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Conductance fluctuations in mesoscopic disordered systems for arbitrary scattering potentials

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We prove that the universality of conductance fluctuations in mesoscopic, weakly disordered systems, so far demonstrated for a contact scattering potential, is preserved in the case of an arbitrary potential. This is so at least to first order in the disorder and as long as the electron lifetime and the transport time are well-defined quantities. We also prove that the conductance fluctuations induced by the motion of a single impurity in two dimensions are not as important and universal as if one changes the entire impurity distribution, in contrast to a recent claim. These conductance fluctuations explicitly depend on the degree of disorder and on the characteristics of the system.

I. INTRODUCTION

The universality of conductance fluctuations in weakly disordered systems of mesoscopic sizes has been put forward by various authors.[1] It was shown that the classical result according to which the variance of the dimensionless conductance behaves as L^{D-4} (L is the linear dimension of the system, D the dimensionality), is modified by quantum-mechanical effects and becomes of order 1. In other words, the conductance fluctuations are independent of the degree of disorder, size, and dimensionality of the system. Such a result was demonstrated in the case of independent impurities, and for an electron impurity scattering potential of the contact type.[1]

We recently[2] examined whether such a result survives in the case of interacting impurities. Indeed weak-localization experiments performed at low temperatures first require quenching the system from a high temperature T_0 so that the impurities are frozen at random. If T_0 could be infinite, the impurities would indeed be completely disordered at random, and be independent. However, in practice, T_0 is always finite. Therefore, a local positional ordering among the impurities always subsists in realistic systems. This transforms the scattering potential into an effective one,[2] depending on the interactions between the impurities and on the scattering angle, even if it was of a contact type to begin with. Two important results were thus obtained:[2] (a) for a weak local impurity ordering and to first order in the electronic disorder, the universality of the conductance fluctuations is preserved because, on a large scale, the conductance fluctuations are insensitive to such a weak local impurity ordering; and (b) on the other hand, moving a single impurity away induces nonuniversal conductance fluctuations in two dimensions, in contrast to what was claimed in Ref. 3 for the independent impurity case. Indeed this is a local perturbation which is certainly sensitive to the local positional ordering of the impurities.

In the present paper, we confine ourselves to the independent impurity case (for simplicity). The results emphasized in the present paper are the following.

(i) We prove that conductance fluctuations are universal not only for a contact scattering potential, but also for any momentum-dependent potential, in the weakly localized regime, and as long as the electron lifetime τ and transport time τ_{tr} are well defined. Actually the interacting impurity case appears as a particular example of this more general one, since, as we stressed above, the effective potential induced by the local impurity ordering is a particular form of momentum (or scattering angle) dependence for the potential. We recently showed[4] that the electrical conductivity of a weakly disordered system may be computed without any knowledge of the particular form of the potential as long as τ and τ_{tr} are well defined. The precise form of the potential is only needed to compute τ and τ_{tr}. The integrability of the potential mentioned in Ref. 4 is actually not necessary. A singular potential[5] may yield well-defined τ and τ_{tr} under certain conditions. Therefore the existence of τ and τ_{tr} is the only necessary condition, in the present paper, for our results to hold.

(ii) On the other hand, and under the same conditions, we examine the extra conductance fluctuations induced when one moves a single impurity away. Here too, as in the interacting impurity problem of Ref. 2, we prove that the extra conductance fluctuations induced that way are not universal in two dimensions. Actually, as stressed above, the effective potential resulting from interactions

between the impurities in Ref. 2 becomes momentum dependent and thus represents a particular example of the arbitrary potentials studied here, i.e., they can all be considered as finite range scattering potentials. It is thus reasonable to expect that the local perturbation induced by the motion of a single impurity is sensitive to such a finite range potential, i.e., to the characteristics of the problem. In other words, the claim made in Ref. 3 that conductance fluctuations induced by the motion of a single impurity are, in two dimensions, as strong and universal as if changing the entire impurity distribution holds only in the particular case of independent impurities *and* a contact scattering potential, and if an extrapolation to strong disorder can be made. In realistic systems, and in the weakly localized regime of interest here as well as in Ref. 3, this claim does not hold. As in Refs. 2, 4, and 5, we use the Born approximation.

II. THE CONDUCTANCE FLUCTUATIONS FOR INDEPENDENT IMPURITIES AND A CONTACT POTENTIAL, RECALLED (THE SUBSCRIPT 0 WILL REFER TO THIS CASE)

Quantum-mechanical effects amount[1] to multiplying the classical value of the variance of the dimensionless conductance, $\mathrm{var}(g_0)=\langle g_0^2\rangle-\langle g_0\rangle^2$, by an integral involving the two types of diffuson (or Cooperon) diagrams displayed in Figs. 1(a) and 1(b), yielding

$$\mathrm{var}(g_0)\sim L^{D-4}\int_{L^{-1}}^{q_0}(q^2)^{-2}d^D\mathbf{q}\ . \tag{1}$$

The conductance G_0 is related to g_0 by $G_0=(e^2/h)g_0$, (e is the charge of the electron, and h the Planck constant). We recall that a diffuson (Cooperon) is an infinite impurity scattering ladder in the particle-hole (particle-particle) channel, obeying the Bether-Salpeter equation corresponding to Fig. 1(c). Reference 6 showed that diagrams with only two diffusons (or Cooperons) are to be considered. In that case the two-Cooperon diagram is deduced from the two-diffuson one just by turning one of the loops by π around a vertical axis in the plane of the figure, so that they make identical contributions. Therefore, in the following, we will speak only in terms of two diffusons. In Eq. (1), q is the momentum common to the two diffusons. q_0 is an upper cutoff given in the case of independent impurities and of a contact scattering potential by

$$q_0=l_0^{-1}\ , \tag{2}$$

where the elastic mean free path is $l_0=k_F\tau_0$ (in atomic units), and k_F is the Fermi momentum. l_0, in this particular case, is altogether the elastic lifetime and the transport time. We recall that it is the lower limit in the integral (1) which essentially matters: the diffuson (or the Cooperon) propagator diverges for vanishing q. Therefore the quantum corrections will play a role only in the neighborhood of $q\sim L^{-1}$. At $q=q_0$ one obtains only a few corrections. Indeed, performing the integral in (1) yields

$$\mathrm{var}(g_0)\sim L^{D-4}\frac{1}{D-4}[q_0^{D-4}-L^{4-D}]$$
$$\sim\frac{1}{4-D}\left[1-\frac{1}{(Lq_0)^{4-D}}\right]\ . \tag{3}$$

