Theory of friction: The role of elasticity in boundary lubrication

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I analyze the sliding motion of an elastic block on a substrate with a layer of lubrication molecules. This model of boundary lubrication illustrates how the surface stress generated by the lubrication layer at the block-substrate interface is transmitted to the upper surface of the block. It shows that it is essential to correctly incorporate the elastic properties of the block and of the substrate; otherwise, incorrect results will result. In light of the theoretical results I discuss the sliding friction measurements of Yoshizawa and Israelachvili.

I. INTRODUCTION

The study of sliding friction is one of the oldest problems in physics and certainly one of the most important from a practical point of view.[1,2] Hence it has been estimated that about 5% of the gross national product in developed countries is "wasted" on friction and related wear. In spite of this, remarkably little is understood about the fundamental, microscopic processes responsible for friction and wear.

Usually, as a "particle" moves slowly (velocity v) in some medium it will experience a friction or drag force F proportional to the velocity v. For example, the drag force on a small spherical particle moving slowly in a fluid is given by the Stokes formula and is proportional to the velocity v. Similarly, on a heavy charged particle moving slowly in a metal acts an electronic friction which is proportional to v for $v \ll v_F$, where v_F is the Fermi velocity of the metal electrons. In the context of surface physics, the electronic and phononic friction forces acting on a heavy adsorbate moving slowly parallel to the surface are both proportional to v if $v \ll v_F$ and $v \ll c$, where c is the sound velocity of the solid.[3] The basis for this is that the friction force is due to the creation of excitations in the surrounding media and must vanish continuously as $v \rightarrow 0$. Therefore, in a Taylor expansion of the drag or friction force (assuming analyticity for $v=0$), $F(v)=av+bv^2+\cdots$, the leading contribution as $v \rightarrow 0$ will in most cases be proportional to v.

In the sliding of a macroscopic block on a substrate, the friction force is found to be velocity independent for low velocities v, in apparent conflict with the discussion above. But it was realized by Tomlinson[4] that this can be understood if, during sliding, rapid processes occur somewhere in the system even if the center of mass of the block moves arbitrarily slowly relative to the substrate. The fundamental problem in sliding friction is to understand the microscopic origin of these rapid processes, and to relate them to the macroscopic motion of the block.

In some cases the rapid processes are quite well understood. For example, during sliding of a metal block on a metal substrate without a lubricant fluid, the friction force is due mainly to the shearing of cold-welded contact "points" (junctions).[1] After a junction has been formed it is first elastically deformed, which occurs very slowly if v is low, followed by plastic deformations involving rapid motion of dislocations and other rapid, nonadiabatic, rearrangement processes. As the block slides on the substrate, junctions are continuously "broken" and "formed," at a rate proportional to the sliding speed; hence the friction force is velocity independent. As another example, consider two clean insulator surfaces (or two metal surfaces passivated by layers of chemisorbed molecules, e.g., fatty acid molecules), sliding slowly relative to each other. In this case the rapid processes may correspond to local slip events, as indicated in Fig. 1. That is, atomic groups on the two surfaces will "interlock," deform elastically, and finally undergo rapid slip processes where an atomic group moves or flips from one potential well to another nearby. The rapid local motion cannot occur adiabatically, but will generate sound waves which ultimately are "damped," giving rise to irregular heat motion. This type of atomistic slip has been observed directly with the atomic force microscope.[5]

It is very hard experimentally to probe directly the nature of rapid processes which occur at a sliding interface. However, some information can be inferred indirectly by performing sliding friction measurements on well-defined systems, and registering the macroscopic (e.g., center of mass) motion of the block as a function of time. For lu-

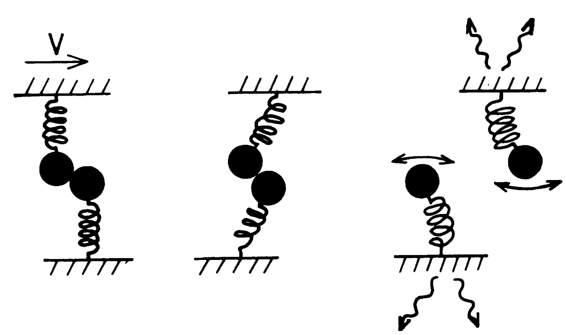

FIG. 1. As two surfaces slide relative to each other, atomic groups will interlock (a), deform elastically (b), and finally undergo a rapid slip process. The rapid local motion is damped by the emission of sound waves.

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bricated surfaces, such measurements have been performed during the last few years, in particular by Yoshizawa and Israelachvili,[6] Gee *et al.*,[7] and Israelachvili,[8] and by Granick[9] and Reiter *et al.*[10] These studies usually use mica surfaces which can be produced atomically smooth (e.g., without a single step) over macroscopic areas. Figure 2 shows a schematic experimental setup: a mica block is slid on a mica substrate with an intervening lubrication fluid. A spring is connected to the mica block and the "free" end of the spring is moved with some velocity v_s, which typically is kept constant but sometimes is allowed to change in time. The force in the spring is registered as a function of time and is the basic quantity measured in most of these friction studies. It is obvious that the time dependence of the spring force (and its dependence of v_s) contains information about the nature of processes occurring at the sliding interface, but this information is very indirect. For example, as will be shown below, it is crucial to take into account the elastic properties of the block itself, a fact which has been overlooked, or at least not properly accounted for, in several earlier studies of sliding friction.

The sliding friction probed in an experiment of the type illustrated in Fig. 2 can be considered as resulting from the process of eliminating (or "integrating out") physical processes which vary rapidly in space and time in order to obtain an effective equation of motion for the long-distance and long-time behaviors of the system. For example, in the case of boundary lubrication considered in this paper, the first step may be to eliminate those processes in the lubricant film which vary rapidly in space and time, to obtain the effective surface stress that the lubricant layer exerts on the lower surface of the block. The next step is to study how this surface stress is transmitted to the upper surface of the block, and finally how this affects the spring force studied in a typical sliding friction measurement.

In this paper I study a simple model which illustrates how the surface stress generated by the lubrication layer at the block-substrate interface is transmitted to the upper surface of the block. The study of this problem turns out to have important implications for the understanding of boundary lubrication, in particular during "starting" and "stopping;" it shows, e.g., that results and conclusions of some earlier computer simulations of sliding friction, where the elastic properties of the block and substrate were not properly taken into account, are qualitatively incorrect. In the light of the present model study, I analyze the sliding friction measurements of Yoshizawa and Israelachvili; their results for the dependence of the spring force on time and on v_s are in good general agreement with the theoretical predictions.

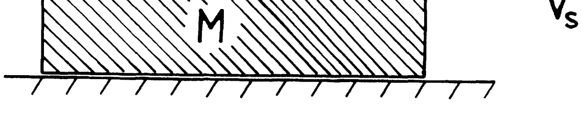

FIG. 2. Sliding of a block on a substrate. The free end of the spring moves with the velocity v_s.

In Sec. II I present the basic model and study its properties with emphasis on "starting" and "stopping." Section III contains a general discussion about the origin of stick-and-slip motion, and a comparison of theoretical results with the experimental data of Yoshizawa and Israelachvili. Section IV contains a summary and conclusions. In Appendix A it is shown how the motion of a rigid block can be obtained from the elastic block case, in the limit in which Young's modulus goes to infinity.

II. THEORY

A. The model and basic equations

We consider the model shown schematically in Fig. 3. A spring with the force constant k_s is connected to an elastic block in the shape of a rectangular parallelepiped located on a flat substrate. The block has thickness d, and the side contacting the substrate is a square with an area $A = D \times D$. We assume that a single monolayer of lubrication molecules separates the block from the substrate (boundary lubrication). The spring is connected to the block via a thin rigid and massless sheet (area $A = D \times D$) "glued" to the top surface of the block, see Fig. 3. Hence the force exerted by the spring on the block is assumed to act uniformly on the upper surface, i.e., if F_s is the force in the spring then the tangential stress acting on the upper surface of the elastic medium equals $\sigma_s = F_s / A$. The free end of the spring is assumed to move with the speed v_s, and the basic problem addressed below is to determine the variation of the spring force with time.

Let (x,y,z) be a coordinate system with the $z=0$ plane in the top surface of the block and with the positive z axis pointing toward the substrate; see Fig. 3. The free end of the spring moves with the velocity v_s along the positive x axis. We assume that the block is made from isotropic elastic media and that the $D \gg d$, i.e., that the width and depth of the block are much larger than its height. Under these conditions the displacement field $\mathbf{u}(\mathbf{x},t)$ in the block will to a good approximation depend only on z and t; close to the vertical sides of the block the displacement field will be more complicated, but this region of space can be neglected if $D \gg d$. The field $\mathbf{u}(\mathbf{x},t) = \hat{\mathbf{x}} u(z,t)$ satisfies the wave equation

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial z^2} = 0 \tag{1}$$

where c is the transverse sound velocity. The tangential stress, exerted by the spring on the $z=0$ surface of the block, can be related to $u(z,t)$ via

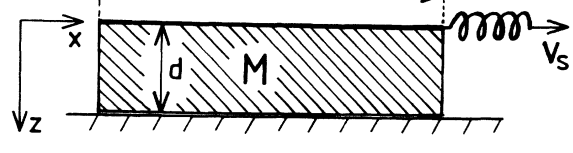

FIG. 3. An elastic block on a substrate. The free end of the spring moves with the velocity v_s.

$$\frac{1}{\kappa}\frac{\partial u}{\partial z}(0,t)=-\frac{F_s(t)}{A}=-\sigma_s(t)\ , \tag{2}$$

where $1/\kappa=\rho c^2$ (ρ is the mass density of the block). If the stress exerted by the lubrication molecules on the surface $z=d$ of the block is denoted by $-\sigma(t)$, then

$$\frac{1}{\kappa}\frac{\partial u}{\partial z}(d,t)=-\sigma(t)\ . \tag{3}$$

Note that $\sigma(t)$ is obtained from the microscopic stress $\sigma(\mathbf{x},t)$ by averaging over (or "integrating out") the rapid (in space and time) fluctuating part of the motion of the lubrication molecules. It is implicitly assumed that no spatial fluctuations of $\sigma(\mathbf{x},t)$ occur on the length scale d or longer; if such fluctuations occurred they would need to be taken directly into account, and the problem we study below would be much more complicated. If $-\sigma(t)$ is the stress exerted by the lubrication molecules on the bottom surface of the block, then, according to Newton's law of action and reaction, the block must exert stress $\sigma(t)$ on the layer of lubrication molecules. This will in general lead to some drift motion of the lubrication layer. If one assumes that the time dependence of $\sigma(t)$ is slow compared with typical relaxation times associated with the motion of the lubricant molecules, then one can treat σ as a constant when determining the relation $\sigma=f(v)$ between σ and the speed v of the adsorbate layer.[11]

I will now briefly discuss the nature of the relation $\sigma=f(v)$ between σ and v, based on the numerical simulations presented in Refs. 12, 3, and 13. These simulations considered a system of point particles, interacting via Lennard-Jones pair potentials and moving on a corrugated substrate. Owing to the adsorbate-substrate coupling, each adsorbate experience a friction force $-m_a\eta\dot{\mathbf{r}}$, proportional to its velocity $\dot{\mathbf{r}}$ and a fluctuating force (arising from the irregular thermal motion of the substrate atoms) related to the friction η and to the substrate temperature T via the fluctuation-dissipation theorem. The drift velocity v was obtained by averaging over all the adsorbates and over time.

