Technique for measuring the Cooper-pair velocity, density, and mass using Doppler splitting of the plasmon resonance in low-dimensional superconductor microstructures

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The creation of a Cooper-pair mass spectroscopy is suggested. The plasmons in low-dimensional superconductor structures (layers or wires in the dielectric background) are theoretically considered for this purpose. The Cooper-pair mass m^* (the parameter m^* in London electrodynamics) can be determined by measurements of the Doppler shift of the plasmon frequency when a direct current is applied through the superconductor. The plasmons with frequency ω lower than the superconducting gap 2Δ can be detected by the same far-infrared absorption technique and grating coupling used previously for investigation of two-dimensional plasmons in semiconductor microstructures and polaritons in condensed-matter physics.

I. INTRODUCTION

The effective mass of quasiparticles related to the dynamic equation of quasimomentum is one of the most important basic ideas in condensed-matter physics. In the physics of superconductivity the mass of the "superconducting electron," m^* (i.e., the effective Cooper-pair mass), was introduced by London and London[1] as early as 1935. In the present renaissance in the physics of superconductivity the knowledge of the Cooper-pair mass can give important information on the mechanism of high-T_c superconductivity. However, the Cooper-pair mass has not recently been investigated.

What is the value of m^* for InO_x or $YBa_2Cu_3O_{7-\delta}$ [Y(1:2:3)], and is this generally a reasonable question? The aim of the present paper is to arouse interest in m^* by suggesting an experimental method for the measurement of the volume density of the Cooper pairs n and as a consequence of m^*. Let us mention that the density n participating in the current response is related to the so-called superfluid density, not to the superconducting condensate density.

"We are unable to determine whether the 'velocity of the electricity' in the wire is great or small.[2]" Really, at the known volume current density $j = e^* n v_{\rm dr}$ the determination of n is possible if we know the drift velocity $v_{\rm dr}$. But how can one measure the velocity of a fluid of identical quantum particles? Can several Cooper pairs be identified to trace their motion? Such an identification can be performed by some quantum excitation of the coherent superconducting state. We suppose that a plasmon in thin low-dimensional superconductors can be used as this identifying mark. Such a drift-velocity meter has been realized for the two-dimensional (2D) electron gas with high mobility in $GaAs/Al_xGa_{1-x}As$ heterostructures.[3]

II. METHOD

Our starting point is the well-known dispersion equation of the 2D plasmon (for an introduction see, for example, the review Ref. 4)

$$\omega_{\rm pl}^2 = 2\pi n^{\rm 2D} e^{*2} q/\varepsilon m^*, \tag{1}$$

where $q = 2\pi/a_{\rm gr}$ is the 2D wave vector of the plasmon fixed by the grating constant $a_{\rm gr}$ (see Fig. 1), d is the thickness of the superconducting layer, $n^{\rm 2D} = dn$ is the number of Cooper pairs per unit area and ε is the dielectric constant of the bulk insulator around the layer. This formula applies if $T \ll T_{\rm KT}, T_c$, $\hbar\omega \ll 2\Delta(0)$, $2\pi/q \gg \xi(0)$, $\omega/q \ll c/\eta$, and $d \ll \lambda$, where T is the temperature, $T_{\rm KT}$ is the Kosterlitz-Thouless (KT) transition temperature, c is the light velocity, $\xi(0)$ is the Ginzburg-Landau (GL) coherence length for $T = 0$, and $\eta = \sqrt{\varepsilon}$. The plasmon dispersion relation is a simple consequence of the London electrodynamics applicable to low frequencies $\omega \ll 2\Delta$. Plasmons for superconducting systems were predicted for thin filaments,[5,6] thin layers and Josephson arrays,[7,8] and even for an arbitrary dimension,[9] but since then no experimental investigations have been made to our knowledge. The plasmon dispersion relation can also be expressed by the experimentally measurable kinetic inductance[10]

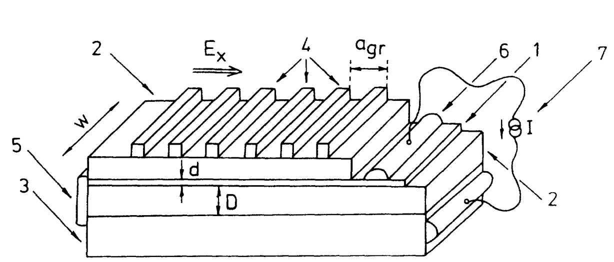

FIG. 1. Gedanken microstructure for the measurement of the Cooper-pair mass m^* (schematic, not to scale). 1, thin superconducting layer, 2, dielectric medium, 3, bulk superconductor, 4, grating strips, 5,6, source-drain electrodes, and 7, current generator. The electric field E_x of the external FIR electromagnetic field is polarized perpendicularly to the grating. The weak magnetic field B of the drift current is parallel to the grating.