The usual conclusion is that

$$\mathrm{var}(g_0)\sim 1\ ,\quad \mathrm{var}(G_0)\sim\left[\frac{e^2}{h}\right]^2\ \text{with}\ Lq_0\gg 1\ , \tag{4}$$

i.e., the conductance fluctuations are universal. Here we wish to make a few remarks concerning the condition $Lq_0\gg 1$ involved in (4). In this weak-localization regime, the disorder is assumed to be weak, but strong enough for q_0^{-1} to be much smaller than L. On the other hand, L itself is not very large since the sample is of mesoscopic dimension. Therefore, suppose for one moment that the condition $Lq_0\gg 1$ is *not* fulfilled: then two remarks must be made.

(i) In dimensions $D<4$, one obtains deviations from universality when Lq_0, although larger than 1, is not extremely large. This could happen, for instance, when the localization is weak enough so that one is close to the ballistic regime. Note that those deviations will be the strongest in $D=3$ (due to the power $4-D$).

(ii) In dimensions $D\geq 4$ (although such a case is not very physical), (3) is evidently not universal, and reads approximately

$$\mathrm{vag}(g_0)\sim(Lq_0)^{D-4}\ ,\quad Lq_0\gg 1\ . \tag{5a}$$

Note that, strictly at four dimensions,

$$\mathrm{vag}(g_0)\sim\ln(Lq_0)\ ,\quad \text{whatever}\ (Lq_0)\ . \tag{5b}$$

In other words, at four dimensions and above, $\mathrm{var}(g_0)$ increases with L.

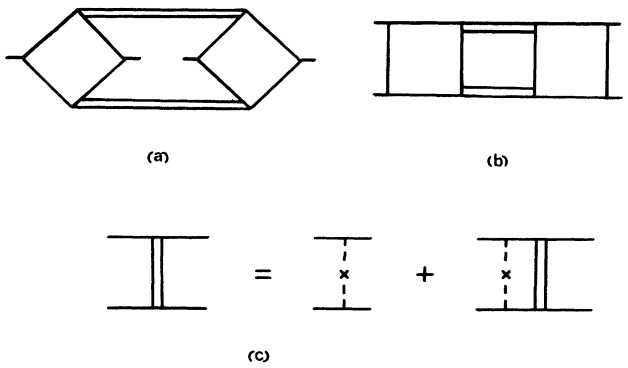

FIG. 1. (a) and (b) Two types of conductance fluctuation diagram, according to Ref. 6. The two conductance loops are linked with two diffusons (or Cooperons) obeying the Bethe-Salpeter equation displayed in (c). The dotted line with a cross denotes a one-impurity scattering.

III. THE CONDUCTANCE FLUCTUATIONS FOR AN ARBITRARY SCATTERING POTENTIAL

In Sec. II we recalled what is known in the literature. In this section we examine a more general case in which

$V(Q)$, the Fourier transform of the scattering potential, depends explicitly on Q. In that case and to first order in the electron disorder, the two diffuson diagrams contributing to the conductance fluctuations are those of Figs. 2(a)–2(f). The extra diffusons appearing as vertex corrections of the external lines obey the Bethe-Salpeter equation of Fig. 2(g). A dotted line with a cross indicates an extra single, impurity line. Such an extra line may link any two opposite sides of each Hikami-type box (the squares) in all possible ways. Therefore there are actually four-diagrams of the type described in Figs. 2(b), 2(c), 2(e), and 2(f).

Since $V(Q)$ depends on the momentum transfer $Q=|\mathbf{k}'-\mathbf{k}|$ (with $\mathbf{k}$ and $\mathbf{k}'$ the electron momenta before and after collision on the impurity), τ and $\tau_{\rm tr}$ are therefore different. Writing $Q^2=|\mathbf{k}'-\mathbf{k}|^2\approx 2k_F^2\,(1-\cos\theta)$, where θ is the scattering angle between $\mathbf{k}$ and $\mathbf{k}'$, we have, within the Born approximation and with $V(Q)\equiv V(\cos\theta)$,