The relation $\sigma=f(v)$ can have two qualitatively different forms. If the adsorbate layer is in a two-dimensional (2D) fluid state, which is always the case in some parts of the (θ,T) (θ is the adsorbate coverage) phase diagram, then the $\sigma=f(v)$ relation has the form schematically indicated in Fig. 4(a). In this case the drift velocity will be nonzero for arbitrarily small σ. This is, of course, exactly what one expects for a fluid: an arbitrarily weak external force can shear a fluid. Furthermore, no hysteresis is observed, i.e., the relation between σ and v does not depend on whether σ decreases from a high value or increases from zero. Hence, if the lubrication layer in a sliding friction experiment is in a 2D-fluid state, smooth sliding is expected (i.e., no stick-and-slip motion) for any spring velocity v_s. This is exactly what is observed experimentally. For example, Yoshizawa and Israelachvili[6] have studied a 12-Å-thick hexadecane film between two smooth mica surfaces and found stick-and-slip motion when the temperature $T=17\,^\circ$C, but smooth sliding for $T=25\,^\circ$C. As will be shown below, stick-and-slip motion is observed when the adsorbate layer is in a pinned solid state at "stick." Hence the melting temperature of the hexadecane film is somewhere between 17 and 25 °C.

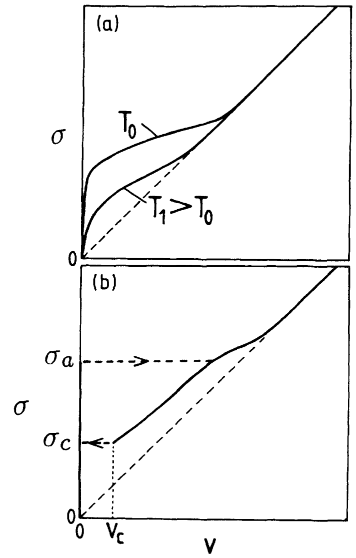

FIG. 4. The drift velocity $\langle v\rangle$ of an adsorbate layer as a function of the external stress $\sigma=n_aF$, where n_a is the adsorbate coverage and F the external force acting on each adsorbate. (a) the adsorbate layer is in a fluid state when $\sigma=0$. (b) The adsorbate layer is in a pinned solid state when $\sigma=0$. From Refs. 12 and 3.

Assume now that instead the system is in a part of the (θ,T) phase diagram, where the adsorbate layer is in a solid state which is commensurate with or at least pinned by the substrate. In this case the $\sigma=f(v)$ relation has the qualitative form shown in Fig. 4(b). If the system is first thermalized with $\sigma=0$ and then σ is increased, the pinned solid structure will remain, and the drift velocity will be zero ($v=0$), until σ reaches some critical stress σ_a. At this point the adsorbate system fluidizes, and the drift velocity increases abruptly from $v=0$ to v_a. If σ increases further the drift velocity continues to increase, as indicated in the figure. If σ is reduced below σ_a the system does not return to the pinned solid state at $\sigma=\sigma_a$, but continues to slide until σ reaches some lower critical stress σ_c where the system abruptly returns to the pinned state.

The hysteresis shown in Fig. 4(b) can have two origins. The first follows from the fact that the temperature in the adsorbate systems during sliding is higher than that of the substrate, and might be so high that the fluid configuration rather than the solid pinned state is stable for $\sigma_c<\sigma<\sigma_a$. However, a more general explanation is the following. First, it has been found that the return to the pinned solid state at $\sigma=\sigma_c$ is a nucleation process. However, a drag force from the surrounding flowing 2D fluid will act on a pinned island.[14] Assuming a circular pinned island, and that the drag force is uniformly distributed on the adsorbates in the island, the drag force is so large that the island will fluidize if $\sigma>\sigma_c=\sigma_a/2$.

The picture presented above has been obtained from numerical simulations on finite systems, but many of the results can be understood based on 2D hydrodynamics and should therefore be very general. These theoretical arguments also indicate that the transition to the pinned state at $\sigma = \sigma_c$ may be more complex than indicated by the simulations. I will now present both theoretical and experimental arguments that the return to the pinned state may occur as indicated in Fig. 5, i.e., the $\sigma = f(v)$ relation has a wide almost horizontal region for $v_c < v < v_b$, where $\sigma \approx \sigma_c$.[15]

As pointed out above, the return to the pinned state is a nucleation process.[16,3] In Ref. 3 it has been shown that the drag force on a pinned circular island (radius R) is proportional to R^2. This drag force acts as a pressure on the periphery of the island. If the pinning potential is small compared to the lateral adsorbate-adsorbate interaction potential (at the nearest-neighbor separation), and if the island is not too large, the drag force will distribute itself more or less uniformly on all the adsorbates in the island, and the argument given above about the stability of a circular island for $\sigma < \sigma_c = \sigma_a / 2$ holds. But as the island grows this condition will finally break down: for a "very" large pinned island the drag force will only act on a thin shell of adsorbates at the periphery of the island, where the local stress will be so large as to give rise to local fluidization. Hence an isolated island will have some large but finite radius R_0. It follows that as σ is reduced to some critical stress σ_b, the system may "flip" from a "smooth" fluid state (for $\sigma > \sigma_b$) to some granular state consisting of a configuration of pinned islands (with radius $\sim R_0$) surrounded by 2D fluid. As σ is reduced further the radius $\sim R_0$ is likely to increase rapidly, and the drift velocity v decreases, giving rise to an almost horizontal region in the $\sigma = f(v)$ relation for $v_c < v < v_b$.

Another reason for the existence of an almost horizontal region in the $\sigma = f(v)$ relation may be as follows. Suppose we reduce the stress so that pinned islands start to occur. Now, if the islands are pinned by both of the sliding surfaces simultaneously, then, since it will take time for an island to grow and since the block and the substrate are in relative motion, during the growth of an island there will be a force on the island building up due to the local (at the island) elastic deformations of the block and substrate. If the force on the island grows large enough, the island will fluidize. On the other hand, if an island is initially pinned by only one of the two sliding surfaces, with different islands being pinned by either of the two differents surfaces, then "collisions" between pinned islands would occur during the sliding process which would result in a fluidization of islands. This should result in a "granular" sliding state for the adsorbate layer, where pinned islands are continuously formed and fluidized. To what extent these effects are important in practice will depend on the nucleation and growth rate of the pinned islands and on the sliding velocity v.

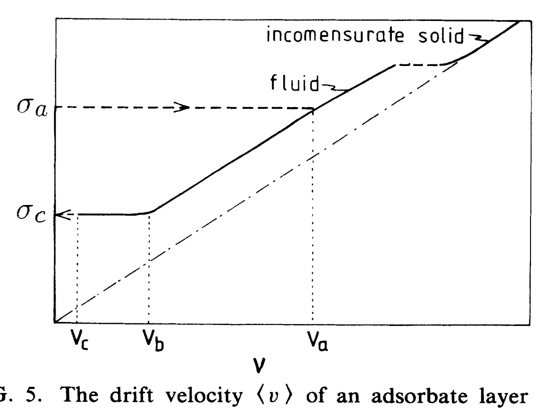

FIG. 5. The drift velocity $\langle v \rangle$ of an adsorbate layer as a function of the external stress σ. The adsorbate layer is in a pinned solid state when $\sigma = 0$.

For both of the processes discussed above, one would, for an infinite system, except the size R_0 of the islands to grow without bound in a continuous manner upon approaching the fully pinned state. But a real contact area has a finite width (area $A = D \times D$), where typically $D \sim 10^5$ Å. Hence, in practice, when R_0 reaches $\sim D$, the system has returned to the fully pinned state, and this might be the origin of the critical velocity v_c. It is important to note that the "horizontal" region of the $\sigma = f(v)$ relation cannot be studied by numerical simulations if the radius of a pinned island is larger than the basic unit used in the simulations.

The general form of the $\sigma = f(v)$ relation presented in Fig. 5 is supported by results of sliding friction measurements (see Sec. III B). Here I only note two facts: First, smooth sliding (i.e., sliding without stick and slip) is observed (if the damping is large enough so that inertia effects can be neglected, see Secs. II C and III) in a large velocity interval $v_c < v_s < v_b$, where the friction force is almost velocity independent; this implies that the $\sigma = f(v)$ curve has a "large," almost horizontal, region as indicated in Fig. 5. Second, direct support for a "granular" state with pinned regions and fluid regions is drawn from the study by Reiter *et al.*,[10] who probed the response of a sliding junction to an oscillatory external force. This study showed that although the dissipative stress in the sliding state was almost independent of sliding velocity (as long as it is not too large) significant (although smaller) elastic stress also persisted, which decreased with increasing deflection amplitude but was almost independent of oscillation frequency. The fact that elastic stresses occurred, and that the elastic component decreased with increasing oscillation amplitude, is strong support for the existence of pinned islands; a large oscillation amplitude would then imply stronger forces on the pinned islands, and hence would tend to fluidize a larger fraction of them as compared with a lower oscillation amplitude.

It is interesting to note that a granular sliding state has been observed by Chen and Zukoski at low sliding velocities in experiments involving a $\sim$1-mm-thick film of a colloidal crystal (a charge-stabilized polystyrene latex solution with particles with a diameter of a few times 10^4 Å).[17] Although this system differs from that involved in boundary lubrication experiments (in the latter systems the film is typically only one or two monolayers thick, while the latex film is about 1000 monolayers thick), the $\sigma = f(v)$ relation for the two systems is similar, e.g., hysteresis of the form indicated in Fig. 5 also occurs in studies involving colloidal crystals.

It is important to have a rough estimate of the magnitudes of σ_a, v_a, and v_c (note: that σ_c and v_b are of the same order of magnitude as σ_a and v_a). The stress σ_a can be deduced directly from the static friction force if the contact area is known. Typically one finds $\sigma_a \sim 10^8$ N/m^2 for lubricated surfaces. The sliding velocity v_a is roughly determined by $n_a m_a \eta v_a \sim \sigma_a$ the [dash-dotted line in Fig. 5], and if we take $\eta \sim 10^{13}$ s^{-1} (see Ref. 18) this gives $v_a \sim 10$ m/s. In the sliding measurement of Gee, McGuiggan, and Israelachvili, the critical velocity v_c has been measured directly to be of order $\sim 10^{-6}$ m/s. In the quantitative estimates presented below, I have used these numerical values, although it is likely that they may vary widely from one sliding system to another.

