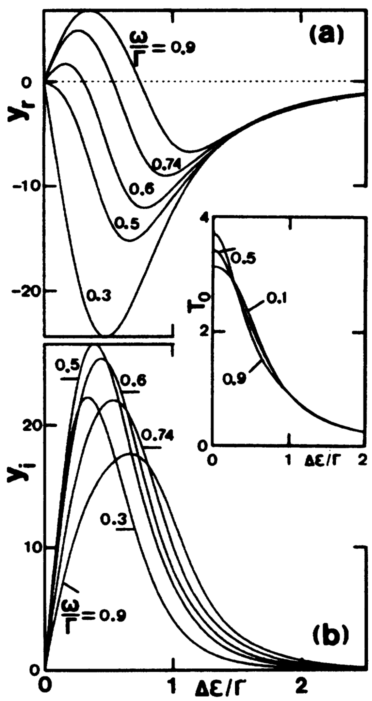

FIG. 3. Real (y_r) and imaginary (y_i) part of the normalized admittance as functions of $\Delta\epsilon/\Gamma\equiv(\epsilon_\perp-\epsilon_c)/\Gamma$ for various values of ω/Γ. The inset shows the broadening and the suppressed peak value of the mean current T_0 as ω/Γ decreases from 0.9 to 0.1.

Therefore, if we fix at a frequency $\omega/\Gamma>\frac{1}{2}$ and increase $\Delta\epsilon$, around $\Delta\epsilon/\Gamma\simeq\omega/\Gamma-\frac{1}{2}$, we expect a change of the conductance $G(E,\omega)$ [or $y_r(\Delta\epsilon,\omega)$] due to the opening of the single-photon emission channel. Since all photon processes are included in (2), this physical picture will be modified but will still be valid qualitatively.

When a photon is emitted coherently, the phase shift π in the electron wave function manifests in the phase $\varphi(E,\omega)$ of the admittance, which is simply $\varphi(E,\omega)=\tan^{-1}[y_i(\Delta\epsilon,\omega)/y_r(\Delta\epsilon,\omega)]$. Since $y_i(\Delta\epsilon,\omega)$ is always positive, the phase shift of π results in the change of sign of $y_r(\Delta\epsilon,\omega)$. While the details of such behavior may be modified by other photon processes, the conclusion remains to be correct as indicated by part (a) of Fig. 3. For a given value of $\omega/\Gamma>\frac{1}{2}$, at certain energy $\Delta\epsilon/\Gamma\geq\omega/\Gamma-\frac{1}{2}$, $y_i(\Delta\epsilon,\omega)$ changes from positive to negative.

Due to the emission and absorption of photons, obviously the mean current peak $T_0(E,\omega)$ will be broadened with the peak value suppressed. Since all multiple photon processes are included in (2), the amount of broadening and suppression is larger for smaller ω/Γ. This is clearly demonstrated by the inset in Fig. 3 for $\omega/\Gamma=0.1$, 0.5, and 0.9, which are calculated with the same parameter value $\alpha eV_1/\Gamma=0.4$.

All above analyses are for DBRTS with 2D emitter. For devices with 3D emitter, the current (6) has contribution from electrons with different energies $\epsilon_\perp$. Then, the result in Fig. 2 is no longer valid. While the transmission probability (2) and the current (6) can be computed numerically without difficulty, we will not show them here because they have already been calculated by Johansson[19] with the same approach as we used here. It is interesting to observe that even for the case of 3D emitter, the crossover of Re $Y(\omega)$ vs applied bias can still be detected, as indicated in Fig. 3 of Ref. 13.

For a DBRTS with 3D emitter, using the relation similar to (7), we find that Re $Y(\omega)$ remains finite for all ω, and so the conductance $G(\omega)\equiv\{\text{Re } Y^{-1}(\omega)\}^{-1}$ and the inductance $L(\omega)\equiv(1/\omega)\text{Im } Y^{-1}(\omega)$ can be defined. Knowing these quantities, an equivalent circuit is constructed as shown by part (b) of Fig. 1, in which the dash-lined box simulates the DBRT diode and the external circuit is represented by C, R, and Z_L. The upper limit of operating frequency $\omega_{\max}$ for the DBRT diode can be calculated from this equivalent circuit. Using the diode of Brown *et al.*[7] we set $\alpha=\frac{1}{2}$ and obtain $\omega_{\max}=187$ GHz for the temperature 300 K, in good agreement with the observed $\omega_{\max}\simeq 200$ GHz.

Since the conductance, the inductance, and Γ all depend on the barrier strength of a DBRTS, their product should be an intrinsic property of individual tunneling process specified by ω/Γ and $\Delta\epsilon/\Gamma$. This conjecture can be easily justified for the case of 2D emitter, because here $G(E,\omega)L(E,\omega)\Gamma=-(\Gamma/\omega)\tan\varphi(E,\omega)$ which depends strongly on ω/Γ and $\Delta\epsilon/\Gamma$. However, for a DBRT diode with a 3D emitter, the product $G(\omega)L(\omega)\Gamma$ has been discussed in the literature and was claimed to be 1 by Brown *et al.*,[7] and to be 2 by Fu and Dudley.[15] Our calculation shows a non-negligible frequency dependence of this product. Corresponding to the above calculated $\omega_{\max}=187$ GHz, we found $G(\omega_{\max})L(\omega_{\max})\Gamma\simeq 1.7$.

In conclusion, we should point out that with presently available DBRT diodes, the $\omega_{\max}$ is still too low to allow a *photon replica* in the *I-V* curve.

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