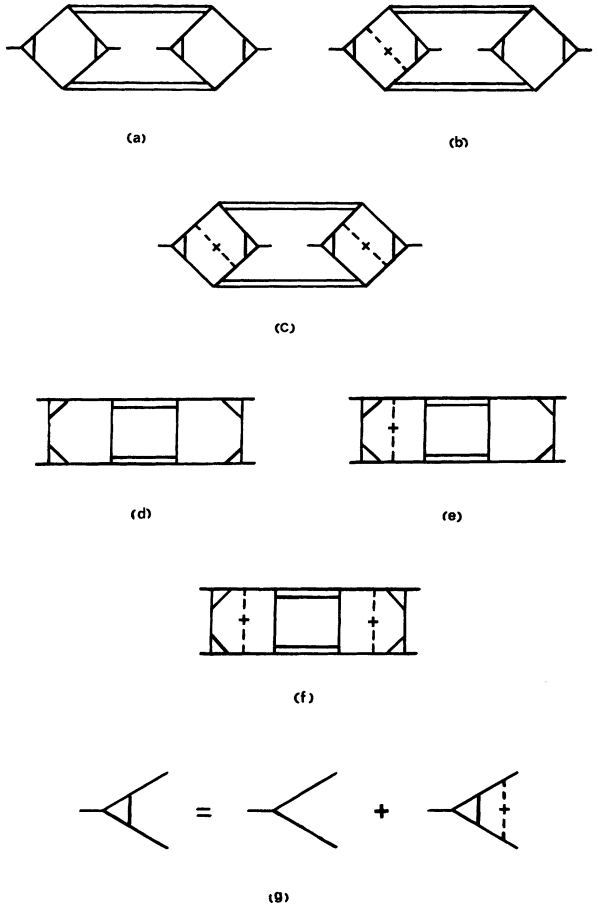

FIG. 2. (a)–(f) Conductance fluctuation diagrams in the general case of an arbitrary electron-impurity scattering potential. The external line vertices are renormalized by diffusons obeying the Bethe-Salpeter equation displayed diagrammatically in (g). The extra impurity lines may link any two opposite sides of the conductance "boxes." There are thus four diagrams like (b), as well as for (c), (e), and (f) (they are not all shown). These are the only diagrams relevant to first order in the electronic disorder.

$$\tau^{-1}=2\pi N(0)n_I\int V^2(\cos\theta)d\Omega\Big/\int d\Omega\,,$$
$$\tau_{\rm tr}^{-1}=2\pi N(0)n_I\int(1-\cos\theta)V^2(\cos\theta)d\Omega\Big/\int d\Omega\,. \tag{6}$$

$d\Omega$ is the angular element of integration. $N(0)$ is the density of states at the Fermi energy, n_I is the impurity density. We use the following results from Ref. 4.

(i) Each of the two diffusons Γ behaves, at vanishing frequency and for small momentum q,

$$\Gamma=\frac{1}{2\pi N(0)\tau^2}\frac{1}{D_{\rm tr}q^2}\,. \tag{7}$$

The diffusion coefficient $D_{\rm tr}$ is given in atomic units by

$$D_{\rm tr}=k_F^2\tau_{\rm tr}/D\,. \tag{8}$$

As recalled in Ref. 4, the divergence of Γ when $q\to 0$ (at zero frequency) follows from the Ward identity, insuring the conservation of total number of particles.[7]

(ii) The upper cutoff on q is given (with $l_{\rm tr}=k_F\tau_{\rm tr}$) by

$$q_0=\min\left[\frac{1}{l'}\frac{1}{\sqrt{ll_{\rm tr}}}\right] \tag{9}$$

instead of (2), which was only valid for a contact potential for which $l\equiv l_{\rm tr}$.

(iii) Each of the diffuson vertices renormalizing the points connected to the various external lines introduces a factor $(\tau_{\rm tr}/\tau)$ for vanishing external frequency.

With all these modifications compared to the standard case of Ref. 1, we compute the diagrams of Fig. 2. But before performing any further calculation, we already know that the results will depend on both τ and $\tau_{\rm tr}$.

Denoting by $K_{(a)}$, $K_{(b)}$, and $K_{(c)}$ the contributions of the diagrams in Figs. 2(a)–2(c) we show that the sum of these contributions remains unchanged compared to the standard case. The calculation is straightforward. We use the fact that the extra single-impurity lines, combined with the scalar product of two momenta characteristic of a current-current correlation function, introduce factors such as $\int\cos\theta V^2(\cos\theta)d\Omega$, which can be expressed in terms of only τ and $\tau_{\rm tr}$, as being proportional to $(\tau^{-1}-\tau_{\rm tr}^{-1})$. Below we indicate only the extra factors which, for each diagram, represent a modification compared to the standard case of independent impurities and a contact potential. Thus we obtain

$$\begin{aligned}
K_{(a)}&\propto\left[\frac{\tau_{\rm tr}}{\tau}\right]^4\frac{1}{(\tau^2\tau_{\rm tr})^2}\tau^6=\left[\frac{\tau_{\rm tr}}{\tau}\right]^2,\\
4K_{(b)}&\propto -2\left[\frac{\tau_{\rm tr}}{\tau}\right]^4\frac{1}{(\tau^2\tau_{\rm tr})^2}\tau^7\left[\frac{1}{\tau}-\frac{1}{\tau_{\rm tr}}\right]\\
&=-2\left[\frac{\tau_{\rm tr}}{\tau}\right]^2\left[1-\frac{\tau}{\tau_{\rm tr}}\right],\\
4K_{(c)}&\propto\left[\frac{\tau_{\rm tr}}{\tau}\right]^4\frac{1}{(\tau^2\tau_{\rm tr})^2}\tau^8\left[\frac{1}{\tau}-\frac{1}{\tau_{\rm tr}}\right]^2\\
&=\left[\frac{\tau_{\rm tr}}{\tau}\right]^2\left[1-\frac{\tau}{\tau_{\rm tr}}\right]^2,
\end{aligned} \tag{10}$$

In (10) the factors $(\tau_{tr}/\tau)^4$ come from the four diffusons renormalizing the external vertices. τ^6, τ^7, and τ^8 come from the integrals in the complex plane of the electron Green's functions involved in each diagram. $1/(\tau^{-1}-\tau_{tr}^{-1})$ to the power 1 or 2 denotes the extra one or two single-impurity lines combined with the scalar product of the two momenta mentioned above. From (10), it turns out that

$$K_{(a)}+K_{(b)}+K_{(c)} \propto \left[\frac{\tau_{tr}}{\tau}\right]^2 \left[1-\left[1-\frac{\tau}{\tau_{tr}}\right]\right]^2 = 1 . \quad (11)$$

In other words, the sum of the contributions of the diagrams in Figs. 2(a)–2(c) is unchanged compared to the contact potential case of Ref. 1. A similar calculation shows that the sum of the contributions of the diagrams in Figs. 2(d)–2(f) is also unchanged.