B. Starting and stopping

Suppose that the free end of the spring in Fig. 3 is pulled very slowly. This will lead to elastic deformation of the block (and the spring) as indicated in Fig. 6(a). As long as the tangential stress at $z=d$ is below σ_a, the block will not move, and the displacement field $u(z,t)$ is given by $u=-\kappa\sigma_s z$ so that the tangential stress in the block equals σ_s everywhere. When σ_s reaches the critical stress σ_a the lubricant layer will fluidize, and the surface $z=d$ of the block starts to move with some velocity v_0 to be determined below. This change in the displacement field will propagate with the sound velocity c toward the upper surface ($z=0$) of the block; see Fig. 6(b). Let us study this elastic wave propagation in detail. Let $t=0$ be the time when the adsorbate layer fluidizes. The displacement field for times $0<t<d/c$ can be written as [see Figs. 6(b) and 6(c)]

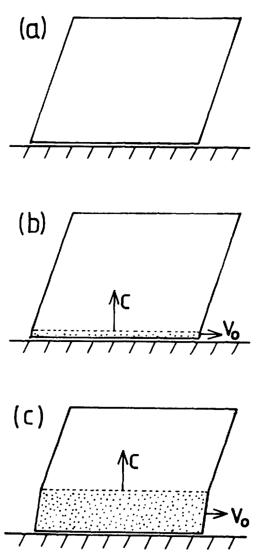

FIG. 6. An elastic block on a substrate. (a) The stress σ_s in the block induced by the external spring is just below the critical stress σ_a necessary to fluidize the adsorbate layer. (b) The spring stress has reached σ_a, the adsorbate layer has fluidized, and the bottom surface of the block moves with the velocity v_0. (c) The region of "motion" propagates with the transverse sound velocity c toward the upper surface of the block. After a time interval d/c the elastic wave has reached the upper surface, and the whole block moves with the velocity v_0.

$$u=-\kappa\sigma_a z, \quad 0<z<d-ct , \tag{4}$$

$$u=u_0-\kappa\sigma_0 z+v_0 t , \quad d-ct<z<d , \tag{5}$$

where u_0, σ_0, and v_0 are constants. The displacement field u must be continuous for $z=d-ct$, which gives

$$u_0-\kappa\sigma_0 z+v_0(d-z)/c=-\kappa\sigma_a z$$

or

$$u_0+v_0 d/c=0 , \tag{6}$$

$$\kappa\sigma_0+v_0/c=\kappa\sigma_a . \tag{7}$$

The displacement field given by (4)–(7) is continuous everywhere, *and* satisfies the wave equation (1). In order to satisfy the correct boundary condition on the surface $z=d$, the stress σ_0 and the velocity v_0 occurring in (5) must satisfy

$$\sigma_0=f(v_0) . \tag{8}$$

In Fig. 7, I show the graphic solution to Eqs. (7) and (8). Note that the system "jumps" [in the (v,σ) plane] from $v=0$ to a finite velocity $v_0 \sim v_a$. It is of crucial importance to note that this is possible because initially only a very thin (in the continuum model, infinitesimal) slab or solid at the bottom of the block ($z=d$) need to be accelerated to the speed v_0. If the block were perfectly rigid, then the whole block would have to change its velocity, which can only occur slowly in order not to generate enormous forces of inertia. But any real solid has a finite elasticity (and a finite sound velocity), and the transition indicated in Fig. 7 may occur practically horizontally and instantaneously.

Let us discuss the magnitude of v_0 in a typical case. In Fig. 7 the dash-dotted line is given by $\sigma=m_a n_a \eta v$. Now, if $\eta \sim 10^{13}$ s^{-1}, $m_a \sim 500u$, and $n_a \sim 0.01$ Å^{-2}, one obtains $m_a n_a \eta \sim 10^7$ kg/m^2 s. On the other hand, relation (7) can be written as $\sigma_0=\sigma_a-\rho c v_0$, where in a typical case $\rho \sim 10^4$ kg/m^3 and $c \sim 1000$ m/s, so that $c\rho \approx 10^7$ kg/m^2 s. It follows that the slope of the $\sigma=f(v)$ curve for $\sigma \sim \sigma_a$ is of a similar magnitude to that associated with the line $\sigma=\sigma_a-\rho c v$. Hence, at the onset of fluidization of the adsorbate layer, the velocity of the surface $z=d$ of the block will increase abruptly from $v=0$ to a finite v_0 of order v_a.

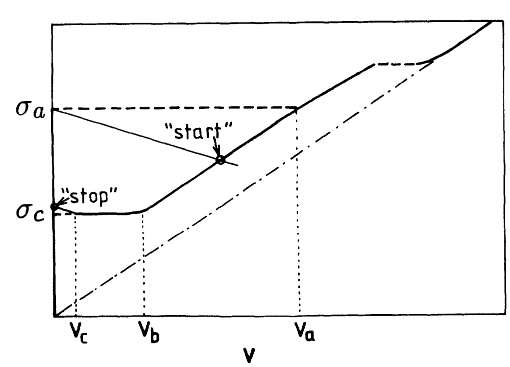

FIG. 7. "Starting": Graphical solution of Eqs. (7) and (8). "Stopping": Graphical solution of Eqs. (12) and (13).

How rapid is the "starting" process? To address this question, note that in the elastic continuum model the bottom surface of the block changes its velocity *instantaneously* when the surface stress at the block-substrate interface changes as a result of the fluidization of the adsorbate layer. For a real system, the continuum model breaks down at very short distances (on the order of an atomic lattice spacing); the layer of atoms of the block in direct contact with the lubrication layer will accelerate within the time period $a/c \sim 10^{-13}$ s, where a is an atomic distance, to reach a final velocity v_0 accurately determined by the elastic continuum model. The present study and all experimental sliding friction measurements performed have until now probed only the properties of the system on time scales much longer than this "atomistic" time; for our purposes the "starting" and "stopping" (see below) processes can be considered instantanous.

According to (4) and (5), at the moment $(t=d/c)$ the elastic wave generated at the bottom surface of the block has reached the upper surface, the whole block moves uniformly with the velocity $v_0=(\sigma_a-\sigma_0)/\rho c$. This is the same motion as exhibited by a rigid block and, in fact, the velocity of the elastic block at $t=d/c$ is exactly the same as that of a rigid block exposed to a surface stress σ_a on the upper surface and $-\sigma_0$ on the lower surface. This follows at once from Newton's equation of motion for the center of mass, $M\ddot{x}=A(\sigma_a-\sigma_0)$, so that $\dot{x}=A(\sigma_a-\sigma_0)t/M$. Substituting $t=d/c$ in this equation gives $\dot{x}=Ad(\sigma_a-\sigma_0)/Mc=(\sigma_a-\sigma_0)/\rho c$, where $\rho=M/Ad$ is the mass density. Hence the motion of the block at time $t=d/c$ is identical to that of a rigid block, but for $t<d/c$ the results differ and, as shown above, this has important implications for the nature of "starting." Furthermore, in the rigid-block limit the stress σ_0 is not defined.

In principle it is possible to follow the motion of the block for all times by simply studying how elastic waves are generated and reflected from the surfaces $z=0$ and d of the block, accounting for the appropriate boundary conditions at these surfaces. But except for the "starting" and "stopping" (see below) time periods of duration d/c, such a treatment is in most cases equivalent to the motion of a rigid block. This is shown in Appendix A, which illustrates how the equation of motion for a rigid block can be obtained from those for an elastic block in the limit $c\rightarrow\infty$.

After the time period d/c, the elastic wave generated at the bottom surface $(z=d)$ has reached the top surface $(z=0)$, and the whole block moves uniformly with the velocity v_0. From here on we can accurately study the motion assuming a rigid block; this is the topic of Sec. II C, where it is shown that, as time increases, the velocity v and the stress σ for $z=d$ both decrease from their initial values v_0 and σ_0. When $\sigma(t)$ has finally decreased to σ_c the lubricant layer returns to its pinned state. But just as in "starting," this "stopping" transition cannot be treated using the rigid-block limit, as the force of inertia required to retard a rigid macroscopic block from the speed v_c to zero over a distance on the order of an atomic lattice spacing would be very high, and would immediately fluidize the pinned structure. However, in the elastic continuum model there is no problem, since we only need to "stop" the motion of the $z=d$ side of the block in order to return to the pinned state; the region of "stopped" motion then propagates as an elastic wave with the sound velocity c toward the upper surface $(z=0)$ of the block, and after the time period d/c the whole block is standing still. In order to study this "stopping" motion, let us assume that $t=0$ corresponds to the time when the adsorbate layer returns to its pinned state. Hence for $0<t<d/c$ the displacement field $u(z,t)$ has the form (see Fig. 8)

$$u=u_c-\kappa\sigma_c z+v_c t\ ,\quad 0<z<d-ct\ , \tag{9}$$

$$u=u_1-\kappa\sigma_1 z\ ,\quad d-ct<z<d\ , \tag{10}$$

where (u_c,σ_c,v_c) and (u_1,σ_1) are constants. The requirement that u is continuous at $z=d-ct$ gives

$$u_c-\kappa\sigma_c+v_c(d-z)/c=u_1-\kappa\sigma_1 z$$

or

$$u_c+v_c d/c=u_1\ , \tag{11}$$

$$\kappa\sigma_c+v_c/c=\kappa\sigma_1. \tag{12}$$

The displacement field given by (9)–(12) is continuous everywhere *and* satisfies the wave equation (1). In order to satisfy the correct boundary condition on the surface $z=d$, the stress σ_c and velocity v_c occurring in (9) must satisfy

$$\sigma_c=f(v_c)\ . \tag{13}$$

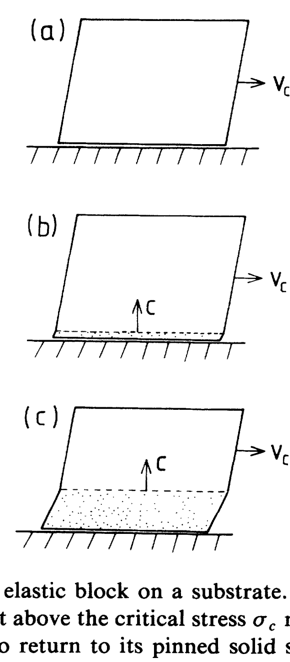

FIG. 8. An elastic block on a substrate. (a) The stress σ in the block is just above the critical stress σ_c necessary for the adsorbate layer to return to its pinned solid state. (b) The stress has reached σ_c, the adsorbate layer has returned to its pinned state, and the bottom surface of the block has stopped moving. (c) The region of "stopped motion" propagates with the transverse sound velocity c toward the upper surface of the block. After the time interval d/c the elastic wave has reached the upper surface, and the whole block has stopped moving.

In Fig. 7, I show the graphic solution to Eqs. (12) and (13) for a typical case. For the same reason as in "starting" the system "jumps" almost instantaneously from $v=v_c$ to $v=0$. Again it is of crucial importance to note that this is possible because initially only a very thin (in the continuum model, infinitesimal) slab of solid at the bottom of the block ($z=d$) needs to be retarded to zero speed. If the block were perfectly rigid then the whole block would have to change its velocity, which can only occur slowly in order not to generate enormous forces of inertial. But any real solid has a finite elasticity (and a finite sound velocity), and the transition indicated in Fig. 7 occurs practically instantaneously.