$$\mathcal{L} = 4\pi\lambda^2/d = m^* c^2/e^{*2} n^{2\mathrm{D}} \quad \text{for} \quad d \ll \lambda,$$
$$\omega_{\mathrm{pl}}^2 = 2\pi c^2 q/\mathcal{L}\varepsilon. \tag{2}$$

Let us mention that in the Gaussian system used $\mathcal{L}$ has the dimension of length (1 nH = 1 cm).

If the thin superconducting layer is spaced at distance D from a bulk superconductor, as is shown in Fig. 1, the plasmon dispersion becomes acoustic,[4]

$$\omega_{\mathrm{pl}} = v_{\mathrm{ac}} q, \quad v_{\mathrm{ac}} = c(4\pi D/\varepsilon\mathcal{L})^{1/2} = (c/\eta\lambda)(Dd)^{1/2}, \tag{3}$$

for $\omega < 2\Delta$, $v_{\mathrm{ac}} \ll c/\eta$, and $D \ll a_{\mathrm{gr}}$. The plasma frequency is in this case (3) $(D/a_{\mathrm{gr}})^{1/2}$ times lower than in the $D \gg a_{\mathrm{gr}}$ case (1),(2). The bulk superconductor also causes a homogeneous distribution of the surface current in the plane of the superconducting layer. The return of the current back through the bulk superconductor, as shown in Fig. 1, is necessary for homogenization of the 2D surface current $j^{2\mathrm{D}}$ in the plane of the layer. The weak magnetic field parallel to the grating, $B = 4\pi j^{2\mathrm{D}}/c$, in the insulator layer between the superconductors can be closed by a U-shaped insulated superconducting covering of the bottom of the microstructure. In this case, the 2D current will be extremely homogeneous.

The polaritons considered are in the meV region (for high-T_c superconductors) and can be investigated by the same far-infrared (FIR) technique used previously for investigations of the 2D plasmons in the 2D electron gas in semiconductors. However, the plasma resonances will be extremely sharp due to the absence of Ohmic dissipation for $k_B T, \hbar\omega \ll 2\Delta(0)$ when there are no thermally excited normal carriers.

The dispersion relations (1)–(3) are written in the coordinate system in which the Cooper pairs are unmoving. In the case of an applied surface constant direct current (dc) $j_{\mathrm{dr}}^{2\mathrm{D}} = dj = e^* n^{2\mathrm{D}} v_{\mathrm{dr}}$ perpendicular to the grating, the drift velocity causes a Doppler shift $\Delta\omega = v_{\mathrm{dr}} q$ of the frequency ω of the plasma resonance. As a consequence, the Doppler shift of the plasmons running in the opposite directions $q = \pm 2\pi/a_{\mathrm{gr}}$ gives a doublet splitting[3] of the plasmon resonance with the frequency difference $2\Delta\omega = 4\pi v_{\mathrm{dr}}/a_{\mathrm{gr}}$; for acoustic plasmons $\Delta\omega/\omega_{\mathrm{ac}} = v_{\mathrm{dr}}/v_{\mathrm{ac}}$. The predicted effect is unfortunately very small and can be observed only in high-technology superconducting microstructures unthinkable in London's time. The relative change of the frequency of the acoustic plasmon (3), for example, is small even for drift velocities v_{dr} comparable with the critical depairing velocity $v_c \cong \hbar/m^*\xi(0)\pi$ which creates the decay of Cooper pairs,

$$\Delta\omega/\omega_{\mathrm{pl}} = v_c/v_{\mathrm{ac}} \cong \eta(m_e/m^*)\,[\delta/\xi(0)]\,[\lambda/(Dd)^{1/2}]$$
$$\gg 1/N, \quad \mathrm{Im}\,\varepsilon/\mathrm{Re}\,\varepsilon. \tag{4}$$

Here m_e and $\delta = \hbar/m_e c$ are the mass and the Compton length of a free electron. This inequality gives the same restriction $\delta/\xi(0) \simeq 10^{-4}$ for the homogeneity of the layer thickness, surface current, dielectric losses, experimental resolution, and also for the minimal number of superconducting grating strips N.