The diagrams displayed in Fig. 2 are the most general ones contributing to the conductance fluctuations for an arbitrary potential $V(Q)$, to first order in the disorder. Therefore the only remaining difference from the standard case[1] is the cutoff q_0. Thus here we obtain the same result as that in formula (3) but with q_0 given by (9) instead of (2):

$$\mathrm{var}(g) \sim \frac{1}{4-D}\left[1-\left[\frac{\sqrt{l l_{tr}}}{L}\right]^{4-D}\right] \quad \text{if } l_{tr} > l , \quad (12a)$$

$$\mathrm{var}(g) \sim \frac{1}{4-D}\left[1-\left[\frac{l}{L}\right]^{4-D}\right] \quad \text{if } l_{tr} < l . \quad (12b)$$

Result (12a) is particularly interesting: even if $l \ll L$, if it happens that $l_{tr} \gg l$ (although $l_{tr} < L$), the deviation from the universal value of 1 may become sensible. However, to the accuracy of such experiments, the deviation may be hard to detect.

IV. THE CONDUCTANCE FLUCTUATIONS INDUCED BY THE MOTION OF A SINGLE IMPURITY; THE CONTACT POTENTIAL CASE RECALLED

A very intriguing suggestion was made in Ref. 3, according to which moving a single impurity away from its original position induces, in two dimensions (2D), conductance fluctuations as large and universal as those obtained when one changes the entire impurity distribution. Here we recall the derivation of this result for use in our case in the Sec. V. More specifically we follow the formalism of Feng *et al.*[3] (FLS). These authors showed that moving a single impurity a distance $\delta\mathbf{r}$ away from its original position amounts to splitting one of the diffusons in two, as shown in Fig. 3. These two diffusons are thus separated by a kernel involving four electron Green's functions, and the modified potential $V'=(|\mathbf{k}'-\mathbf{k}|)$ reads[3]

$$V'^2(|\mathbf{k}'-\mathbf{k}|) = V^2(|\mathbf{k}'-\mathbf{k}|)\mathrm{Re}\{1-e^{i\delta\mathbf{r}\cdot(\mathbf{k}'-\mathbf{k})}\} , \quad (13)$$

where $\mathrm{Re}(x)$ is the real part of x. $\mathbf{k}$ and $\mathbf{k}'$ are the electron momenta before and after scattering on the (moved) impurity. Formula (13) was given in FLS and rederived in the Appendix of Ref. 2. Therefore the initial diffuson has been replaced by

$$\Gamma' = \int\int \frac{d^D\mathbf{k}'}{(2\pi)^D}\frac{d^D\mathbf{k}}{(2\pi)^D} G(\mathbf{k},\tilde{\omega}_{n+\nu})G(\mathbf{k}-\mathbf{q},\tilde{\omega}_{n'})G(\mathbf{k}',\tilde{\omega}_{n+\nu})G(\mathbf{k}'-\mathbf{q},\tilde{\omega}_{n'})V^2(|\mathbf{k}'-\mathbf{k}|)\left\{1-\frac{e^{i\delta\mathbf{r}\cdot(\mathbf{k}'-\mathbf{k})}+e^{-i\delta\mathbf{r}\cdot(\mathbf{k}'-\mathbf{k})}}{2}\right\} . \quad (14)$$

q is the (small) momentum common to the two diffusons Γ. It can be neglected in the Green's functions of (14). The $\tilde{\omega}$'s are the appropriate Matsubara frequencies of the electron lines, including the electron lifetime. The modification brought in by the motion of the impurity is therefore contained in Γ'/Γ. We make use of some algebra that we derived in Ref. 8, which shows that

$$\int \frac{d^D\mathbf{k}}{(2\pi)^D} G(k,\tilde{\omega}_{n+\nu})G(k,\tilde{\omega}_n)e^{\pm i\mathbf{k}\cdot\mathbf{R}} = 2\pi N(0)\tau M_0(k_F R) , \quad (15)$$

with

$$M_0(x) = \begin{cases} \dfrac{\sin x}{x} , & 3\mathrm{D} \\ J_0(x) , & 2\mathrm{D} \\ \cos x , & 1\mathrm{D} . \end{cases} \quad (16)$$

J_0 is the Bessel function of the first kind of index zero.[9] We have

$$M_0(x=0)=1 , \qquad M_0(x\to\infty) , \quad 2\mathrm{D},3\mathrm{D} . \quad (17)$$

(In 1D, and when $x \to \infty$, M_0 is not well defined.)

In the case of Ref. 3, the impurities were assumed to be independent, and the potential to be a contact one, i.e., $V^2(|\mathbf{k}'-\mathbf{k}|)=cst=V^2$. Then Γ' takes the form Γ'_0:

$$\Gamma'_0 = \Gamma_0^2(2\pi N(0)\tau_0)^2V^2\{1-M_0^2(k_F\delta r)\} = \Gamma_0^2(2\pi N(0)\tau_0)^2\frac{1}{2\pi N(0)n_I\tau_0}\{1-M_0^2(k_F\delta r)\} . \quad (18)$$

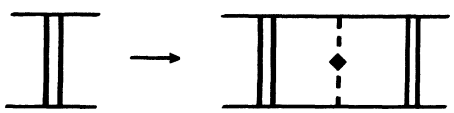

FIG. 3. One of the diffusons which is split in two by the motion of a single impurity, denoted by a black diamond, and which gives rise to the modified scattering potential of Eq. (13).