At this point I would like to comment on the sliding friction study of Thompson and Robbins.[19] They performed molecular-dynamics simulations on a system consisting of a molecular thin film between two atomically flat solid walls. The wall atoms were coupled to the sites of a rigid 3D lattice via stiff springs. Hence, the walls had no long-range elastic properties. In their simulations they found that the transition from smooth sliding to stick-and-slip motion occurred at some critical spring velocity v_c which scaled as $M^{-1/2}$ with the mass of the moving wall. This fact was explained as follows: In order for the wall to stop, its kinetic energy must be converted into potential energy in the film. The maximum potential energy that can be stored in the film is of the order of $\sim aF_a$, where a is a lattice constant of the wall, and F_a the static friction force. Equating this to the kinetic energy at $v=v_c$ gives

$$v_c \sim (aF_a/M)^{1/2} \; . \tag{14}$$

However, for a real physical sliding system this explanation for the occurrence of the critical speed v_c is incorrect, since the block as a whole will never stop abruptly; initially only the bottom surface of the block stops, and the inertia forces involved in this are negligible. The reason that $v_c \sim M^{-1/2}$ is observed in simulations by Thompson and Robbins is the unphysical treatments of the elastic properties of the walls: by assuming that the wall atoms are connected to the sites of a rigid lattice, all the atoms in the block or wall must stop simultaneously at the return to the pinned state. Removing this unphysical assumption by using an elastic block creates a different mechanism for the origin of the critical velocity v_c (see Sec. II A). Hence it should not come as a surprise that the relation $v_c \sim \sqrt{F_a}$ implied by (14) is not observed in a recent sliding friction study for hexadecane molecules between two flat mica surfaces.[6]

C. Motion during slip

The motion of the block between the short "starting" and "stopping" events (both of duration $d/c \sim 10^{-5}$ s) can be treated accurately using the rigid-block approximation if the condition

$$\max|v_s - v(t)| \ll cF_a/k_s d \tag{15}$$

is satisfied (see Appendix A). Here F_a is the static friction force and $v(t)$ the velocity of the center of mass of the block. This condition results from the requirement that the change in the spring force, during the time $2d/c$ it takes for an elastic wave to propagate from the bottom surface to the top surface and back again, is small compared with F_a.

The nature of the motion of the block depends on the initial velocity v_0 occurring after "starting." In this section, as an illustration, I assume that $v_0 \approx v_a$, i.e., that the "starting" (and "stopping") transitions in Fig. 7 are almost horizontal. In this case $\max|v_s - v| \sim v_a$ and (15) reduces to $v_a k_s d \ll cF_a$. If this condition is satisfied, the position of the block, $x(t)$, can be accurately determined by Newton's equation for a rigid block:

$$M\ddot{x} = k_s(l + v_s t - x) - Af(\dot{x}) \; . \tag{16}$$

Here M is the mass of the block and $A\sigma = Af(\dot{x})$ is the friction force given by the product of the surface area A and the frictionless stress. We assume that time $t=0$ occurs just after "starting," where the spring force must equal $A\sigma_a$, and where the block has the velocity $v_0 \approx v_a$. Hence if we choose $x(0)=0$ then we must have $k_s l = A\sigma_a$, which determines the parameter l. If we measure $x(t)$ in units of l, time in units of l/v_a, velocity in units of v_a, and stress in units of σ_a, then (16) takes the form

$$\ddot{x} = Q[1 + v_s t - x - f(\dot{x})] \; , \tag{17}$$

where

$$Q = \frac{A\sigma_a l}{Mv_a^2} = \frac{F_a l}{Mv_a^2} = \frac{F_a^2}{k_s M v_a^2} = \frac{F_a}{k_s d}\frac{\sigma_a}{\rho v_a^2} \; . \tag{18}$$

But the present treatment is valid only if (15) holds, i.e., only if

$$Q \gg \frac{\sigma_a}{\rho v_a c} \; . \tag{19}$$

Since I have assumed that the "starting" transition is almost horizontal, the inequality (see Sec. II B) $\sigma_a/\rho v_a c \gg 1$ must be satisfied. Hence (19) implies $Q \gg 1$ in order for the rigid-block model to be valid. This corresponds to overdamped motion, which is almost Q independent and determined accurately by neglecting the second-order time derivative term in (17), i.e., by setting the term $[\cdots]$ in (17) equal to zero. If the condition (15) is not met (but $\sigma_a/\rho v_a c \gg 1$ still holds), a more detailed calculation in necessary to account for the elastic properties of the block for all stages in the motion.

I have calculated the spring force F_s for a few different forms of the function $f(v)$. In all cases I have chosen $Q=100$, but the results are essentially independent of Q for large Q. In Fig. 9, I show the relation $\sigma=f(v)$ for four different cases [Figs. 9(a)–9(d)], for which I have solved (17) by numerical integration. The resulting spring force is plotted in Fig. 10 as a function of time, and for several values of the spring velocity v_s. The latter is measured in units of v_c, which is the lowest spring velocity which allows smooth sliding. As expected, for $v_s < v_c$ stick-and-slip motion occurs, while the motion is smooth for $v > v_c$. The important qualitative effect in

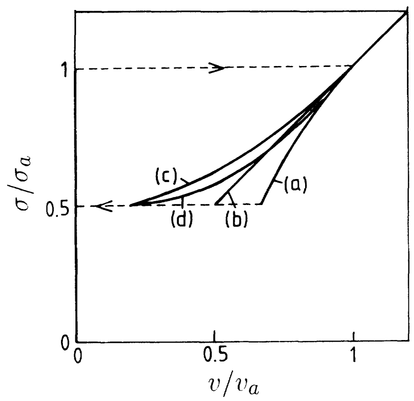

FIG. 9. The relation $\sigma=f(v)$ for four different cases (a)–(d), where I have assumed $\sigma_c=\sigma_a/2$.

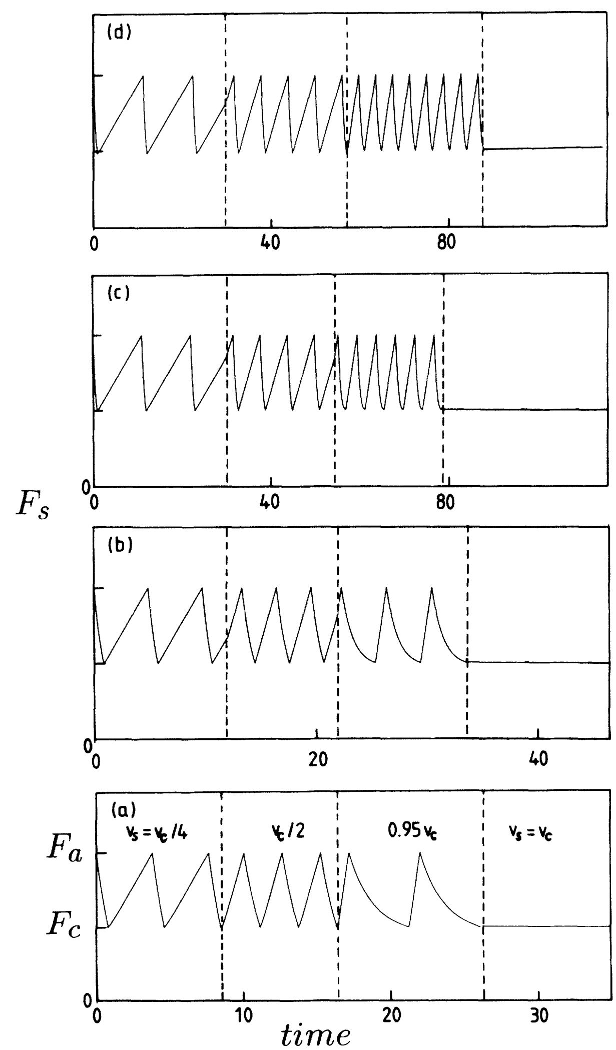

FIG. 10. The spring force F_s as a function of time, for several values of the spring velocity v_s. The four cases (a)–(d) have been calculated assuming the relation between σ and v indicated in Fig. 9 by cases (a)–(d). In the calculations, $Q=100$, and I assumed that the "starting" and "stopping" transitions occur horizontally in the (v,σ) plane.

Fig. 10 is the fact that in cases (c) and (d) the stick-and-slip frequency increases continuously as $v_s \rightarrow v_c$ from below, while it first increases and then decreases in cases (a) and (b). Furthermore, in the latter case the slip part of the motion has a rather slowly decaying tail when $v_s \sim v_c$ while this tail is much shorter in cases (c) and (d). As discussed in Sec. II only cases (c) and (d) are consistent with experimental data, i.e., the relation $\sigma=f(v)$ has the qualitative form indicated in Fig. 5.

In a more general case, where the initial velocity v_0 is less than v_a, the sliding process will be more complex. In particular, instead of a continuous drop in the sliding velocity during the slip period, the block may first accelerate (though the maximum velocity is always below v_a). Furthermore, due to inertial effects, stick-and-slip motion may occur at spring velocities higher than v_c, and the amplitude of the oscillations in the spring force during a stick-and-slip period will not equal F_a-F_c as above, but will be larger (see Sec. III B).

III. COMPARISON WITH EXPERIMENTS AND DISCUSSION

In experimental sliding systems, the block in Fig. 2 is never in contact with the substrate over the whole apparent contact area A. This fact must be taken into account when analyzing experimental data. In this section I study this problem in some detail.

A. Qualitative discussion

Consider the sliding configuration shown in Fig. 11. A small block with mass M^*, to be referred to as the miniblock, is connected via a spring (with bending force constant k_s^*) to a large block with mass M. The large block has a (bottom) surface area A and the miniblock a surface area δA, where $\delta A \ll A$. A spring with a force constant k_s is connected to the large block, and the free end of the spring is moved with the speed v_s, which we assume is below v_c. The miniblock is sliding on a lubricated surface. The big block is acted upon not only by the forces from the springs k_s and k_s^*, but in addition by a viscous force $-M\gamma\dot{x}$, which we assume is proportional to the center-of-mass velocity $\dot{x}$. This force may result, e.g.,

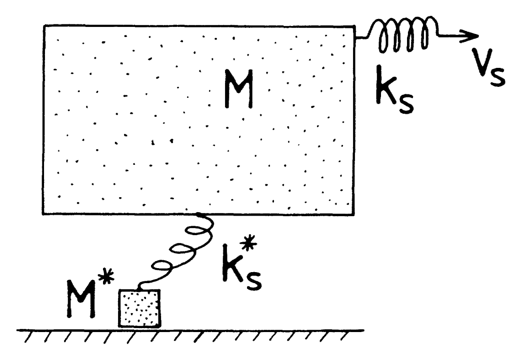

FIG. 11. A small block of mass M^* is connected to a big block with mass M by a spring with the bending force constant k_s^*. The free end of the spring k_s moves with the velocity v_s.

from the viscous drag from a fluid surrounding the big block. I assume that the motion of the miniblock is overdamped, as is likely to be the case in the applications considered in Secs. III B and III C.

Now, assume that $k_s^* \gg k_s$ and $M \gg M^*$, and I will discuss qualitatively the nature of the motion of the miniblock and the big block.