The Cooper-pair density is unchanged by the drift when the current density is much smaller than the GL theoretical depairing critical current[11] $j_c = H_c/4\pi\lambda$ (H_c is the thermodynamic critical field). The measurement of the Doppler splitting $2\Delta\omega$ gives the drift velocity $v_{\mathrm{dr}} = a_{\mathrm{dr}}\Delta\omega/2\pi$. In the absence of screening by a bulk superconductor, the distribution of the 2D current density is homogeneous if the width w of the superconducting strip is much less than the 2D screening length $\mathcal{L}$. This $a_{\mathrm{gr}} \ll w \ll 4\pi\lambda^2/d$ condition can be reached only for extremely thin layers. Under this condition the 2D current density $j^{2\mathrm{D}} = I/w$ can be expressed by the experimentally measured current I through the "source-drain" electrodes of the microstructure shown in Fig. 1. For a simultaneously measured current density $j = j^{2\mathrm{D}}/d$ we can determine the volume density of the Cooper pairs, $n = j/e^* v_{\mathrm{dr}}$. The Cooper-pair mass can be expressed then by $m^* = 4\pi n e^{*2}\lambda^2/c^2$. However, a more natural way is (1) determination of $n^{2\mathrm{D}} = j^{2\mathrm{D}}/e^* v_{\mathrm{dr}}$, (2) expression of $\mathcal{L}$ by the plasmon dispersion relation [Eq. (3)], for example, $\mathcal{L} = 4\pi D q^2/\varepsilon\omega_{\mathrm{ac}}^2$, and (3) finally determination of $m^* = e^{*2}\mathcal{L} n^{2\mathrm{D}}/c^2$. The 2D plasmons can be used in principle for the determination of the penetration depth too.

If the grating period is much bigger than the superconducting strip width $a_{\mathrm{gr}} \gg w$ the plasmon becomes one dimensional (1D) (cf. Ref. 12),

$$\omega_{\mathrm{pl}} = (c/\lambda\eta)(A/2\pi)^{1/2}\,[\ln(1/qw)]^{1/2}\,q, \tag{5}$$

where A ($= wd$ for the considered microstructure) is the cross-sectional area of the 1D superconductor. Further lowering of the plasmon frequency can be achieved if a thin superconducting wire is separated from a bulk superconductor by a thin insulator layer with thickness $D \ll a_{\mathrm{gr}}$. In this case, the logarithm must be replaced by a constant of order unity, and the 1D plasmon dispersion is purely acoustic.

The Doppler shift of the plasmons in low-dimensional superconductors is considered as an important ingredient of the supposed solid state two-stream instability.[13] An appropriate layered superconducting microstructure can be prepared by contemporary technology for growing thin high-T_c layers[10] and even superlattices[14] with $d = 12$ Å. The development of Cooper-pair mass spectroscopy can be expected in the near future. On this account we will describe the renormalization of m^* by disorder for conventional dirty alloys.

III. THE PIPPARD–LANDAU THEORY

Let us write the kinetic energy of the Cooper pairs in the spirit of the GL theory[15]

$$E_{\rm kin} = \int\int d^3x\, d^3y\, \{[-i\hbar\nabla_x - e^*\mathbf{A}(\mathbf{x})]\}^* \cdot [\mathbf{K}(\mathbf{x},\mathbf{y})/2m^*_{\rm pure}] \cdot [-i\hbar\nabla_y - e^*\mathbf{A}(\mathbf{y})]\,\psi(\mathbf{y}), \tag{6}$$

where: $\mathbf{x}$ and $\mathbf{y}$ are displacement vectors, $\mathbf{A}$ is the vector potential, $m^*_{\rm pure}$ is the Cooper-pair mass for the clean material, and ψ is the GL superconducting wave function.[15] The comparison of the current density given in this theory by the variational derivative

$$\mathbf{j}(\mathbf{x})/c = -\delta E_{\rm kin}/\delta\mathbf{A}(\mathbf{x}) \tag{7}$$

with the well-known formula of Pippard nonlocal electrodynamics[16] fixes the kernel

$$\mathbf{K}(\mathbf{x},\mathbf{y}) = (3/4\pi\xi_0)(\mathbf{R}\otimes\mathbf{R}/R^4)\exp(-R/\xi_p),$$
$$1/\xi_p = 1/\xi_0 + 1/l, \quad R = |\mathbf{R}|, \quad \mathbf{R} = \mathbf{y} - \mathbf{x}, \tag{8}$$

where ξ_0 is the Pippard coherence length for the pure metal, within several percent accuracy $\xi_0 = \xi(0)$ for pure isotropic metals. For gauge transformations $\psi \leftarrow \exp[i\chi(\mathbf{x})]\psi$ the kernel receives phase multipliers

$$\mathbf{K}(\mathbf{x},\mathbf{y}) \leftarrow \exp[i\chi(\mathbf{x})]\mathbf{K}(\mathbf{x},\mathbf{y})\exp[-i\chi(\mathbf{y})]. \tag{9}$$

Probably the most direct confirmation of this unification of the GL and Pippard theories comes from the experiment[17] on the fluctuation diamagnetic moment $M_{\rm fl}$ in an external magnetic field H just at the critical temperature T_c. The corresponding state law for $M_{\rm fl}(T_c, H)$ is a reflection of the existence of a kernel $\mathbf{K}$ general for all isotropic conventional superconductors.