The quantity $\{1-M_0^2(k_F\delta r)\}$ is called $\alpha(k_F\delta r)$ in FLS, although these authors derived it only in 3D (with a slight mistake: see the comment in Ref. 5 of Ref. 2). $\Gamma_0=[1/(2\pi N(0)\tau_0^2)][D/(k_F^2\tau_0 q^2)]$ for the vanishing frequency of Γ_0, and $\tau_0^{-1}=2\pi N(0)n_I V^2$. In such a case FLS evaluated the diagram corresponding in the present paper to Fig. 1(a), with one Γ_0 replaced by the Γ_0' given by (18). Γ_0' can replace either of the two Γ_0's. Compared to $\mathrm{var}G_0$ obtained from (1) in the case of FLS without moving an impurity,

$$\mathrm{var}G_0 \sim \left[\frac{e^2}{h}\right]^2 L^{D-4}\int_{L^{-1}}^{q_0}\frac{q^{D-1}dq}{(q^2)^2}\sim\left[\frac{e^2}{h}\right]^2, \quad Lq_0\gg 1 , \tag{19}$$

we will call $\delta G_0'$ the conductance fluctuation obtained when moving an impurity a distance δr away:

$$\begin{aligned}(\delta G_0')^2 &\sim \left[\frac{e^2}{h}\right]^2 L^{D-4}\frac{D}{k_F^2\tau_0^2}\frac{1}{n_I}\{1-M_0^2(k_F\delta r)\}\\ &\quad\times\int_{L^{-1}}^{q_0}\frac{q^{D-1}dq}{(q^2)^3}\\ &\sim\left[\frac{e^2}{h}\right]^2\frac{L^2}{n_I l_0^2}\frac{L^2}{n_1 l_0^2}\{1-M_0^2(k_F\delta r)\} ,\end{aligned} \tag{20}$$

with $l_0=(k_F\tau_0)$ the elastic mean free path for this contact potential and the independent impurity case. Equation (20) may be rewritten as

$$(\delta G_0')^2\sim\left[\frac{e^2}{h}\right]^2\frac{L^D}{n_I l_0^D}\left[\frac{L}{l_0}\right]^{2-D}\{1-M_0^2(k_F\delta r)\} . \tag{21}$$

With $L^D\equiv$ the volume Ω, and $1-M_0^2(k_F\delta r)\equiv\alpha(k_F\delta r)$, Eq. (21) is identified with formula (3) of FLS. FLS argue that the individual impurity scatters so strongly the $(\Omega/n_I l_0)\sim 1$, and in the case where $k_F\delta r\gg 1$, $\alpha(k_F\delta r)\to 1$, so that

$$\begin{aligned}&(\delta G_0')^2\sim\left[\frac{e^2}{h}\right]^2 , \quad D=2 , \quad k_F\delta r\gg 1 ,\\ &\left[\frac{\Omega}{n_I l_0^D}\right]\sim 1 .\end{aligned} \tag{22}$$

Therefore, in 2D, $\delta G_0'$ is as strong and universal as $\mathrm{var}G_0$ given above in (19). However, in their erratum,[3] FLS emphasize that the hypothesis $(\Omega/n_I l_0^D)\sim 1$ holds only when $k_F l_0\sim 1$.

Indeed, in 2D

$$\frac{\Omega}{n_I l_0^2}=\frac{1}{n_I}\frac{L^2}{l_0^2}$$

and

$$\frac{1}{l_0}\sim\frac{n_I}{\Omega}\frac{1}{k_F}\sim\frac{n_I}{k_F L^2}$$

so that

$$\frac{\Omega}{n_I l_0^2}\sim\frac{n_I}{(k_F L)^2}\sim\frac{1}{k_F l_0} .$$

Therefore $\Omega/(n_I l_0^2)\sim 1$, if and *only if* $k_F l_0\sim 1$. This occurs only in the regime of strong disorder near the metal-insulator transition. In other words, FLS assumed that they could extrapolate their result derived in the weakly localized regime, i.e., to lowest order in $1/(k_F l_0)$, as such, to the strongly localized regime $(k_F l_0)\sim 1$. This does not appear very reasonable: when the disorder increases, higher-order terms in $1/(k_F l_0)$ become important, more diagrams are involved, and it is hard to guess the result when $k_F l_0\to 1$. We will come back to that point in Sec. V.

V. THE CONDUCTANCE FLUCTUATIONS INDUCED BY THE MOTION OF A SINGLE IMPURITY IN THE CASE OF AN ARBITRARY POTENTIAL

In this section we reexamine the modified diffuson propagator given in (14) when the Fourier transform of the scattering potential is momentum dependent, i.e., the general case studied in this paper. We first show that for an arbitrary potential even within the same (unreasonable) hypothesis $\Omega/(n_I l_0^D)\sim 1$ of FLS, the result for the corresponding quantity $\delta G'$ is modified compared to $\delta G_0'$, while, as we saw in Sec. III, $\mathrm{var}G$ is unchanged compared to $\mathrm{var}G_0$ given in (19) for $Lq_0\gg 1$. Second, releasing the unreasonable extrapolation to $k_F l_0\cong 1$, we show that $\delta G'$ is in reality much smaller than $\mathrm{var}G$ by an amount $1/(k_F l_{\mathrm{tr}})$.

In our general case $V^2(Q=|\mathbf{k}'-\mathbf{k}|)$ is the Fourier transform of some function $F(R)$:

$$V^2(Q)=\int F(R)e^{i\mathbf{Q}\cdot\mathbf{R}}d^D\mathbf{R} . \tag{23}$$

In the absence of anisotropy, $F(R)$ depends only on the magnitude of R. Inserting (23) into (14), with $\mathbf{Q}\equiv\mathbf{k}'-\mathbf{k}$ we obtain

$$\begin{aligned}\Gamma'=\Gamma^2\int d^D\mathbf{R}\,F(R)\int\int\frac{d^D\mathbf{k}}{(2\pi)^D}\frac{d^D\mathbf{k}'}{(2\pi)^D}G(\mathbf{k},\tilde{\omega}_{n+\nu})G(\mathbf{k},\tilde{\omega}_{n'})G(\mathbf{k}',\tilde{\omega}_{n+\nu})G(\mathbf{k}',\tilde{\omega}_{n'})\\ \times\{e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{R}}-\tfrac{1}{2}[e^{i(\mathbf{k}'-\mathbf{k})\cdot(\mathbf{R}+\delta\mathbf{r})}+e^{i(\mathbf{k}'-\mathbf{k})\cdot(\mathbf{R}+\delta\mathbf{r})}]\} .\end{aligned} \tag{24}$$