The motion of the miniblock will be as discussed in Sec. II C. That is, the miniblock first elastically deforms until a critical surface stress σ_a has been reached [point A in Figs. 12(a) and 12(b)], at which point the lubrication film fluidizes. During a very short-time period the miniblock reaches its maximum velocity v_0 of order v_a (point B), after which the velocity decreases monotonically (overdamped motion) while the velocity of the big-block increases. After the short-time period τ, the average velocity $v(t)$ of the surface $z=d$ of the big block (which corresponds to the spring velocity v_s^* for the miniblock) and the velocity of the miniblock will (nearly) coincide; I denote $v(\tau)=v^*$. If v^* is above the critical speed v_c (which I assume to be the case in what follows), the miniblock will not return to its pinned state but will continue to slide at a velocity close to $v(t)$.

Let us now consider the motion of the big block. We consider two limiting cases, namely zero damping $\gamma=0$ (underdamped motion) and a large damping γ such that inertia effects associated with the motion of the big block can be neglected (overdamped motion).

Consider first zero damping, $\gamma=0$. Just before the slip starts, the $z=d$ surface of the big block is acted upon by the (average) tangential surface stress $\sigma=\sigma_a \delta A/A$. Since the ratio $\delta A/A$ is assumed to be much smaller than unity, this surface stress is much lower than the one which acts on the miniblock. This has the following important consequence. While after fluidization the miniblock almost instantaneously reaches the speed $v_0 \sim v_a$, the big block will, after a time period $\sim d/c$, have a velocity $v \sim (\delta A/A)(\sigma_a/\rho c)$ which is below v_b. But since the mass of the big block is so large, it has not had time to move any appreciable distance. This implies that the force in the spring k_s is still close to the value $F_a = \delta A \sigma_a$ it possessed at the moment the adsorbate layer was fluidized. Hence, after the initial rapid relaxation of the small block the big block will be acted upon by the net force $\sim(F_a - F_c) > 0$ (where $F_c = \delta A \sigma_c$), and the block will accelerate. Since the spring k_s^* is very stiff, the motion of the miniblock (after the initial rapid relaxation) will closely follow that of the big block. The motion of the miniblock is indicated in Fig. 12(a). A full stick-and-slip cycle involves the sequence $A \to B - C - D - E \to F$.

I note that in many practical cases, the velocity v^* is negligibly small compared with the highest velocity attained by the block at later times. That is, the motion of the big block will, after the initial rapid relaxation of the miniblock, start from $v = v^* \approx 0$ and only gradually increase and then decrease again. It is easy to prove that during this motion the inequality (15) is usually satisfied (see below), and that the motion of the big block can therefore be studied by solving Newton's equation for a rigid block, just as for the miniblock, but with a different initial condition, namely $v \approx 0$ at the start of the slip. As I discuss in Sec. II B, because of inertia effects, stick-and-slip motion occurs even if $v_s > v_c$, and the amplitude of the variation in the spring force during a stick-and-slip cycle is roughly $2(F_a - F_c)$ rather than $F_a - F_c$, as expected if inertia effects can be neglected (see Sec. III C).

Next, let us consider the case where a viscous force acts on the big block, which is so large that inertia effects can be neglected (overdamped motion). In this case the following scenario occurs: After the initial rapid motion (time period τ) where the miniblock relaxes toward lower velocities while the big block accelerates, the big block and miniblock reach the same velocity v^*. But from here on, because of the viscous force, the velocity of the big block will monotonically decrease with increasing time until $v(t)$ reaches v_c, at which point the system returns to the pinned state. If $v^* \gg v_c$ the shape of the time dependence of the spring force will be similar to that discussed in Sec. II C. That is, neglecting the short initial time

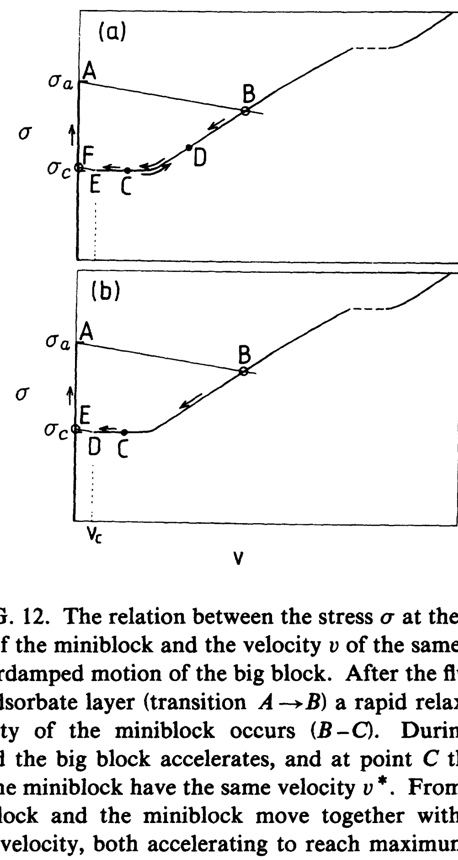

FIG. 12. The relation between the stress σ at the bottom surface of the miniblock and the velocity v of the same surface. (a) Underdamped motion of the big block. After the fluidization of the adsorbate layer (transition $A \to B$) a rapid relaxation of the velocity of the miniblock occurs ($B - C$). During this time period the big block accelerates, and at point C the big block and the miniblock have the same velocity v^*. From here on the big block and the miniblock move together with almost the same velocity, both accelerating to reach maximum velocity at point D. The system then retards and when the velocity reaches the critical velocity v_c (point E) the bottom surface of the miniblock returns to the pinned state (transition $E \to F$) and the whole cycle $F - A \to B - C - D - C - E \to F$ repeats iself. (b) Overdamped motion of the big block. After the fluidization of the adsorbate layer (transition $A \to B$) a rapid relaxation of the velocity of the miniblock occurs (B–C). During this time period the big block accelerates, and at a point C the big block and the miniblock have the same velocity v^*. From here on the big block and the miniblock move together with almost the same velocity, both retarding (C–D). When the velocity reaches the critical velocity v_c (point D) the bottom surface of the miniblock returns to the pinned state (transition $D \to E$) and the whole cycle $E - A \to B - C - D \to E$ repeats itself.

period τ where the big-block accelerates (the time period τ is very short and usually not resolved experimentally), the slip will start with a rapid, almost linear (in time), drop in the spring force followed by a slower time variation. The motion of the miniblock is indicated in Fig. 12(a). A full stick-and-slip cycle involves the sequence $A \rightarrow B - C - D \rightarrow E$. In this case stick-and-slip motion occurs only if $v_s < v_c$, and the amplitude of the variations in the spring force equals $F_a - F_c$, as expected if inertia effects can be neglected (see Sec. III C).

In Secs. III B and III C I consider the limiting cases of underdamped and overdamped motion of the big block in more detail. The former case occurs in many practical cases, e.g., when a metal block is slid onto a metal substrate while the latter case seems to prevail in sliding friction studies such as those in Refs. 6–8, involving two mica surfaces with a single contact "point" (junction).

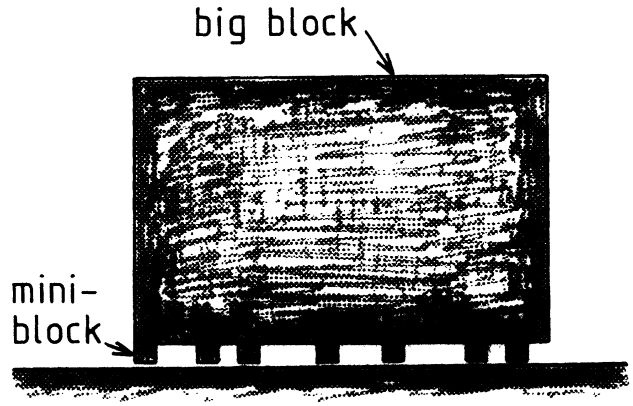

FIG. 13. A block on a substrate. The miniblocks represent the surface roughness of the block which contact the substrate.

B. Underdamped motion

I will now discuss in detail the sliding problem considered above when the viscous damping vanishes (i.e., $\gamma = 0$). The stick-and-slip motion discussed above in Sec. II C arises from the "forbidden gap" in the $\sigma = f(v)$ relation (i.e., no stationary motion of the adsorbate layer is possible for $0 < v < v_c$), but for underdamped motion stick-and-slip motion has a different origin related to the fact that the friction force during sliding is lower than the static friction force. To discuss this important point, consider a metallic block on a metallic substrate with a lubrication fluid. Now, the actual contact between the block and the substrate does not occur over the whole apparent surface area, but only at some contact points (junctions) where surface roughness "touch." The sum of all the junctions is called the area of real contact δA. The rest of the apparent area of contact is usually much larger than the real area of contact, but plays essentially no part in determining the overall interaction between the two solids.

It has been found that the size of the real area of contact in most practical cases can be estimated accurately by assuming that plastic deformation has occurred at each junction, and that all junctions are in a state of incipient plastic flow. This assumption at once gives $\delta A = L/\sigma_c$, where L is the load and σ_c (the penetration hardness) the largest compressive stress that the materials can bear without plastic yielding. As an example, for a steel cube with a 10-cm side on a steel table, one obtains $\delta A \approx 0.1\ \mathrm{mm}^2$, i.e., the actual contact area is only a fraction $\delta A / A \sim 10^{-5}$ of the apparent contact area A. I also note that various experiments, e.g., visual inspection of two surfaces after contact has occurred, have shown that a junction typically has a diameter of 10^5 Å, so that the number of junctions is ~ 1000.

Let us refer to those surface roughness structures (surface protrusions) of the block which contact the substrate as miniblocks, and the rest as big blocks (see Fig. 13). The miniblocks have some typical mass M^* and are connected to the big block via some effective "springs" with force constants k_s^* which have their origin in the fact that when a tangential force acts on a miniblock it will displace relative to the big block by local elastic deformation of the big block (see Appendix B).

The motion of a miniblock will be as discussed in Sec. II C. That is, the miniblock first elastically deforms until the critical surface stress σ_a has been reached, at which point the lubrication film fluidizes. During a very short-time period of order $d^*/c \sim 10^{-8}$ s, the miniblock reaches its maximum velocity v_0 of order v_a, after which the velocity monotonically decreases (overdamped motion) while the velocity of the big block increases. The average velocity $v(t)$ of the surface $z = d$ of the big block (which corresponds to the spring velocity v_s^* for miniblocks) and the velocity of the miniblocks will coincide after some short time τ; I denote this velocity by v^*. I assume that v^* is above the critical speed v_c. Hence the miniblock will not return to its pinned state, but will continue to slide at a velocity close to $v(t)$.

Let us now consider the motion of the big block. Just before slip starts, the $z = d$ surface of the big block will be acted upon by the (average) tangential surface stress $\sigma = \sigma_a \delta A / A$. Since the ratio $\delta A / A$ typically equals 10^{-5} or so (see above), this surface stress is much lower than the one which acts on the miniblocks. This has the following important consequence. While after fluidization a miniblock almost instantaneously reaches a speed v_0 which is of order $v_a \sim 10$ m/s, the big block will (after the time period $\sim d/c$) have a velocity $v \sim (\delta A / A)(\sigma_a / \rho c) \sim 10^{-4}$ m/s which is practically zero. Hence the motion of the big block will start from $v \approx 0$ and only gradually increase and then (if stick-and-slip motion occurs) decrease again. It is easy to prove that during this motion inequality (15) is usually satisfied (see below) and the motion of the big block can therefore be studied by solving Newton's equation for a rigid block, just as for the miniblocks, but with a different initial condition, namely $v \approx 0$ at the starting of the slip.