For smooth space changes of the wave function ψ, the local GL approximation of the kernel by the Dirac δ function $\mathbf{K}(\mathbf{R}) = (\xi_p/\xi_0)\delta(\mathbf{R})\mathbb{1}_{3\times3}$ gives the renormalization of the Cooper-pair mass by the disorder (cf. Ref. 18)

$$m^*_{\rm dirty} = (1 + \xi_0/l)m^*_{\rm pure}. \tag{10}$$

In Ref. 18, "mass" is indicated by quotation marks. The BCS theory gives temperature-dependent multipliers of the order of unity which are omitted in this formula. The main properties of the post-BCS dirty-alloy theory, such as increase of the upper critical magnetic field H_{c2}, the GL parameter κ, and Ginzburg's number $\mathcal{G}$, with the disorder $1/l$ can be well explained in the framework of the presented nonlocal phenomenological theory.

Qualitatively, the increase of $m^* \propto \rho$ proportionally to the electric resistivity continues even in the strong scattering limit. For 100 Å of thin disordered InO_x film[19] the KT transition temperature $T_{\rm KT}$ is considerably below the GL one T_c, i.e., $(T_c - T_{\rm KT}) \simeq T_c$. Then, the number of Cooper pairs per unit area at $T_{\rm KT}$ has the same order as the surface density n_e of the normal electrons $n(T_{\rm KT}) \cong (1 - T_{\rm KT}/T_c)(n_e/2) \simeq n_e$.

On the one hand, it is well known that electrons of every superconductor are highly degenerate, $\hbar^2 n_e/2m_e \ll k_B T_c$, but on the other hand the equation for $T_{\rm KT}$ (Ref. 19)

$$\hbar^2 n(T_{\rm KT})/2m^* = k_B T_{\rm KT}/\pi \tag{11}$$

shows that after superconducting pairing the Cooper pairs are almost a classical gas. It seems that the Cooper-pair mass $m^* \gg m_e$ reaches typical hadronic values for these strongly disordered films. Another question waiting for an experimental solution is the following. Are the heavy-fermion superconductors also heavy-boson materials (for the electrodynamics of correlated metals see, for example, Ref. 20 and references therein)? We hope that understanding of the superconductivity begins with the London electrodynamics. Following a "harmless survey"[21] we come to the conclusion that the (electromagnetic current response) Cooper-pair mass (let us call it the London mass) is a typical key problem remaining unresolved experimentally.

IV. CHOICE OF PARAMETERS

Let us now consider whether such mass spectroscopy is possible for $Y(1{:}2{:}3)/PrBa_2Cu_3O_{7-\delta}$ (Ref. 14) or $Y(1{:}2{:}3)/Nd_{1.83}Ce_{0.17}CuO_x$ (Ref. 22) layered microstructures of high-T_c superconductors, which are in the limelight of the physics of superconductivity. Let us take for an estimate $j_{\rm dr} = 30$ MA/cm^2, $j_c(4.2\text{ K}) = 70$ MA/cm^2 (Ref. 22), $n = 2\times10^{21}$ cm^{-3}, $d = D = 24$ Å, $T_c = 70$ K, $\Delta T = 0.1$ K, $\lambda = 2000$ Å, $\xi_0 = 17$ Å, $T = 4$ K, $\varepsilon = 4.8$, $\hbar\omega_{\rm ac} = 10$ meV, $f_{\rm ac} = \omega_{\rm ac}/2\pi = 2.41$ THz, and $Na_{\rm gr} = 2$ mm. Under these conditions we obtain $j_{\rm dr}/j_c = 0.4$, $v_{\rm dr} = 940$ m/s, $v_c = 2$ km/s, $v_{\rm Fermi} = 130$ km/s, $v_{\rm ac} = 1.64$ Mm/s, $c = 300$ Mm/s, $v_{\rm ac}/v_{\rm dr} = 1.7\times10^3$, $2\Delta(0) = 3.5\,k_BT_c = 21$ meV, $a_{\rm gr} = f_{\rm ac}/v_{\rm ac} = 680$ nm, $1/a_{\rm gr} = 15\times10^3$ cm^{-1}, $N = 3\times10^3$, $2/N = 0.7\times10^{-3}$, $Nv_{\rm dr}/v_{\rm ac} = 1.7$, $\Delta T_c/T = 1.4\times10^{-3}$, and $2\Delta\omega/\omega_{\rm ac} = 3.4\times10^{-3} = 2.4\,\Delta T_c/T$.