With (15), (24) reads

$$\Gamma'=\Gamma^2\int d^D\mathbf{R}\,F(R)(2\pi N(0)\tau)^2[M_0^2(k_FR)-\tfrac{1}{2}M_0^2(k_F|\mathbf{R}+\delta\mathbf{r}|)-\tfrac{1}{2}M_0^2(k_F|\mathbf{R}-\delta\mathbf{r}|)]\ . \tag{25}$$

Here

$$\Gamma=\frac{1}{2\pi N(0)\tau^2}\frac{D}{k_F^2\tau_{\rm tr}q^2}$$

with τ^{-1} and $\tau_{\rm tr}^{-1}$ given in (6).

In what follows, we compare (25) with (18), and more precisely its contribution to (22), when $k_F\delta r\gg1$. First, the terms depending on δr in (25) give vanishing contributions when $k_F\delta r\gg1$. This is not possible to demonstrate analytically in 2D, but can be done straightforwardly in 3D. The derivation is given in the Appendix. It is reasonable to believe that the same result also holds in 2D. Taking this for granted, we are left with only the first term in (25) or, equivalently, the first term in (13), yielding, with the help of (6),

$$\Gamma'\xrightarrow[k_F\delta r\to\infty]{}\Gamma^2[2\pi N(0)\tau]^2\frac{1}{2\pi N(0)\tau n_I}\ , \tag{26}$$

with $\Gamma=[1/(2\pi N(0)\tau_0^2)][D/(k_F^2\tau_0q^2)]$. Compared to the result of Sec. III, the extra contribution is contained in $2\Gamma'/\Gamma$. (The factor 2 comes from the fact that Γ' may replace either one of the two diffusons Γ. It is also contained in FLS. Thus, for the conductance fluctuations induced when moving a single impurity a distance $\delta r\gg1/k_F$, away, in 2D and 3D, we find

$$(\delta G')^2\sim\left[\frac{e^2}{h}\right]^2L^{D-4}\frac{D}{k_F^2\tau\tau_{\rm tr}n_I}\int_{L^{-1}}^{q_0}\frac{q^{D-1}dq}{(q^2)^3}\ ,\quad D=2,3 \tag{27a}$$

$$\sim\left[\frac{e^2}{h}\right]^2\frac{L^2}{n_Ill_{\rm tr}}\sim\left[\frac{e^2}{h}\right]^2\frac{L^2}{n_Il^D}\left[\frac{L}{l}\right]^{2-D}\frac{l}{l_{\rm tr}}\ ,\quad k_F\delta r\gg1\ . \tag{27b}$$

The 1D case is rather peculiar (see the Appendix), as it always retains a dependence in $(k_F\delta r)$ and vanishes periodically for certain values of that parameter.

The last expression in (27b) has been written under the form (21) of FLS. Within the same hypothesis used by FLS, $L^D/(n_Il^D)=\Omega/(n_Il^D)\sim1$, i.e., making the extrapolation $k_Fl\sim1$, we obtain

$$(\delta G')^2\sim\left[\frac{e^2}{h}\right]^2\left[\frac{l}{l_r}\right]\ ,\quad D=2\ ,$$
$$k_F\delta r\gg1\ ,\quad\frac{\Omega}{n_Il^D}\sim1\ , \tag{28}$$

a result evidently different from $(\delta G_0')^2\sim(e^2/h)^2$ in Eq. (22).

To summarize, comparing our results for an arbitrary potential with the result obtained for independent impurities and a contact potential, we obtain the same universal result when an impurity is *not* moved away:

$$\mathrm{var}G=\mathrm{var}G_0\sim\left[\frac{e^2}{h}\right]^2\quad(Lq_0\gg1)\ . \tag{29}$$

Instead, when *moving* an impurity a distance $\delta r\gg1/k_F$ away, we obtain a different result:

$$\left[(\delta G')^2\sim\left[\frac{e^2}{h}\right]^2\frac{l}{l_{\rm tr}}\right]\neq\left[(\delta G_0')^2\sim\left[\frac{e^2}{h}\right]^2\right]\ ,$$
$$D=2\ ,\quad\frac{\Omega}{N_Il^D}\sim1\ . \tag{30}$$

Clearly the inequality switches to an equality when $l=l_{\rm tr}=l_0$, which only happens in the case of a contact potential. In contrast, when $l_{\rm tr}\gg l_0$, $\delta G'$ may be diminished considerably. In any case, the result for $\delta G'$, although independent of L within the assumption $\Omega/(n_Il^D)\sim1$, depends on the characteristics of the system via l and $l_{\rm tr}$, in particular on the range of the potential. As a particular example, let us consider the case of Ref. 11 with a Yukawa type of scattering potential behaving like $\exp(-\gamma k_Fr)/r$. With $\gamma=2\sinh(\varphi/2)$, it was shown that $(l/l_{\rm tr})=(\tau/\tau_{\rm tr})=1-e^{-\varphi}$. Therefore when φ (or γ) decreases from infinity (the contact potential case), the potential range increases and $l/l_{\rm tr}$ deceases below 1.

In some recent experiments on GaAs-$Ga_{1-x}Al_xAs$ heterojunctions,[12] for different samples one finds $l=0.18$ μm with $l_{\rm tr}=8.7$ μm, or $l=0.07$ μm with $l_{\rm tr}=4$ μm; in both cases $l/l_{\rm tr}\approx2\times10^{-2}$, a value much smaller than 1. In earlier experiments[13] on GaAs/$Ga_{1-x}Al_xAs$ heterojunctions, for two different samples one obtains $l/l_{\rm tr}\sim1.1\times10^{-1}$ and 2.6×10^{-1}.