Now, the $\sigma = f(v)$ relation is assumed to have a wide, almost horizontal, region for $v_c < v < v_b$, where the frictional stress equals $\sigma_c \approx \sigma_a / 2$. If the velocity of the big block is below v_b during the whole sliding process (this is often the case in boundary lubrication experiments), the friction force acting on a miniblock, during most of the slip period of the big block, will, to a very good approximation, equal σ_c. Hence the kinetic friction force on the big block will be $F_c = \sigma_c \delta A$, and the static friction force will be $F_a = \sigma_a \delta A$.

To study the motion of the big block, I treat it as a rigid block (see above) with mass M, and assume that a spring with force constant k_s is connected to it as indicated in Fig. 3. The free end of the spring is assumed to move with the constant velocity v_s. Let $x(t)$ be the position coordinate of the block at time t, and assume that at $t=0$ the block is stationary relative to the substrate [$x(0)=0$ and $\dot{x}(0)=0$] and that the spring force vanishes. Now, for "low" spring velocities v_s the solid block is either in a pinned state relative to the substrate (i.e., $\dot{x}=0$) in which case the friction force is $F_f \le F_a$, or else in a sliding state where the friction force equals $F_c < F_a$. We assume that the sliding velocity is so low that we can neglect the velocity dependence of the kinetic friction force. The equation of motion for the block takes the form

$$M\ddot{x} = k_s(v_s t - x) - F_f \ , \tag{20}$$

where

$$F_f \le F_a \quad \text{if } \dot{x}=0 \ ,$$

$$F_f = F_c \quad \text{if } \dot{x} > v_c \ .$$

Since v_c is such a small velocity we can set it equal to zero in the present case. If we measure time in units of $(M/k_s)^{1/2}$, distance in units of F_a/k_s, velocity v in units of $F_a(Mk_s)^{-1/2}$, and friction force F_f in units of F_a, then

$$\ddot{x} = v_s t - x - F_f \ . \tag{21}$$

In Fig. 14, I show the spring force $F_s = v_s t - x(t)$ when $F_c = F_a/2$ in a case where the velocity increases from 0.05 to 0.1 to 0.2. The result in this figure can be understood as follows: Initially, as time increases the spring will extend, but the solid block will not move until the force in the spring reaches F_a. At this point the adsorbate structure "fluidizes" (see Sec. II A) and the two surfaces can slide relative to each other. The friction force is now $F_c < F_a$, and since the spring force is F_a at the onset of sliding the box will initially accelerate to the right. Note that owing to the inertia of the block, the velocity $\dot{x}$ is initially lower than v_s, and the spring will continue to extend for a while before finally $\dot{x} > v_s$. The maximum spring force will therefore not occur exactly at the onset of sliding but slightly later, where the spring force is greater than F_a. In Fig. 14 this inertia effect is very small (this is typically the case sliding friction experiments) and the maximum spring force is nearly equal to F_a. When the sliding velocity $\dot{x} > v_s$ the spring force decreases, but the analysis presented below shows that the sliding motion does not stop when the spring force equals F_c but continues until the spring force reaches $\approx 2F_c - F_a$, where the motion stops and the whole cycle repeats itself. This model is simple enough that one can analytically evaluate the amplitude ΔF of the oscillations in the spring force which occur during sliding, as well as the stick-and-slide time periods. Assume that the sliding process starts at $t=0$ and let t_0 denote the time when the slip starts so that $v_s t_0 = F_a$. The general solution of (21) valid during the first slip period is

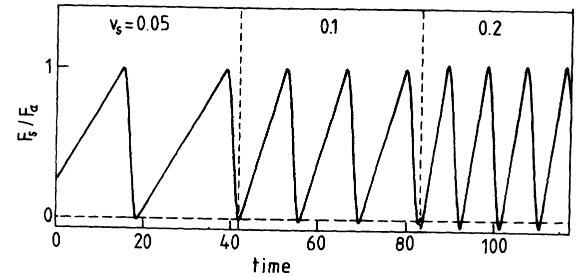

FIG. 14. The spring force F_s as a function of time for underdamped motion of the big block. The static and kinetic friction forces are denoted by F_a and F_c, respectively.

$$x(t) = v_s t - F_c - A \sin(t+\alpha) \ , \tag{22}$$

and the spring force is

$$F_s(t) = v_s t - x(t) = F_c + A \sin(t+\alpha) \ . \tag{23}$$

At $t=t_0$ both x and $\dot{x}$ vanish, so that

$$x(t_0) = v_s t_0 - F_c - A \sin(t_0+\alpha) = 0 \ ,$$

$$\dot{x}(t_0) = v_s - A \cos(t_0+\alpha) = 0 \ ,$$

where $v_s t_0 = F_a$. Hence

$$A = [v_s^2 + (F_a - F_c)^2]^{1/2} \ , \tag{24}$$

$$t_0 + \alpha = \arcsin[(F_a - F_c)/A] \equiv \phi \ . \tag{25}$$

Now, during the first slip period $t+\alpha$ increases continuously from ϕ to $2\pi - \phi$ [note that $\cos(2\pi-\phi)=\cos\phi$]. Hence slightly after the begining of the sliding, for $t+\alpha = \pi/2 > \phi$, the spring force takes its largest value $F_c + A$ [see (23)]; this delay is caused by inertia effects. Similarly, slightly before the end of the sliding, for $t+\alpha = 3\pi/2 < 2\pi - \phi$, the spring force takes its smallest value $F_c - A$. Hence the amplitude ΔF of the oscillations in the spring force is $\Delta F = 2A$ or, using (24),

$$\Delta F = 2[v_s^2 + (F_a - F_c)^2]^{1/2} \ .$$

The time between a maximum and the following minimum of F_s equals π. Similarly, the time between a minimum and the following maximum in F_s equals $2(F_a - F_c)/v_s$. Finally, let us prove that inequality (15) is satisfied, as assumed implicitly. From (22) we obtain $|v_s - v(t)| \le F_a - F_c \approx F_a/2$ or, returning to coordinates with dimension, $|v_s - v(t)| \le F_a(Mk_s)^{-1/2} = (cF_a/k_s d)(d^2 k_s/Mc^2)^{1/2}$. However, in a typical case $d=0.1$ m, $k_s \sim 1000$ N/m, $M \sim 10$ kg, and $c \sim 2000$ m/s, so that $d^2 k_s/Mc^2 \sim 10^{-7}$ and inequality (15) is satisfied.

The time dependence of the spring force during stick-slip motion, as shown in Figs. 14 and 15, is often seen in experiments. However, the discussion above is valid only if the maximum of the sliding velocity is below v_b. If this

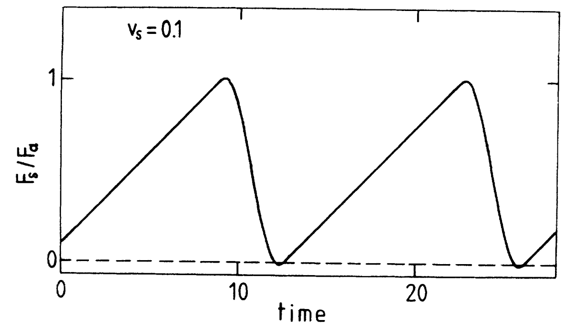

FIG. 15. Magnification of a "stick-and-slip" period shown in Fig. 14.

condition is not satisfied the frictional stress during sliding will not equal σ_c, but will vary with time. The maximum slip velocity in the model calculation above equals

$$v_{\max}=v_s+[v_s^2+(F_a-F_c)^2/Mk_s]^{1/2} ,$$

and the condition for the validity of the discussion above is that $v_{\max}<v_b$. Since for a block on a substrate without an "external" load, F_a and F_c are both proportional to M (since $F=fL=fMg$), it follows that $(F_a-F_c)^2/Mk_s \sim M/k_s$, and the condition $v_{\max}<v_b$ will be satisfied if the spring is stiff enough (i.e., k_s is large enough) or the mass of block M small enough.

C. Overdamped motion

In this section I discuss the limit of overdamped motion, i.e., I assume that a viscous drag force acts on the big block, which is so large that inertia effects can be neglected.

Mica has a layer structure which makes it possible to prepare macroscopic surface areas with atomistic smoothness, e.g., without a single step. Mica surfaces are therefore ideal for model studies of sliding friction. In this section I discuss the sliding friction measurements of Yoshizawa and Israelachvili[6] for thin hexadecane films between two shearing mica surfaces. In these measurements thin mica crystals are glued on curved glass substrates. By pressing the two mica-covered surfaces together a small circular contact area will arise, the diameter of which depends on the load, but which in most experiments is somewhere in the range 10–100 μm. In the sliding friction measurements presented in Ref. 6, a large fraction of the sliding apparatus is immersed in the lubricating fluid, and this generates a viscous drag force which might be the reason why in this case the sliding motion is observed to be overdamped.

To discuss the sliding process, let us define a miniblock as the dotted volume element in Fig. 16, and the rest as the big block. The width, depth, and height of the miniblock is on the order of the diameter of the contact area to the substrate, which we take to be $D^*\sim 6\times 10^{-5}$ m. The miniblock is connected to the big block via an effective spring with force constant k_s^*, which has its origin in the fact that when a tangential force acts on the miniblock it will displace relative to the big block by local elastic deformation of the big block (see Appendix B).

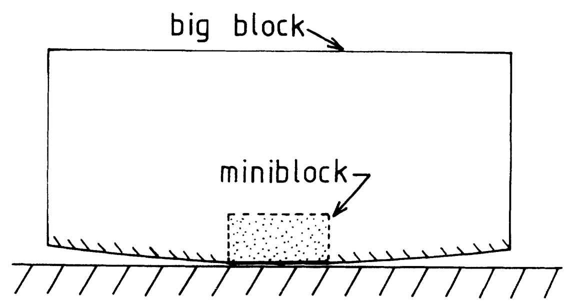

FIG. 16. A mica block on a mica surface. The mica block has a slightly curved surface with a single contact junction with the substrate. The dashed volume element defines the miniblock.

I assume that the motion of the miniblock is overdamped (i.e., that inertia effects are negligible). That is, the miniblock first deforms elastically until the critical surface stress σ_a has been reached at which point the lubrication film fluidizes. During a very short-time period, of order $d^*/c\sim 10^{-8}$ s, the miniblock reaches its maximum velocity v_0 of order v_a, after which the velocity monotonically decreases (overdamped motion) while the velocity of the big block increases. The average velocity $v(t)$ of the surface $z=d$ of the big block (which corresponds to the spring velocity v_s^* of the miniblock) and the velocity of the miniblock will coincide after some short time τ; I denote this velocity by v^*. I assume that v^* is above the critical speed v_c. Hence the miniblock will not return to its pinned state but continue to slide at a velocity close to $v(t)$.