The main problem, as a practical consideration, is that the most homogeneous films ever produced have a transition width ΔT_c of about 0.1 K, which is limited by uniformity and not thermodynamic fluctuations. It is believed that this is caused by stoichiometric fluctuations, mainly in the oxygen, and is the origin of the ever-present flux-pinning phenomenon. Thus even in the best cases the fractional width of the plasmon resonance smeared by the disorder, $\Delta T_c/T_c$, would be 10^{-3}. In our example, the fractional Doppler splitting $2\Delta\omega/\omega_{\rm ac}$ exceeds the natural fractional linewidth $\Delta T_c/T_c$ of the plasmon resonance by a large enough factor of 2.4 (cf. Ref. 3) for the transport current $j_{\rm dr}$ to be considerably smaller than the critical current j_c. Critical currents observed experimentally for similar layered microstructures[22,23] are of the order of the GL theoretical j_c. Consequently, the intrinsic difficulty connected with the disorder is likely to be surmounted simply by possible technical improvements in sample preparation.

The homogeneous electric field $E(t) = E_0\cos\omega t$ of a far-infrared laser is weakly modulated by a dielectric grating and the layer experiences the electric field $E(t) = E_0(1 + \beta\cos qx)\cos\omega t$ where the grating coupling constant is extremely small, $\beta \ll v_{\rm dr}/v_{\rm ac} \ll 1$. Far from the plasmon resonance the homogeneous electric field creates a homogeneous 2D current $j_\omega^{2D} \simeq (cE_0/4\pi)\cos\omega t$ in the thin superconducting layer and the reflection coefficient $R = 1$. Just at the plasmon resonance the coherent electromagnetic (e.m.) field creates giant current oscillations $j_{\omega q}^{2D} \simeq (cE_0/4\pi\beta)\cos\omega t\cos qx$ but again $R = 1$. These current and Cooper-pair density oscillations can be detected by the recently suggested acoustic phonon emission of plasmons[24] or, as is convenient in the physics of polaritons, by the nonlocality of the reflection. We can illuminate only half of the grating microstructure and observe the e.m. emission from the unilluminated region. The intensity of the phonon or e.m. emission as a function of the frequency ω has the split resonance behavior[3] shown in Fig. 2. The main technical problem is the resolution of this split plasmon resonance. If a FIR laser with fixed frequency $\approx \omega_{\rm ac}$ is used, we can tune the plasmon resonances by small changes of the dc current-$j_{\rm dr}^{2D}$ and current dependent superfluid density $n(j_{\rm dr}^{2D})$.

Recently, a measurement of the effective mass of the Cooper pair was performed by Fiory *et al.*[25] Their mass spectroscopy is based on an electrostatic charge modulation of the kinetic inductance of epitaxial Y(1:2:3) films. Hydrodynamic relations in superconductivity, such as the Bernoulli principle and the London Hall effect give other possibilities for creation of Cooper-pair mass spectroscopy.[26]

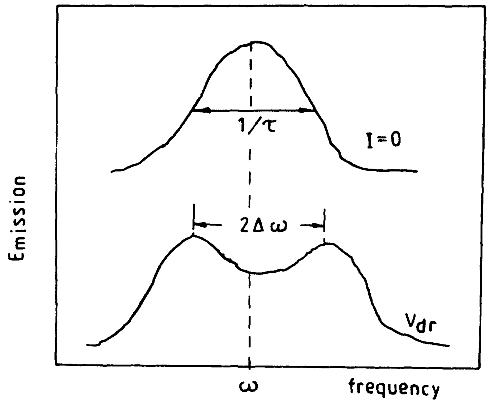

FIG. 2. Expected phonon or FIR emission spectrum from the shaded region of the grating. The upper curve is the plasmon resonance emission for zero source-drain current I. The lower curve presents (after Ref. 3) the Doppler doublet splitting for large enough drift velocities $v_{\rm dr}$. The Raman spectrum for the backscattering perpendicular to the layer will be qualitatively the same. The depth between maxima depends on the dimensionless parameter $Nv_{\rm dr}/v_{\rm ac} > 1$.

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