On the other hand, and, more importantly, as already mentioned, the extrapolation made by FLS that $\Omega/(n_Il^D)\sim1$ is questionable. If we release this assumption, it is clear from (27b) and from FLS that $\delta G'$ and $\delta G_0'$ depend on the characteristics of the system. Therefore in the weakly localized regime, using $\Omega/(n_Il^D)\sim[1/(k_Fl)^{D-1}]$, we obtain

$$(\delta G')^2\sim\left[\frac{e^2}{h}\right]^2\frac{1}{(k_Fl)^{D-1}}\left[\frac{L}{l}\right]^{2-D}\left[\frac{l}{l_{\rm tr}}\right]\ ,$$
$$k_F\delta r\gg1\ , \tag{31}$$

and, in 2D

$$(\delta G')^2 \sim \left[\frac{e^2}{h}\right]^2 \frac{1}{k_F l_{\rm tr}} . \tag{32}$$

We believe that formula (32) is the correct one to use in the weakly localized regime. The result is therefore not universal and can be considerably smaller than the universal value $(e^2/h)^2$.

In the experiments of Ref. 12, for instance, in the sample for which $l_{\rm tr}=4$ μm and $k_F^{-1}=65$ Å^{-1}, one obtains $1/(k_F l_{\rm tr}) \approx 1.6\times 10^{-3}$.

VI. CONCLUSION

We have examined the conductance fluctuations of a weakly disordered mesoscopic system in the general case of an arbitrary momentum-dependent scattering potential, and to first order in the electronic disorder. We have showed that these fluctuations remain universal, as in the case of the contact potential in Ref. 1. Thus we have generalized the demonstration of that universality independently of the specific form of the potential. The only condition required is that τ and $\tau_{\rm tr}$ are well defined.

However, we have also shown that conductance fluctuations induced by the motion of a single impurity are *not* the same universal ones in 2D if one changes the entire impurity distribution. They depend on the degree of disorder and the characteristics of the system via τ and $\tau_{\rm tr}$. Universality would hold only within the hypothesis of Ref. 3 of a contact potential, independent impurities, and in the limit of a strong disorder. Otherwise the universal value is multiplied by the factor $(k_F l_{\rm tr})^{-1}$, a quantity $<<1$ in the metallic regime. This factor increases with the disorder; as such it would extrapolate to 1 at the Anderson localization transition. However, the validity of that extrapolation is not obvious: outside the weakly localized regime (where the present calculations as well as those of Ref. 3 are made), it is difficult to presume whether $k_F l_{\rm tr}$ will still appear as it stands here or under a more complicated form.

The two results above can be understood physically: on a large scale conductance fluctuations are insensitive to the specific structure of the potential (at least to first order in the electronic disorder), since this potential is the same on each impurity. But when one moves a single impurity away, one induces an extra local perturbation which is certainly affected by the specific nature and range of the scattering potential.

Our second result implies that it is necessary to reexamine the conjecture of Ref. 3 concerning possible interpretation of $1/f$ noise at higher temperatures. According to our result, one would expect the noise to increase with disorder, going from the metallic regime toward the Anderson transition, where it would possibly approach the universal value $(e^2/h)^2$. This tendency has indeed been observed in Ref. 14, which represents an experimental attempt to study the noise as a function of the degree of disorder.

APPENDIX

1. Calculation of Eq. (23) in 3D

With (16), (24) reads

$$\Gamma' = \Gamma^2 (2\pi N(0)\tau)^2 Y , \tag{A1}$$

$$Y = \int d^3\mathbf{R}\, F(R) \left\{ \frac{\sin^2(k_F R)}{(k_F R)^2} - \frac{1}{2}\frac{\sin^2(k_F|\mathbf{R}+\delta\mathbf{r}|)}{(k_F|\mathbf{R}+\delta\mathbf{r}|)^2} \right\} - \tfrac{1}{2}\frac{\sin^2(k_F|\mathbf{R}-\delta\mathbf{r}|)}{(k_F|\mathbf{R}-\delta\mathbf{r}|)^2}$$

$$= Y_1 - \tfrac{1}{2}Y_2 - \tfrac{1}{2}Y_3 . \tag{A2}$$

As indicated in the text, Y_1 is more straightforwardly obtained from the first term in (24), and yields

$$Y_1 = \frac{1}{2\pi N(0)\tau n_I} . \tag{A3}$$

On the other hand, it is easy to verify that

$$Y_2 = Y_3 . \tag{A4}$$

We thus have to calculate Y_2:

$$Y_2 = \int d^3\mathbf{R}\, F(R) \frac{\sin^2(k_F|\mathbf{R}+\delta\mathbf{r}|)}{(k_F|\mathbf{R}+\delta\mathbf{r}|)^2}$$

$$= 2\pi \int R^2 dR\, F(R) \int_0^\pi \sin\theta \frac{\sin^2(k_F\sqrt{R^2+\delta r^2+2R\,\delta r\cos\theta})}{k_F^2(R^2+\delta r^2+2R\,\delta r\cos\theta)} . \tag{A5}$$