To study the motion of the big block, I treat it as a rigid block with mass M and assume that a spring with force constant k_s is connected to it as indicated in Fig. 3. The free end of the spring is assumed to move with the constant velocity v_s. Let $x(t)$ be the position coordinate of the block at time t. For $v_s<v_c$ the block is either in a pinned state relative to the substrate (i.e., $\dot{x}=0$), in which case the friction force is $F_f\le F_a$, or else in a sliding state where the friction force equals $F_c<F_a$. We assume that the sliding velocity is so low that we can neglect the velocity dependence of the kinetic friction force. With $l=F_a/k_s$ the equation of motion for the big block takes the form

$$M\ddot{x}=k_s(l+v_st-x)-F_f-M\gamma\dot{x} ,$$

where

$$F_f\le F_a \quad \text{if } \dot{x}=0 ,$$

$$F_f=F_c \quad \text{if } \dot{x}>v_c .$$

If we measure time in units of $(M/k_s)^{1/2}$, distance in units of F_a/k_s, velocity v in units of $F_a(Mk_s)^{-1/2}$, friction force F_f in units of F_a, and γ in units of $(M/k_s)^{-1/2}$, then

$$\ddot{x}=1+v_st-x-F_f-\gamma\dot{x} . \tag{26}$$

In the measurements of Yoshizawa and Israelachvili, $M\approx 0.02$ kg and $k_s\approx 500N$, so that the units of time, distance, velocity, and γ become ~ 0.01 s, 10^{-5} m, 10^{-3} m/s, and 100 s^{-1}, respectively. The initial conditions can be determined as follows. After fluidication has occurred, the miniblock will accelerate almost instantaneously to the velocity $v_0 \sim v_a$ and then retard, so that after a short-time interval τ it reaches the velocity v^*. Since $M \gg M^*$ the big block will not move very much during the short time τ, and we can set $x(\tau)=0$. However, the big block has picked up some momentum (impulse) which can be determined from the equation of motion of the miniblock. The coordinate $q(t)$ of the miniblock is obtained from the corresponding Newton's equations, with the initial conditions $q(0)=0$ and $\dot{q}(0)=v_0$. The momentum $M\dot{x}(\tau)$ of the big block, to be used together with $x(\tau)=0$ as initial conditions for the motion of the big block, is obtained from $q(t)$ by integrating the spring force from $t=0$ to $t=\tau$:

$$M\dot{x}(\tau)=Mv^* \approx \int_0^\tau dt\, k_s^*[l^*-q(t)] \; .$$

It is easy to solve (26) to obtain the coordinate $x(t)$ and the spring force during the slip:

$$x(t)=a+v_s t+\frac{v^*-v_s}{\lambda_+-\lambda_-}(e^{-\lambda_- t}-e^{-\lambda_+ t})$$
$$-\frac{a}{\lambda_+-\lambda_-}(\lambda_+ e^{-\lambda_- t}-\lambda_- e^{-\lambda_+ t}) \; ,$$
$$F_s=1+v_s t-x(t) \; ,$$

where $a=1-F_c-\gamma v_s$ and $\lambda_\pm=\gamma/2\pm(\gamma^2/4-1)^{1/2}$. In Fig. 17, I show the spring force $F_s=1+v_s t-x(t)$ when $F_c=\frac{1}{2}$, $v_c=0.005$ (or in coordinates with dimension, $v_c \approx 1$ μm/s), $v_s=0.8v_c$, $v^*=0.1$, and $\gamma=3$.

The time dependence of the spring force found above differs in three important ways from that in the undamped case studied in Sec. III B: (a) At the beginning of the slip, the spring force has a sharp falloff (linear in time), followed by a slower decay. (b) Smooth sliding occurs when the spring velocity v_s is above the critical velocity v_c. (c) The amplitude of the variation of the spring force during a stick-and-slip cycle is F_a-F_c rather than $2(F_a-F_c)$, as observed in the undamped case.

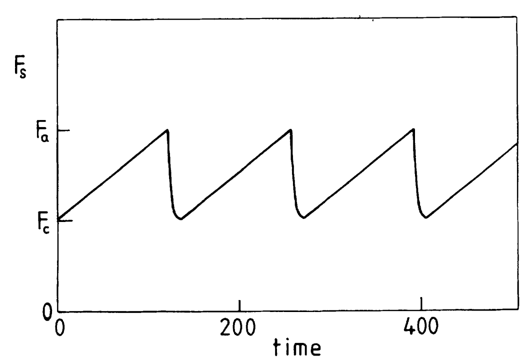

FIG. 17. The spring force F_s as a function of time for overdamped motion of the big block. The static and kinetic friction forces are denoted by F_a and F_c, respectively.

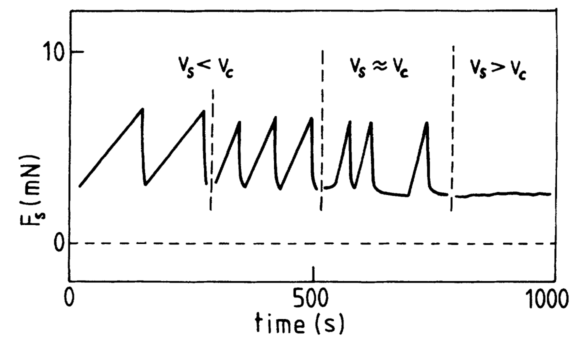

FIG. 18. The spring force as a function of time for a hexadecane film between two mica surfaces. Above some fairly sharply defined critical velocity $v_c \approx 0.4$ μm/s, the stick-slip spikes disappear and the sliding occurs smoothly. From Ref. 6.

These three attributes are exactly what are observed in the experiments by Yoshizawa and Israelachvili.

In Fig. 18 I show a typical result of a sliding friction measurement of Yoshizawa and Israelachvili. For $v > v_c$ smooth sliding occurs, while stick-and-slip motion occurs for $v < v_c$. The shape of the spring force during slip in Fig. 17 is similar to that in Fig. 18.

IV. SUMMARY AND CONCLUSION

I have presented a simple model study of the motion of an elastic block on a substrate with a layer of lubrication molecules. The study illustrates how the surface stress generated by the lubrication layer at the block-substrate interface is transmitted to the upper surface by the block. During "starting," initially an infinitesimally thin layer at the bottom surface of the block starts to move with a finite velocity. Similarly, during "stopping", initially an infinitesimally thin layer at the bottom surface of the block stops moving. This implies that there are no "problems" associated with inertia effects in "starting" or "stopping." I have also shown that stick-and-slip motion can have two different origins depending on the nature of the sliding system.

The discussion in this paper has been based on the assumption that the adsorbate layer can be in two different states, a solid pinned state and a fluidized flowing state, and that during a stick-and-slip cycle the adsorbate system oscillates between these two states. However, sometimes the lubrication molecules bind so strongly to the surfaces that no fluidization can occur. This seems to be the case with fatty acid molecules which are often used as boundary lubrications. But even in this case one would expect the $\sigma=f(v)$ curve to have the schematic form shown in Fig. 5 (but the static and kinetic stresses σ_a and σ_c may now have very similar magnitudes). Hence the discussion presented in this paper about the role of elasticity in boundary lubrication should be very general.

Finally, let me point out that the model studied in this paper may need extensions in some cases. First, I have assumed that $\sigma=f(v)$ relates the stress $\sigma(t)$ to the velocity $v(t)$ at the same time. This assumes that the important relaxation processes in the lubrication film occur rapidly compared with the typical times involved in the sliding processes (e.g. the "slip time" during a stick-and-slip cycle). This assumption may not always be true, since molecules on a surface may exhibit large rearrangement barriers which imply long relaxation times. Hence, in a more general case one may have to replace the local (in time) relation between v and σ with a more general nonlocal relation,[20] where the velocity at time t depends on the stress at earlier times, $t' \leq t$. This extension of the theory may be particularly important when the external force acting on the sliding system is oscilling in time, as in the experiments by Granick[9] and Reiter *et al.*[10]

Another extension, which may be important in some practical cases, is to include elastic coupling between the miniblocks (see Sec. III B) as in the study by Carlson and Langer.[21] It would be interesting to repeat their model study using the relation $\sigma = f(v)$ discussed in Sec. II A instead of the quite arbitrary friction law they used.

APPENDIX A

In this appendix, I show how equations of motion for an elastic block go over into those of a rigid block as the sound velocity c (or, equivalently, the Young's modulus E) goes to infinity. As in Sec. II, we assume that the thickness d of the block is small compared with its extent in the x and y directions, and we can therefore neglect the modification of the displacement field close to the vertical surfaces of the block. Hence the displacement u in the x direction of a material point in the block will depend only on z and t, i.e., $u = u(z,t)$, and satisfy the wave equation

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial z^2} = 0 ,$$

where c is the transverse sound velocity. The tangential stress is given by

$$\sigma = \frac{1}{\kappa} \frac{\partial u}{\partial z} ,$$

where $1/\kappa = \rho c^2$, ρ being the mass density.

Assume that a spring is connected to the block (see Fig. 2) and that the free end of the spring moves with the velocity v_s. The block will not start to move until the stress at the $z=d$ surface of the block reaches the critical value σ_a, at which point the lubrication layer fluidizes and the motion starts. As discussed in Sec. II, the initial motion consists of an elastic wave propagating with the (transverse) sound velocity c from the $z=d$ surface toward the $z=0$ surface. We now consider the motion of the block at an arbitrary time t during the slip. The motion of the block can be considered to result from elastic waves in the block, and it turns out to be enough to consider elastic waves with sharp steplike wave fronts. Let us call the time period $2d/c$ it takes for an elastic wave to propagate from the surface $z=0$ to $z=d$ and back again to $z=0$ for a "cycle."