Changing variable by setting $y=\sqrt{R^2+\delta r^2+2R\,\delta r\cos\theta}$, we obtain

$$Y_2 = 2\pi \int R^2 dR\, F(R) \int_{|R-\delta r|}^{R+\delta r} \frac{y\,dy}{R\,\delta r}\frac{\sin^2(k_F y)}{k_F^2 y^2}$$

$$= \frac{1}{2\pi k_F^2 \delta r} \int V^2(Q) Q\, dQ [\, Y_2' - (Y_2'' - Y_2''')\,] , \tag{A6}$$

with

$$Y_2' = \int_0^\infty \sin(QR) dR \ln\frac{R+\delta r}{|R-\delta r|} = \frac{\pi}{Q}\sin(Q\delta r) , \tag{A7}$$

$$Y_2'' = \int_0^\infty \sin(QR) dR\, Ci[k_F(R+\delta r)] , \tag{A8}$$

$$Y_2''' = \int_0^\infty \sin(QR) dR\, Ci[k_F|R-\delta r|] , \tag{A9}$$

where $Ci(x)$ is the cosine integral.[9] A slightly lengthy but simple algebra yields

$$Y_2'' - Y_2''' = \frac{\pi \sin(Q\delta r)}{Q}[\theta(Q-k_F)+\tfrac{1}{2}\delta(Q-k_F)] , \tag{A10}$$

where $\delta(x)$ is the delta function[9] of x. Putting (A7) and (A10) back into (A6), we obtain

$$Y_2 = \frac{1}{2k_F^2\delta r}\left\{\int_0^{k_F} V^2(Q)\sin(Q\delta r)dQ - V^2(k_F)\sin(k_F\delta r)\right\} . \tag{A11}$$

$V^2(k_F)$ is a finite quantity, and $[\sin(k_F\delta r)/(k_F\delta r)]\to 0$, when $k_F\delta r \gg 1$.

On the other hand, the integral in (A11) is bounded, and the first term in (A11) also vanishes when $k_F\delta r \gg 1$. Thus we have proved that when $k_F\delta r \gg 1$, Y in (A2) reduces to its first term given in (A3), and thus one obtains formula (26) in the text. We wish to add a remark concerning the 1D case.

2. *Calculation of Eq. (24) in 1D*

This case is special due to the fact that, as noted after Eq. (17), $M_0(x)$ is not well defined when $x\to\infty$, while in 2D and 3D $M_0(x\to\infty)\to 0$. In 1D $M_0(x\to\infty)$ does not necessarily vanish. Then Y reads

$$\begin{aligned} Y &= \int_{-\infty}^{+\infty} dR\, F(R)\{\cos^2(k_F R)-\tfrac{1}{2}\cos^2[k_F(R+\delta r)] \\ &\qquad -\tfrac{1}{2}\cos^2[k_F(R-\delta r)]\} \\ &= Y_1 - Y_2 - Y_3 . \end{aligned} \tag{A12}$$

First, in contrast to the other two dimensionalities,

$$Y_2 \neq Y_3 . \tag{A13}$$

Furthermore, in the sum Y_2+Y_3, $\cos^2[k_F(R+\delta r)]+\cos^2[k_F(R-\delta r)]$ contains a constant term which cancels the one in $\cos^2(k_f R)$. Finally,

$$\begin{aligned} Y &= \int_{-\infty}^{+\infty} dR\, F(R)\cos(2k_F R)\sin^2(k_F\delta r) \\ &= \int_{-\infty}^{+\infty} dR\, F(R)\cos(2k_F R)[1-M_0^2(k_F\delta r)] . \end{aligned} \tag{A14}$$

Now let us consider a particular example. Suppose that the potential is

$$V(R) = \frac{V\varepsilon}{\pi}\frac{1}{\varepsilon^2+R^2} ; \tag{A15}$$

such a form allows us to recover the contact potential when $\varepsilon\to 0$, since $\lim_{\varepsilon\to 0}(V\varepsilon/\pi)[1/(\varepsilon^2+R^2)] = V\delta(R)$. The Fourier transform of (A15) is

$$V(Q) = \int_{-\infty}^{+\infty}\frac{V\varepsilon}{\pi}\frac{\varepsilon}{\varepsilon^2+R^2}e^{iQR}dR = Ve^{-\varepsilon Q} \tag{A16}$$

Then

$$F(R) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} V^2(Q)e^{iQR}dQ = \frac{2\varepsilon V^2}{\pi}\frac{1}{R^2+4\varepsilon^2} . \tag{A17}$$

Inserting (A17) into (A14) yields

$$\begin{aligned} Y &= [1-M_0^2(k_F\delta r)]\int_{-\infty}^{+\infty}\frac{2\varepsilon V^2}{\pi}\frac{\cos(2k_f R)}{R^2+4\varepsilon^2}dR \\ &= [1-M_0^2(k_F\delta r)]V^2 e^{-4k_F\varepsilon} , \end{aligned} \tag{A18}$$

with, we recall, $M_0^2(k_F\delta r)=\cos^2(k_F\delta r)$.

First we note that when $\varepsilon\to 0$, one recovers the contact potential result of FLS. But second, for $\varepsilon\neq 0$, Y is not proportional to $1/\tau(=2N(0)n_I[V^2(Q=0)+V^2(Q=2k_F)])$, but $(1/\tau_{\rm tr})=2\pi N(0)n_I V^2(Q=2k_F)$. If we discard (Y_2+Y_3) in (A12), Y_1 would indeed be proportional to $1/\tau$. The difference again comes from the fact that $M_0^2(x\to\infty)$ is not well defined in 1D so that Y_2+Y_3 do not vanish when $(k_F\delta r)\to\infty$. In the 1D case the equivalent of (27b) would be

$$(\delta G')_{\rm 1D}^2 \sim \left[\frac{e^2}{h}\right]^2\frac{L^2}{n_I l_{\rm tr}^2}\sin^2(k_F\delta r) , \tag{A19}$$

with no well-defined limit when $k_F\delta r \gg 1$. $(\delta G')_{\rm 1D}$ is a periodic function of $(k_F\delta r)$ and vanishes whenever $(k_F\delta r)=n\pi(n=0,1,2,\ldots)$. We found the same dependence in $\sin^2(k_F\delta r)$ in the interacting impurity problem of Ref. 2 [see formula (28) in Ref. 2].

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