Let t be an arbitrary but fixed time, and consider the time interval from t to $t + d/c$. Assume that the displacement field has the form (see Fig. 19)

$$u = u_0 - \kappa\sigma_0 z + v_0(t+\tau) , \quad 0 < z < d - c\tau ,$$

$$u = \bar{u}_0 - \kappa\bar{\sigma}_0 z + \bar{v}_0(t+\tau) , \quad d - c\tau < z < d ,$$

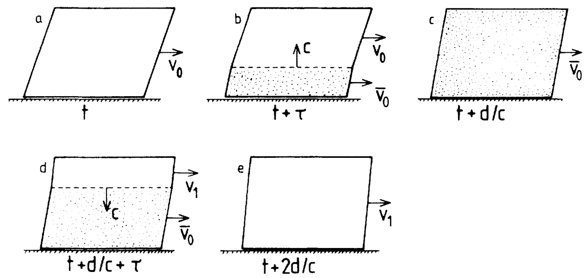

FIG. 19. Elastic wave propagation in a block sliding on a substrate. d is the thickness of the block, and c the transverse sound velocity.

where (u_0, σ_0, v_0) and $(\bar{u}_0, \bar{\sigma}_0, \bar{v}_0)$ are constants, which must be chosen so that u is continuous at $z = d - c\tau$, i.e., so that

$$u_0 - \kappa\sigma_0 z + v_0[t + (d-z)/c] = \bar{u}_0 - \kappa\bar{\sigma}_0 z + v_0[t + (d-z)/c]$$

or

$$u_0 + v_0(t + d/c) = \bar{u}_0 + \bar{v}_0(t + d/c) , \tag{A1}$$

$$\kappa\sigma_0 + v_0/c = \kappa\bar{\sigma}_0 + \bar{v}_0/c . \tag{A2}$$

The displacement field given above satisfies the wave equation and is continuous everywhere. Now, at the surface $z=d$, the stress and velocity must obey the relation $\sigma = f(v)$; see Fig. 5. Since in the present case $\sigma = \bar{\sigma}_0$ and $v = \bar{v}_0$ for $z = d$, this implies

$$\bar{\sigma}_0 = f(\bar{v}_0) . \tag{A3}$$

Next, consider the time interval from $t + d/c$ to $t + 2d/c$, where (see Fig. 19)

$$u = u_1 - \kappa\sigma_1 z + v_1(t + d/c + \tau) , \quad 0 < z < c\tau$$

$$\bar{u} = \bar{u}_0 - \kappa\bar{\sigma}_0 z + \bar{v}_0(t + d/c + \tau) , \quad c\tau < z < d .$$

The condition that u is continuous for $z = c\tau$ implies

$$u_1 - \kappa\sigma_1 z + v_1(t + d/c + z/c) = \bar{u}_0 - \kappa\bar{\sigma}_0 z + \bar{v}_0(t + d/c + z/c)$$

or

$$u_1 + v_1(t + d/c) = \bar{u}_0 + \bar{v}_0(t + d/c) , \tag{A4}$$

$$\kappa\sigma_1 - v_1/c = \kappa\bar{\sigma}_0 - \bar{v}_0/c . \tag{A5}$$

Now, note that the time $2d/c$ of a "cycle" is very short (we are interested in the $c \to \infty$ limit). Hence we can consider the spring force F_s as constant during a "cycle." The spring will act with a surface stress $\sigma_s = F_s/A$ on the surface $z=0$, which we take to be

$$\sigma_a \left[1 + \frac{v_s t}{l} - \frac{u_0 + v_0 t}{l} \right]$$

during the time interval from $t - d/c$ to $t + d/c$. But during this time interval, the surface stress equals σ_0, so that we obtain

$$\sigma_0=\sigma_a\left[1+\frac{v_st}{l}-\frac{u_0+v_0t}{l}\right] .$$

Similarly, during the time interval from $t+d/c$ to $t+3d/c$, we take the spring force to be

$$\sigma_a\left[1+\frac{v_s(t+2d/c)}{l}-\frac{u_1+v_1(t+2d/c)}{l}\right] ,$$

which must equal σ_1. Hence

$$\sigma_1-\sigma_0=\sigma_a\left[\frac{v_s2d/c}{l}-\frac{u_1-u_0+(v_1-v_0)t+v_12d/c}{l}\right] . \tag{A6}$$

Now the time period $\epsilon=2d/c\to 0$ as $c\to\infty$ and (u_0,σ_0,v_0) must deviate less and less from (u_1,σ_1,v_1) as $c\to\infty$. Hence we take the continuum limit by introducing $[u(t),\sigma(t),v(t)]$, which takes the value (u_0,σ_0,v_0) at time t, (u_1,σ_1,v_1) at time $t+\epsilon$, and so on. Using

$$\sigma_1-\sigma_0\approx\dot\sigma(t)2d/c ,$$

$$u_1-u_0\approx\dot u(t)2d/c ,$$

$$v_1-v_0\approx\dot v(t)2d/c ,$$

we obtain, from (A6),

$$\dot\sigma=\sigma_a\left[\frac{v_s}{l}-\frac{\dot u+\dot v t+v}{l}\right] . \tag{A7}$$

Next, using (A1) and (A4) gives

$$u_0+v_0(t+d/c)=u_1+v_1(t+d/c)$$

or

$$\dot u+\dot v t=0 \tag{A8}$$

to leading order in $2d/c$. Using (A7) and (A8) gives

$$\dot\sigma=\sigma_a(v_s-v)/l . \tag{A9}$$

Adding (A2) and (A5) gives

$$\kappa(\sigma_0+\sigma_1)-(v_1-v_0)/c=2\kappa\bar\sigma_0 ,$$

or, using $\rho c^2=1/\kappa$,

$$\bar\sigma_0=\sigma-\rho d\dot v \tag{A10}$$

to leading order in $2d/c$. Similarly, subtracting (A5) from (A2) gives

$$\kappa(\sigma_0-\sigma_a)+(v_0+v_1)/c=2\bar v_0/c ,$$

or, using $\kappa\sim 1/c^2$,

$$\bar v_0=v \tag{A11}$$

to leading order in $2d/c$. Substituting (A10) and (A11) into (A3) gives

$$\sigma-\rho d\dot v=f(v) . \tag{A12}$$

Next, note that

$$u(z,t)=u(t)-\kappa\sigma(t)z+v(t)t ,$$

and hence

$$\frac{\partial u}{\partial t}(z,t)=\dot u-\kappa\dot\sigma z+\dot v t+v .$$

Using (A8), this gives

$$\frac{\partial u}{\partial t}(z,t)=v \tag{A13}$$

to leading order in $2d/c$. Substituting this into (A9) and integrating with respect to time gives

$$\sigma=\sigma_a[l+v_st-u(z,t)]/l .$$

Finally, substituting this into (A12) gives

$$\rho d\dot v=\sigma_a[l+v_st-u(z,t)]/l-f(v) . \tag{A14}$$

Since in the limit $c\to\infty$ the elastic displacement term $-\kappa\sigma z$ in $u(z,t)$ vanishes, Eq. (A14) is just Newton's equation of motion for the center of mass of a rigid block. In particular, introducing the mass $M=\rho dA$ and denoting the center-of-mass position by $x(t)$, (A14) takes the form

$$M\ddot x=F_a(1+v_st/l-x/l)-F_f(\dot x) ,$$

where $F_a=A\sigma_a$ and $F_f=Af(\dot x)$.

APPENDIX B

Assume that isotropic elastic media occupy the half space $z>0$. On the surface $z=0$ I assume that a tangential stress acts within a small circular region with radius R that is centered at a material point P. This will give rise to an elastic deformation of the solid, and P will displace parallel to the surface by the amount (see, e.g., Ref. 22)

$$u=\frac{1+\nu}{2\pi E}\sigma\int_{r<R}d^2x\frac{1}{r}\left[2(1-\nu)+2\nu\frac{x^2}{r^2}\right] ,$$

where $r^2=x^2+y^2$, and where ν is Poisson's ratio and E Young's modulus. The integral is trivial to perform, giving

$$u=(2+\nu-\nu^2)\sigma R/E=(1-\nu/2)(\sigma R/\rho c^2) ,$$

where c is the transverse sound velocity. If we define a force constant by $k_s^*u=F$, where $F=\pi R^2\sigma$ is the total force, then

$$k_s^*=\pi\rho c^2R/(1-\nu/2)=2\rho c^2D^* ,$$

where $D^*=(\pi/2)R/(1-\nu/2)$. For metals, $\nu\approx 0.3$, so that $D^*\approx 2R$.

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[11]Actually, the velocity v of the bottom surface of the sliding block is not identical to the drift velocity of the adsorbate layer. For example, if the block and the substrate are made of identical material, e.g., mica, then the drift velocity of the adsorbate layer will be half the velocity of the bottom surface of the block. This follows directly from the symmetry: In a reference frame where the substrate moves with the velocity $-v/2$, and the block with the velocity $v/2$, the drift velocity of the adsorbate layer must, by symmetry, vanish. In general there will be some scaling factor κ (between zero and one) relating the velocity v(block) of the bottom surface of the block to the drift velocity v(ads.) of the adsorbate layer, $v(\text{ads.})=\kappa v(\text{block})$. However, this linear transformation will not change the general form of the relation $\sigma=f(v)$.

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[14]I assume that initially a "small" island is pinned only by one of the two sliding surfaces. This assumption is plausible if the two surfaces are of different material giving rise to different pinning potentials, but it may also hold for identical materials, e.g., mica, if (as is always the case in practice) the two lattices are rotated relative to each other.

[15]In Fig. 5 the upper horizontal dashed line indicates a sharp transition from a 2D-fluid state to a 2D incommensurate (IC) solid state, which has been observed in molecular dynamics simulations [B. N. J. Persson (unpublished)]. This transition also occurs in Fig. 4(b), but because of the small unit cell (and relative high temperature) used in the simulations on which this figure is based, the transition is smeared out. Note that the IC-solid phase slides on the surface just as if no surface corrugation occurred (dash-dotted line). For sliding involving nonconducting surfaces, e.g., mica, where the friction coefficient η must have its origin in phonon emission (i.e., no electron-hole pair creation), it is very likely that the IC-solid sliding state constitutes a so-called "superlubric" state, where the friction vanishes. The origin of this is as follows: In the simulations I performed on each adsorbate, the substrate-induced friction force $-M\eta\dot{\mathbf{r}}$ acts independently of the nature of the motion of all the other adsorbates. This is likely to be a very good approximation in the 2D-fluid state, where the motion of the adsorbates is relatively incoherent. But in the IC-solid state this is no longer the case: here the whole adsorbate layer slides in a coherent manner over the substrate and this gives rise to a coherent emission of substrate phonons along the surface. However, as shown by Aubry and Sokoloff [J. B. Sokoloff, Phys. Rev. B **42**, 760 (1990)] [see also K. Shinko and M. Hirano, Surf. Sci. **283**, 473 (1993)], for an IC sliding layer the phonons emitted will undergo destructive interference and the net result is that no phonon energy at all is radiated into the bulk. Hence the sliding friction is likely to vanish in this sliding state. Molecular-dynamics simulations have shown, however, that even a low concentration of surface imperfections will "kill" the incommensurate sliding state and convert it into a 2D-fluid state [B.N.J. Persson (unpublished)]. For sliding on metallic surfaces, the electronic friction is always nonzero and no "superlubricating" state is possible.

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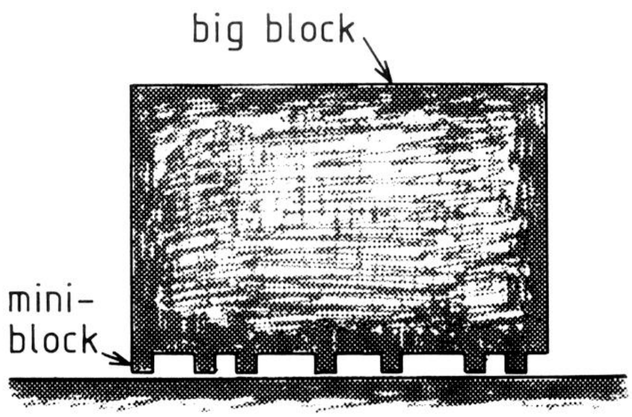

FIG. 13. A block on a substrate. The miniblocks represent the surface roughness of the block which contact the substrate.